Numerical Study of Depressurization Rate during Blowdown Based on Lumped Model Analysis

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To study the thermodynamic aspects of blowdown, the depressurization rate equation has been numerically solved. The equation, derived from macroscopic mass and energy balances in the pressure vessel, consisted of the energy and volumetric discharges terms multiplied by the decrease rate of residual coolant. By applying a dimensional analysis, dimensionless equations were obtained together with dimensionless parameters of blowdown. Blowdown calculations starting at typical BWR operating conditions indicated that the decrease rate of coolant increased for the liquid and two-phase mixture, and decreased for the vapor discharge. Further, the energy discharge term made a larger contribution to the depressurization rate in the case of vapor escape, while the volumetric discharge term did so in the case of liquid and two-phase mixture escape blowdowns. In the lumped model analyses, the averaged specific enthalpy and entropy of the residual coolant increased for the liquid discharge, remained almost constant for the two-phase mixture discharge, and decreased for the vapor discharge blowdown.

**KEYWORDS:** blowdown, depressurization rate, macroscopic balance equation, Pi term, numerical analysis, BWR type reactors, thermodynamic properties, energy balance, reactor operation

I. INTRODUCTION

Simulation tests of the Loss of Coolant Accident (LOCA) for Light Water Reactors (LWR) have been made at several facilities and large amounts of blowdown data have been accumulated. Although a number of papers have been published on the blowdown test results, it is difficult to understand fully the thermodynamic aspects of blowdown processes. For instance, none give a lucid explanation of the difference in pressure changes between liquid and vapor discharges blowdowns.

Harris appears to have shown first the characteristics of pressure and coolant mass changes during blowdowns for different fluid expansions assumed in the pressure vessel. Based on lumped mass and heat balance equations, he obtained numerical results for pressure and coolant mass changes for complete disengaging (no mixing of vapor and liquid in the pressure vessel) or no disengaging (complete mixing). Following Nahavandi's study, a theoretical vessel blowdown analysis based on the lumped model was completed by Moody who provided graphed solutions for typical saturated water system conditions. He considered three ideal discharge blowdowns, that is, liquid phase coolant discharge, vapor phase coolant discharge, and two-phase mixture discharge with complete mixing in the pressure vessel. Although the results were useful to understand subsequent blowdown tests, the mechanism to yield different pressure and coolant mass changes depending on the discharging coolant state was not clarified.

With the development of numerical analysis tools, interest in the succeeding
studies has turned towards improving the analytical models employed in the codes. Through the comparisons with ROSA-I test data, Sobajima(14) improved the discharge and mixing models in the RELAP-3 code. Similarly, Leach et al.(15) and Naff & Schwartz(16) modified the assumption of instantaneous mixing of vapor and liquid in the RELAP-4 code by investigating LOFT and Semiscale test data. Verification studies of RELAP-4 were also performed by several other groups(17)~(20). The LOFT test data were used for the verification studies of TRAC-PI by Pyun & Williams(21). Although these verification studies have proved the codes to be powerful tools and have demonstrated the safety of commercial power plants against LOCA's, they have contributed little towards advancing the physical understanding of blowdown phenomena.

One way to advance our understanding of blowdown can be based on straightforward analysis, for instance as in the lumped model(6)~(8) which is derived from macroscopic balance equations for which physical meanings are readily apparent. Hence, numerical analyses have been done on the liquid, two-phase mixture, and vapor discharges blowdowns based on the macroscopic energy and mass balances equations to examine the principal term for the three different discharge blowdowns. In order to generalize the discussion, the depressurization rate equation together with the mass balance equation was changed into a dimensionless form by application of dimensional analysis. This application also made it possible to obtain dimensionless parameters of blowdown processes which are necessary to compare the experimental data between differently scaled test facilities.

II. THEORETICAL

1. Derivation Synopsis for Depressurization Rate Equation

The lumped model description of the depressurization rate is obtained by application of the macroscopic energy and mass balance equations to the control volumes of the pressure vessel and the break nozzle as shown in Fig. 1. References (8) and (22)~(25) have derived the depressurization rate equation of the control volume "1" (pressure vessel). Since the stagnation enthalpy of the flowing out fluid was assumed to be equal to the averaged stagnation enthalpy of the fluid in "1", \( h_0 \) in Refs. (22) and (24), the term associated with the change of the internal energy per unit mass of coolant, was ignored in the depressurization rate equation. In the present work for which one purpose is clarification of the differences between liquid and vapor discharges blowdowns, the depressurization rate equation derived in Refs. (8), (23) and (25) is used. (The equations in those references are identical except for the misprint in sign of Ref. (25).)

The static pressure in control volume "1" can be functionally expressed with the internal energy per unit mass of coolant \( U \) and the specific volume \( v \) by

\[
p = p(U, v).
\]  
(1)

The depressurization rate is then given by
If the kinetic and potential energies are ignored, the total energy $E$ of system "1", satisfies the relation, $E=U=M \dot{U}$ ($M$: coolant mass) and with $v=V/M$ ($V$: vessel volume), the following equations are obtained:

\[
\frac{d \dot{U}}{dt} = \frac{d}{dt} \left( \frac{E}{M} \right) = \frac{1}{M} \left( \frac{dE}{dt} - \dot{U} \frac{dM}{dt} \right),
\]

\[
\frac{dv}{dt} = \frac{d}{dt} \left( \frac{V}{M} \right) = -\frac{v}{M} \frac{dM}{dt}.
\]

The $d\dot{U}/dt$ is given by two terms: the first originated in the change in total energy of the fluid in the control volume "1"; and the second originated in the change in coolant mass of the fluid in "1". The second term indicates that a decrease in $M$ increases $\dot{U}$, so essentially the term redistributes energy over the coolant remaining in control volume "1". Since the vessel volume is constant, $dv/dt$ is given only by the term due to the change in coolant mass. The storage rate terms $dE/dt$ and $dM/dt$ in Eqs. (3) and (4) are replaced by the outflow rate terms in the macroscopic balance equations of energy and mass, and then the equations are inserted into Eq. (2) to yield the following depressurization rate equation:

\[
\frac{dp}{dt} = \frac{A_B G_C}{M} \left( -(h_{s(out)} - U) \left( \frac{\partial p}{\partial \dot{U}} \right)_v + v \left( \frac{\partial p}{\partial v} \right)_\theta \right),
\]

where no heat generation is assumed in control volume "1". The $A_B$ and $G_C$ in Eq. (5) denote the break area and the critical mass flux, respectively. The mass balance equation in "1" is given by

\[
\frac{dM}{dt} = -A_B G_C.
\]

The term $A_B G_C/M$ in Eq. (5) expresses the ratio between the discharge mass flow rate and the residual mass in control volume "1", and is considered to be the decrease rate of coolant in "1". The first term in the braces expresses the pressure change due to the net discharge of energy (energy discharge term) and the second, the pressure change due to the volumetric discharge of coolant (volumetric discharge term).

The quantities appearing in Eq. (5) are evaluated assuming a saturated state of the two-phase mixture in the vessel. The static quality, $x_s$, is evaluated by the following two equation:

\[
v = M_i \frac{v_i}{v_i},
\]

\[
x_s = \frac{v-v_f(p)}{v_f(p)}.
\]

Equation (7) is obtained from $v \cdot M = V$ (constant), and the subscript "i" in $v_i$ and $M_i$ denotes the value at the initial state. The relations used in the evaluation of other quantities in Eqs. (5) and (6) are summarized in APPENDIX.

2. Application of Dimensional Analysis

Equations (5) and (6) are rearranged as follows with $M=V/v$ where $V$ denotes the vessel volume
The dimensional analysis is applied to Eqs. (9) and (10). The secondary quantities which appear are \( p, t, A_B, V, v \) and \( G_c \). (The \( h_{0(out)} \) and \( \dot{U} \) in Eq. (9) are not considered because their dimensions are cancelled by that of \( \langle \partial p/\partial U \rangle_v \).) Since the three primary quantities, that is, length \( L \), mass \( M \) and time \( T \) constitute the secondary ones, the Pi theorem indicates that there are three independent dimensionless groups. The dimensional matrix, \( M \) is given by

\[
M = \begin{bmatrix}
-1 & 0 & 3 & 3 & -2 & L \\
1 & 0 & 0 & 0 & -1 & 1 & M \\
-2 & 1 & 0 & 0 & 0 & -1 & T \\
\end{bmatrix}
\]

The rank of the dimensional matrix \( M \) is confirmed to be three by taking the determinant of the matrix consisting of the \( v, G_c \) and \( t \) columns of Eq. (11). The selected groupings of the secondary quantities are the \( v, G_c, t \) plus one of the remaining quantities \( p, A_B \) or \( V \), that is,

\[
(p, v, G_c, t), \ (A_B, v, G_c, t) \text{ and } (V, v, G_c, t).
\]

Application of the Pi method yields the following dimensionless parameters:

\[
\pi_1 = \frac{p}{vG_c^2}, \quad \pi_2 = \frac{A_B}{v^3G_c^2}, \quad \pi_3 = \frac{V}{v^3G_c^2}.
\]

Since \( A_B \) and \( V \) always appear in the form of \( A_B/V \) in Eqs. (9) and (10), \( \pi_2 \) and \( \pi_3 \) are lumped together becoming

\[
\pi_5 = \frac{\pi_2 \pi_3}{\pi_1} = \frac{A_BvG_c^2}{V}.
\]

To express Eqs. (9) and (10) with the above two Pi terms, the following operations are performed. From Eq. (9) multiplied by \( t/p \)

\[
\frac{t}{p} \cdot \frac{dp}{dt} + \pi_5 \frac{h_{0(out)}}{\dot{U}} = \frac{\dot{U}}{p} \frac{\partial p}{\partial \dot{U}} - \pi_5 v \frac{\partial p}{\partial v} = 0.
\]

From Eq. (10) multiplied by \( t/v \)

\[
\frac{t}{v} \cdot \frac{dv}{dt} - \pi_5 = 0.
\]

The term \( \pi_1 \) does not appear in Eqs. (14) and (15). Although the dimensionless Eqs. (14) and (15) are obtained, \( \pi_5 \) changes with time due to the dependence of \( v \) and \( G_c \) on \( p \) and further, the physical meaning of \( \pi_5 \) is not explicit. Hence, variables in Eqs. (14) and (15) are replaced by the reduced variables with reference values

\[
\frac{p}{p_0} = p', \quad \frac{t}{t_0} = t', \quad \frac{G_c}{G_{c0}} = G', \quad \frac{\dot{U}}{\dot{U}_0} = \dot{U}', \quad \frac{h_{0(out)}}{h_0'} = h_{0(out)}', \quad \frac{v}{v_0} = v'.
\]

As the reference, the initial values may be adopted. Using the new variables defined above, Eqs. (14) and (15) become

\[
\frac{dp'}{dt'} + \frac{A_B}{V} (vG_c \left( h_{0(out)} - \dot{U} \right) - v (\frac{\partial p'}{\partial \dot{U}})) = 0,
\]

\[
\frac{dv'}{dt'} - \frac{A_B}{V} v \dot{G}_c = 0.
\]
where

\begin{align*}
\pi^2_i &= \frac{A_B G_{co} t_0}{V} = \frac{A_B G_{co} t_0}{M_0}.
\end{align*}

The \( \pi^2_i \) is the ratio of the discharging mass flow rate in the time unit of \( t_0, A_B G_{co} t_0 \) to the coolant mass in the vessel \( M_0 \) and is considered to be the fractional decrease rate of coolant. Further, if the following variable is adopted:

\begin{align*}
t'' &= \pi^2_i t' = \frac{A_B G_{co} v_0 t}{V},
\end{align*}

then, Eqs. (17) and (18) are finally rearranged as follows:

\begin{align*}
\frac{d p'}{dt''} + f(p') &= 0, \\
\frac{d v'}{dt''} + g(p') &= 0,
\end{align*}

where

\begin{align*}
f(p') &= G_i' v' \left( (h_{i(out)}') - U' \left( \frac{\partial p'}{\partial U'} \right)_v - v' \left( \frac{\partial p'}{\partial v'} \right)_U \right), \\
g(p') &= -G_i' v'.
\end{align*}

The functions \( f(p') \) and \( g(p') \) change with time \( t'' \), since pressure \( p' \) changes with \( t'' \). Therefore, Eqs. (21) and (22) indicate that similarities of pressure and coolant mass changes exist between the blowdowns for which \( f(p') \) and \( g(p') \) are identical to each other and from Eq. (20) the dimensionless parameter \( \pi^2_i \) determines the change rates of pressure and mass with respect to \( t' \). The \( \pi^2_i \) becomes \( A_B / V \) between blowdowns where the same fluid is used since \( G_{co}, v_0 \) and \( t_0 \) are equal to each other.

If heat generation exists in the pressure vessel, Eq. (5) becomes

\begin{align*}
\frac{d p}{dt} - \frac{\Sigma q_{in}}{V} v \left( \frac{\partial p}{\partial U} \right)_v + \frac{A_B}{V} v G_c (h_{i(out)}') - \left( \frac{\partial p}{\partial U} \right)_v = 0.
\end{align*}

In the same way as the case for no heat generation, the following equations are finally obtained:

\begin{align*}
\frac{d p'}{dt''} - \pi^2_i (\Sigma q_{in})' v' \left( \frac{\partial p'}{\partial U'} \right)_v + \pi^2_i G_i' v' \left( (h_{i(out)}') - U' \left( \frac{\partial p'}{\partial U'} \right)_v - v' \left( \frac{\partial p'}{\partial v'} \right)_U \right) &= 0, \\
\pi^2_3 &= \frac{(\Sigma q_{in}) t_0 \theta_0}{M_0 U_0} = \frac{(\Sigma q_{in})_0 t_0 \theta_0}{V U_0^3}.
\end{align*}

Equation (27) indicates that the dimensionless parameter \( \pi^2_3 \) expresses the fractional increase rate of energy due to heat generation. The final equations discussed are obtained by the introduction of the variable \( t'' \) into Eq. (26) as,

\begin{align*}
\frac{d p'}{dt''} - \pi^2 (\Sigma q_{in})' h(p') + f(p') &= 0,
\end{align*}

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The dimensionless parameter \( p_0 \) denotes the ratio between the fractional change rate of energy and that of mass which becomes \( \Sigma q_{in}/A_B \) for the blowdowns with the same fluid as derived in our former study\(^{(27)}\).

### III. RESULTS AND DISCUSSION

To simplify the discussions, no heat generation is once again assumed, that is, Eqs. (21) and (22) are used in the numerical analyses. The Homogeneous Equilibrium Model (HEM)\(^{(28)}\) is used in the calculation of \( G_c \). The calculational flow is given in Fig. 2. The calculation starts at 6.9 MPa (almost the BWR rated pressure) and the break area over the vessel volume is \( 10^{-4} \text{m}^{-1} \) (corresponding to a 25\% break of the recirculation line of a BWR-5)\(^{(29)}\).

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**Fig. 2** Calculational flow of depressurization rate during blowdown

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The pressure vessel is initially filled with water.

Figure 3~8 show the changes of each term in Eqs. (23) and (24) during blowdown calculations for the liquid, two-phase mixture, and vapor discharges of water. In the case of the liquid discharge calculation, the coolant remaining in the vessel superheats after \( p' = 0.67 \) and \( v' = 29 \) and therefore, the discharging coolant is changed to vapor. Since \( h_{(out)} \) is larger than \( U' \) for the vapor discharge as shown in APPENDIX, the energy discharge term contributes to the depressurization rate as shown in Fig. 3. In the liquid or two-phase mixture blowdown \( h_{(out)} \) is less than or equal to \( U' \), so the energy discharge rate becomes small and is in the order of the pressure-volume work. Although the discharge rate of the total energy is larger for the liquid or two-phase mixture discharge than for the vapor discharge (i.e. \( G_{e1}h_1 > G_{e2}h_2 \)), the energy per unit mass of coolant in the liquid phase or two-phase mixture is small and its effect upon depressurization is offset by the energy redistribution due to the mass change in the vessel.

As shown in Fig. 4, the average specific volume \( v' \) increases rapidly for the liquid or two-phase mixture discharge blowdown because of the high critical mass flux \( G_c \) through the break nozzle. While the volumetric flow rate through the break is larger for the vapor than for the liquid discharge, the volume discharge term becomes dominant in the case of the liquid discharge blowdown since the depressurization rate depends not on the specific volume of the flowing-out coolant, but on the average specific volume of the coolant in the pressure vessel.

Figure 5(a) and (b) indicate that both \( \frac{\partial p'/\partial U'}{v'} \) and \( -\frac{\partial p'/\partial v'\partial U'}{v'} \) decrease with pressure for any discharging case. The decrease rate of coolant \( v'G_c \) increases for the liquid and two-phase mixture, and decreases for the vapor discharge depending on \( v' \) (Figs. 6 and 8(b)). The principal term influencing the depressurization rate is the energy discharge term for the vapor and the volumetric discharge term for the liquid or two-phase mixture as shown in Fig. 7(a) and (b). The depressurization rate itself \( f(p') \) is larger at an early stage of blowdown and is smaller at a later stage for the vapor than for the liquid or two-phase mixture discharge (Fig. 8(a)). In the latter discharge, the increases in \( v' \) and \( v'G_c \) compensate for the decreases in other quantities at low pressure and keep the depressurization rate in the same order.
The changes of the specific enthalpy $h$ and entropy $s$ of the residual coolant in the vessel during blowdowns are shown in the Mollier Chart of Fig. 9. The specific enthalpy and entropy are evaluated with $x_s$ in Eq. (8) and the saturated properties of water. From Fig. 9, in the case of the liquid discharge, both $h$ and $s$ increase and equal those for the saturated state of steam after $p'=0.67$. The $h$ and $s$ in the case of the two-phase mixture discharge remain almost constant since the discharging coolant has the average properties of the coolant in the vessel and the dependence of $h$ and $s$ of water on pressure is small. The state of residual coolant during vapor discharge blowdowns changes approximately along the liquid saturation curve since the change of $x_s$ is small and $x_s=0$.

From the above discussion, it is understood that the rapid depressurization at an initial stage of vapor discharge blowdown is attributed to the large energy discharge term. Further, the small depressurization rate at low pressure in the case of steam line breaks or small liquid line breaks after the operation of the Automatic Depressurization System (ADS) (discharging vapor), is caused by the decrease of $v'G'_c$ which in the case of large liquid line breaks, increases and together with $v'$, compensates for the decrease of the other quantities. Finally, the change in the depressurization rate after the uncovering of the break for

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Fig. 5(a), (b) Changes of $(\partial p'/\partial U')_v'$ and $-(\partial p'/\partial v')_v$ in Eq. (23) during three different discharge blowdowns ($p'=p/p_o$, $p_o=6.895$ MPa)

Fig. 6 Changes of $v'G'_c$ in Eq. (23) during three different discharge blowdowns ($G'_c=G_c/G_{c0}$, $G_{c0}=3.42 \times 10^4$ kg/s/m$^2$, $G_c=1.62 \times 10^4$ kg/s/m$^2$)
IV. SUMMARY

Dimensionless equations were obtained for the depressurization rate and mass balance during blowdown. Three dimensionless parameters were given to represent the fractional change rate of mass, that of the energy due to heat generation, and the ratio between the two parameters. Numerical analyses were made for the dimensionless equations of blowdown.
down. It was shown that the decrease rate of coolant in the vessel increased for the liquid and two-phase mixture, and decreased for the vapor discharge. The energy discharge term was seen to make a larger contribution to the depressurization rate in the case of vapor discharge, while the volumetric discharge term did so in the case of liquid discharge blowdown. In the lumped model analyses presented here, the thermodynamic potential functions (i.e. the specific enthalpy and entropy) of the residual coolant increased for the liquid discharge, remained almost constant for the two-phase mixture discharge, and decreased for the vapor discharge blowdown.

**[NOMENCLATURE]**

\( A \): Area \( (m^2) \)  
\( E \): Total energy \( (J) \)  
\( G \): Mass flux \( (kg/m^2 \cdot s) \)  
\( h \): Enthalpy \( (J/kg) \)  
\( M \): Mass \( (kg) \)  
\( \rho \): Pressure \( (Pa) \)  
\( s \): Entropy \( (J/kg \cdot K) \)  
\( t \): Time \( (s) \)  
\( U \): Internal energy \( (J) \)  
\( V \): Volume \( (m^3) \)  
\( x \): Quality  
\( ^\wedge \): Specific property  
\( ^\prime \): Reduced  
\( ^0 \): Stagnation or reference 
\( B \): Break  
\( c \): Critical  
\( f \): Liquid  
\( g \): Vapor  
\( i \): Initial  
\( s \): Static  

**REFERENCES**

Relations Used in Blowdown Calculations

If the equilibrium saturated state is assumed in the pressure vessel, the average enthalpy $h$ is given by

$$h = h_f(p) + x_s h_{fg}(p) . \tag{A1}$$

The enthalpy of flowing-out fluid $h_{o(out)}$ is assumed to be

$$h_{o(out)} = \begin{cases} h_f(p), & \text{(Liquid discharge)} \\ h_v(p), & \text{(Vapor discharge)} \\ h_f(p) + x_s h_{fg}(p), & \text{(Two-phase mixture discharge).} \tag{A2} \end{cases}$$

Partial derivatives of $(\partial p/\partial U)_v$ and $(\partial p/\partial v)_p$ are evaluated by

$$\left( \frac{\partial p}{\partial U} \right)_v = \left( \frac{\partial h}{\partial p} \right)_v \frac{1}{\gamma - 1} , \tag{A3}$$

$$\left( \frac{\partial p}{\partial v} \right)_p = - \left( \frac{\partial h}{\partial v} \right)_p / \left( \frac{\partial h}{\partial p} \right)_v \frac{1}{\gamma - 1} , \tag{A4}$$

which are derived from the definition $h = U + pv$. Partial derivatives in Eqs. (A3) and (A4) are obtained from Eq. (A1) and given by

$$\left( \frac{\partial h}{\partial p} \right)_v = \frac{dh_f}{dp} - \frac{dh_{fg}}{dp} \frac{h_{fg}}{v_{fg}} + v \frac{dh_{fg}}{dp} \frac{1}{v_{fg}} , \tag{A5}$$

$$\left( \frac{\partial h}{\partial v} \right)_p = \frac{h_{fg}(p)}{v_{fg}(p)} . \tag{A6}$$