In-Service Calibration Method of Electromagnetic-Flowmeter for LMFBR Utilizing Cross-Correlation of Output Voltage Fluctuations

Examination from Viewpoint of Flow Velocity Distribution in Pipe

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In liquid metal cooled fast breeder reactors, electromagnetic-flowmeters (EMFs) have been extensively used. To calibrate any performance degradation, it is desired to develop an appropriate method without taking the EMF out of the system. A correlation method is applied for calibration in in-service conditions. The flow rate is obtainable from the propagation velocity of the voltage fluctuation. However, the propagation velocity of the fluctuation is larger than the mean flow velocity.

In this paper, the relationship between the flow velocity distribution and the fluctuation frequency is considered and a method of correcting the propagation velocity is proposed based on fluid mechanics. In the case of the 12 in. diameter EMF in "JOYO" reactor, the fluctuation propagates with the average velocity in the region from the pipe center to 0.79 times the inner radius. Then the calculated correction factor which transforms the propagation velocity of the fluctuation to the mean flow velocity is 0.937 and fairly agreed with the measured data. The calibration error of the correlation method does not exceed ±2%. The accuracy of 2% is comparable with that of the calibration by real flow in test loop.

KEYWORDS: fluid flow, propagation time, flow velocity, correlations, cross-correlation function, fluctuation, electromagnetic-flowmeter, calibration, JOYO reactor, measuring methods, accuracy

I. INTRODUCTION

In liquid metal cooled fast breeder reactors (LMFBRs), electromagnetic-flowmeters (EMFs) have been extensively used to measure the coolant flow rate because it is easy to assure tightness of the coolant boundary and because of their stable and linear characteristics. In consideration of the EMF degradation, which may affect its stability, it is necessary to calibrate the EMF at appropriate time intervals. However, because of the difficulty in removing the EMF from the system, the calibration by real flow in a test loop is almost impossible. Therefore, it is desired to develop a simple and accurate calibration method without removing the EMF from the system.

One of the attractive in-service EMF calibration methods is the correlation method. In this method, a cross-correlation function is computed between the output voltage fluctuations, which are observed by a pair of electrodes attached to the pipe along the fluid flow.
direction. These voltage fluctuations are caused by flow velocity fluctuations. The propagation time of the fluctuation, i.e., the delay time which maximizes the cross-correlation function is considered to be the passing time of the fluid between the electrodes. The propagation velocity is obtained by dividing the distance between the electrodes by the propagation time, and the flow rate is determined by multiplying the propagation velocity by the cross section of the flow pipe.

Although the usual cross-correlation method is based on the hypothesis that the flow velocity fluctuation propagates with the mean flow velocity of the fluid, the propagation velocity of the output voltage fluctuations of the EMF does not directly indicate the mean flow velocity because the fluid flow has a velocity distribution in the pipe and the output voltage of EMF is expressed an integral of the flow velocity multiplied by a weighting function over the whole cross section of the pipe.

In the present paper, this situation is investigated in detail and applicability of the correlation method to the EMF calibration is discussed, theoretically and experimentally. The process in which the turbulent behavior of the flowing fluid is transformed into the voltage fluctuation is considered and the propagation velocity of the output voltage fluctuations of the EMF is related to the mean flow velocity from the viewpoint of the flow velocity distribution in a pipe. Then a practical method of the EMF calibration using cross-correlation of the output voltage fluctuations is established. An application of the method to the calibration of the 12 in. diameter saddle coil type EMF in the primary cooling system of “JOYO”, the first sodium cooled experimental fast reactor in Japan, is shown, and effectiveness of this method is proved by experiment.

II. DERIVATION OF THEORETICAL EQUATION

First, the cross-correlation function of the voltage fluctuations is expressed using flow velocity fluctuations. Then, the relationship between the flow velocity distribution in a pipe and the frequency of the flow velocity fluctuation is considered. Finally, from these results, the correction factor which transforms the propagation velocity of the fluctuation to the mean flow velocity is deduced.

1. Formulation of Correlation Function

The coordinate system for expression of turbulent flow in a pipe is shown in Fig. 1. The inner radius of the pipe is a, and the variable p is defined as $p = r/a$. Then $u(t, p, \theta, z)$ is defined as the instantaneous velocity at point $(p, \theta, z)$ along the z-axis, and the local mean flow velocity $\bar{u}(p, \theta, z)$ and the velocity fluctuation $u'(t, p, \theta, z)$ at the point $(p, \theta, z)$ are expressed by

$$\bar{u}(p, \theta, z) = \frac{1}{T} \int_0^T u(t, p, \theta, z) dt, \quad (1)$$

$$u'(t, p, \theta, z) = u(t, p, \theta, z) - \bar{u}(p, \theta, z). \quad (2)$$

Often we will omit some variables in the parentheses whenever no confusion will result. The output voltage fluctuation, $v_i(t)$ is represented by the following formula:

$$v_i(t) = S \int_{-\infty}^{\infty} \frac{u'(t, p, \theta, z)}{P(t)} dp. \quad (3)$$

FIG. 1 Coordinate system to express turbulent flow
where \( W(p, \theta) = \frac{\rho (1 + \rho^2 \cos 2\theta)}{1 + 2\rho^2 \cos 2\theta + \rho^4} \), \( B \): Magnetic flux.

It is assumed that the electrical conductivity of the pipe material is zero. This is a reasonable assumption for simplicity since the liquid metal coolant has an electrical conductivity several times larger than that of the pipe material and little current circulates in the pipe wall. The cross-correlation function, \( \varphi_{12}(\tau) \) of the voltage fluctuations \( v_1(t) \) and \( v_2(t) \) is derived as follows:

\[
\varphi_{12}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} v_1(t)v_2(t+\tau)dt
\]

\[
= \frac{4B_r}{\pi^2} \sum_{p=1}^{\infty} \sum_{\theta_1=0}^{\pi} R_L(\tau, p_1, p_2, \theta_1, \theta_2)W(p_1, \theta_1)W(p_2, \theta_2)d\rho_1 d\rho_2 d\theta_1 d\theta_2,
\]

for

\[
R_L(\tau, p_1, p_2, \theta_1, \theta_2) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} u'(t, p_1, \theta_1, \rho) u'(t+\tau, p_2, \theta_2, \rho + L)dt.
\]

Here, we assume the following ideal relation though there is a component of the velocity fluctuation along the radius direction:

\[
R_L(\tau, p_1, p_2, \theta_1, \theta_2) = 0 \quad \text{for} \quad p_1 \neq p_2 \text{ or } \theta_1 \neq \theta_2,
\]

then we obtain

\[
\varphi_{12}(\tau) = \frac{4B_r}{\pi^2} \sum_{p=1}^{\infty} \sum_{\theta=0}^{\pi} R_L(\tau, p, \theta, \theta, \theta)W^2(p, \theta)d\rho d\theta.
\]

In the hypothetical case where the flow velocity is uniformly distributed in the pipe, several simplifications are possible. The \( u' \) propagates with \( \bar{u} \) equal to \( U \), the total cross-sectional mean flow velocity results in \( U = \frac{L}{\tau_m} \). Here, \( \tau_m \) is equal to \( \tau \) which maximizes \( R_L(\tau, p, \theta, \theta, \theta) \) or \( \varphi_{12}(\tau) \), and is the optimum prediction of the propagation time of \( u' \) between the electrodes in the sense of the least square method. In the actual case where the flow velocity is non-uniformly distributed, we can not directly find the distinct relationship between \( U \) and \( \tau_m \) as in the uniform case.

However, if the fluctuations make dominant contribution to \( R_L \) in the region from the pipe center to some distance, where the flow velocity distribution shows moderate variation with the distance from the pipe center as shown in Fig. 2, \( \tau_m \) becomes nearly equal to the passing time of the fluid near the center between electrodes. Then we can determine \( U \) from the flow velocity near the center based upon a knowledge of fluid

![Fig. 2 Flow velocity distribution](image-url)
mechanics. Fortunately, the velocity correlation where $\bar{u}$ changes rapidly along the pipe wall will approach zero for an appropriate $L$, because the maximum frequency of the velocity fluctuations showing a strong correlation decreases as $p$ increases and the frequency of the velocity fluctuation near the pipe wall is higher than near the center, as shown later.

In the subsequent section, we consider the quantitative relationship between the local mean flow velocity and the frequency of the velocity fluctuation to determine the area where the velocity fluctuations dominantly contribute to the cross-correlation function.

2. Components of Fluctuation Contributing to Correlation

From the discussion in the previous section, it is seen that the voltage fluctuation is a superposition of the flow velocity fluctuation weighted by $W(p, \theta)$ and the frequency components of the voltage fluctuation are same as those of the velocity fluctuations but the magnitudes of the each component differ from each other. Thus, we can determine the frequency components contributing to the cross-correlation function by considering the frequency of the flow velocity fluctuation.

The voltage fluctuation shows the most strong correlation or $\varphi_{12}(r)$ has the largest value at $r=r_0$ when $v_1(t)=v_2(t+\tau_0)$, i.e. the magnitude and direction of $u'$ at any point $(p, \theta)$ are preserved during the propagation between the electrodes. The correlation becomes weaker as $u'(t+\tau_0, p, \theta, z)$ varies from $u'(t, p, \theta, z)$. Then, there can not be the strong correlation in the case where a number of changes in the sign of $u'$ occur during the propagation between the electrodes. Since the upper bound frequency without change of the sign is given by $\bar{u}(p)/2L$, the maximum frequency $\nu(p)$ of the variation showing a strong correlation will be represented by

$$\nu(p)=k \cdot \bar{u}(p)/2L,$$

where $k$ is a correction factor and will be determined from the measured coherence function experimentally.

On the other hand, according to fluid mechanics, the derivative $d\bar{u}(p)/dp$ of the local mean flow velocity $\bar{u}(p)$ is represented for any $\theta$ and $z$, if the flow is fully developed, as follows:

$$\frac{d\bar{u}(p)}{dp} = 50 \frac{u^*}{\lambda} \cdot \frac{p^{1/3}}{(p-1)(p+1)(3p^2+7)},$$

where $u^*$ is the shear velocity. Integrating and rearranging Eq. (9), we obtain the following equation as shown in APPENDIX:

$$\bar{u}(p) = \left[ 1 + \left| I(p') \right|_{p'=0} \right] \sqrt{\frac{\lambda}{8}} U,$$

for

$$F(p) = -2.5 \ln \left( \frac{1 - \sqrt{p}}{1 + \sqrt{p}} \right) - 5 \arctan \left( \sqrt{p} \right) + 10 \left( \frac{3/7}{\sqrt{8}} \right) \left( \frac{1}{2} \ln \left( \frac{q^2 + \sqrt{2} q + 1}{q^2 - \sqrt{2} q + 1} \right) + \arctan \left( \frac{\sqrt{2} q}{1 - q^2} \right) \right),$$

$$q = \left( \frac{3/7}{\sqrt{8}} \right) \frac{1}{\sqrt{p}},$$

$$I(p') = \frac{2}{p^{12}} \int_0^{p'} pF(p) dp,$$

where $\lambda$ is the friction coefficient and the relationship $u^* = \sqrt{\frac{\lambda}{8}} U$ is used. Integration of the right-side of Eq. (13) is calculated in APPENDIX and $I(p')|_{p'=4} = 4.070$. 

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Considering that the velocity fluctuation is caused by a repetitive process in which the turbulence is generated and disappears after running the mixing length $l$, the frequency $f(p)$ of the velocity fluctuation at $(p, \theta, z)$ should be given on the average as follows:\(^{(15)}\):

$$f(p) = \frac{\sqrt{u'^4(p)}}{l}.$$  

(14)

Substituting Prandtl's mixing length\(^{(9)}\) for $l$ in Eq. (14), we obtain

$$f(p) = \frac{\sqrt{u'^4(p)}}{\sqrt{u'''}(p)v''(p)} \int_0^s \frac{d\bar{u}(s)}{ds},$$  

(15)

where $s=1-p$ and $v''(p)$ is a component of the velocity variation along the radius direction. From Eqs. (9) and (15), $f(p)$ is expressed by

$$f(p) = \frac{25}{2} \frac{s^{1/2}}{\sqrt{u'''}(p)v''(p)} \frac{\sqrt{\lambda}}{a} \frac{p^{1/2}}{(p-1)(p+1)(3p^2+7)} U.$$  

(16)

It is seen from Eq. (16) that $f(p)$ increases with $p$ because $\sqrt{u'^4(p)}/\sqrt{u'''}(p)v''(p)$ is roughly constant over the whole cross section. Contrary to this, $v''(p)$ decreases as $p$ increases according to Eq. (8) since $a(p)$ decreases as does $p$. Thus we can consider the $p$ establishing $v''(p) = f(p)$ as the boundary beyond which the velocity fluctuation does not contribute to $R_L$.

3. Derivation of Correction Factor

Describe the value of $p$ which establishes $v''(p) = f(p)$ and the cross-sectional mean flow velocity in the region from the pipe center to $p$ as $p_o$ and $U(p_o)$, respectively. Then, the following relationship results:

$$U(p_o) = L/\tau_m.$$  

(17)

Averaging $\bar{u}(p)$ over the range, $0 \leq p \leq p_o$ and rearranging leads to

$$U(p_o) = \left[1 + \{4.07 - I(p_o)\} \sqrt{\lambda}/8\right] U.$$  

(18)

From Eqs. (17) and (18), the total cross-sectional mean flow velocity $U$ is derived as follows:

$$U = \frac{1}{1 + \{4.07 - I(p_o)\} \sqrt{\lambda}/8} \frac{L}{\tau_m},$$  

(19)

where $\lambda$ can be obtained from the following equation\(^{(9)}\):

$$1/\sqrt{\lambda} = 2.0 \times \log (Re \sqrt{\lambda}) - 0.8,$$  

(20)

if the Reynolds number

$$Re = 2aU \rho / \mu$$  

(21)

is known. Here $\rho$ and $\mu$ are the density of and viscosity of the coolant, respectively. As $\lambda$ does not appreciably change with $U$ for greater values of $Re$, we can calculate the approximate value of $\lambda$ using $L/\tau_m$ for $U$ to obtain $Re$. (If a more exact value is required, iterative calculations are recommended.) Therefore, we can obtain $U$ from Eq. (19), and the flow rate $Q$ is obtained as follows:

$$Q = K \pi a^2 \frac{L}{\tau_m},$$  

(22)

where

$$K = \frac{1}{1 + \{4.07 - I(p_o)\} \sqrt{\lambda}/8}.$$  

(23)
III. EXPERIMENTS

Experiments were made to examine the calibration method for the 12 in. diameter saddle coil type EMF. The EMF used in the experiment is vertically installed in the primary cooling system of "JOYO" as shown in Fig. 3. The specification of the EMF is shown in Table 1. The fluctuation signals involved in the output voltage of the EMF were recorded onto magnetic tape and regenerated for analysis. Through the recordings and analyses, the fluctuation signals are filtered through bandpass filters in the frequency range of 0.1~30 Hz. The signal processing was performed by a digital device, where the A/D converters have 12 bits resolving power. For all analyses, the fluctuation signals were sampled through a rectangular window.

The signal recordings were made at approximately 2 to 5 m/s flow velocity and about 250 to 260°C temperature.

To clarify the properties of the fluctuation signals, some analyses of these signals were performed before the propagation time measurement.

Table 1 Specification of EMF used in experiment

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Liquid Na</th>
<th>Maximum operating pressure</th>
<th>Output voltage</th>
<th>Pipe size</th>
<th>Type</th>
<th>Electrodes interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum operating temperature</td>
<td>450°C</td>
<td>7×10^6 Pa</td>
<td>10 mV/1,400 m³/h</td>
<td>12B Sch 20S (305.5 mm I.D.)</td>
<td>Saddle coil type D.C. excited</td>
<td>150 mm</td>
</tr>
<tr>
<td>Normal operating temperature</td>
<td>370°C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum flow rate</td>
<td>1,400 m³/h</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flow rate range</td>
<td>150~1,400 m³/h</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Properties of Fluctuation Signal

(1) Wave Form and Amplitude Distribution

An example of the fluctuation wave form involved in the output voltage is shown in Fig. 4. It can be confirmed from the figure that the fluctuations are mutually related and there is a time delay between the electrodes corresponding to the flow rate.

Figure 5 shows an example of the amplitude distribution of the voltage fluctuation. In the figure, the dots are the measured values and the curve represents a normal distribution function, the mean value and the variance of which are identical with the ones of the measured amplitude distribution. It is seen from Fig. 5 that the amplitudes of the fluctuation are scattered around the normal distribution.
(2) Power Spectrum Densities

The power spectrum density function is defined as follows:

\[
\Phi_{ij}(f) = \int_{-\infty}^{\infty} \varphi_{ij}(\tau) e^{-j2\pi f \tau} d\tau, \quad \text{for } i, j = 1, 2. \tag{24}
\]

The autopower spectrum density \(\Phi_{ii}(f)\) was obtained to investigate the harmonic components of the fluctuation signal. Some examples of the results are shown in Fig. 6. According to the results, the frequency which gives maximum power is at most 2.5 Hz and decreases as the flow rate does. The frequency is almost proportional to the flow rate. This property is consistent with Eq. (16) because \(\lambda \) and \(\sqrt{u'^2(p)}/\sqrt{u'(p)v'(p)}\) hardly change with \(U\). In the same manner, the frequency which gives \(-30\) dB power is at most 25 Hz and decreases as does the flow rate.
The frequency distribution of the flow fluctuation is calculated by Eq. (16) using $1.8 \sqrt{u'^2(p)/u'(p)v'(p)}$ although the value is that for $Re\approx 5.5 \times 10^5$ (9). The results show that the frequencies for the region from the pipe center to 0.77$a$ are in the range of 0~25 Hz for the case of 1,298.5 m³/h, and the calculation agreed well the experiment.

The crosspower spectrum density was obtained to relate the contribution of each harmonic component of the fluctuation to the propagation time. The cross-correlation function can be expressed using the magnitude $|\Phi_{12}(f)|$ and the phase angle $\phi(f)$ of $\Phi_{12}(f)$ as follows:

$$\psi_{12}(\tau) = \int_{-\infty}^{\infty} |\Phi_{12}(f)| e^{i\phi(f)} e^{-2\pi i \tau f} df.$$  \hspace{1cm} (25)

Equation (25) shows that the propagation time obtained by the correlation method is the average of one of the harmonic components weighted by $|\Phi_{12}(f)|$.

The measured $|\Phi_{12}(f)|$ was very similar to the autopower spectrum density. An example of the measured phase angle $\phi(f)$ of the crosspower spectrum is shown in Fig. 7. In the figure, a line corresponding to 28.27 ms of the propagation time, which is measured by the cross-correlation function, is also shown. The propagation time of each frequency component is given by $\phi(f)/2\pi f$. It is clear that the propagation velocity of the lower frequency components is larger than that of the higher frequency components. Therefore, it is satisfactory to consider that the lower and higher frequency components are caused by the velocity fluctuations generated near the center and near the wall of the pipe, respectively.

(3) Coherence

To obtain the correlativity of each frequency component of the fluctuations and

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Fig6.png}
\caption{Autopower spectrum densities}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Fig7.png}
\caption{Phase angle of crosspower spectrum density}
\end{figure}
the maximum frequency giving strong correlation the coherence function:

\[
\Gamma_{12}(f) = \frac{\Phi_{12}(f)}{\Phi_{11}(f}\Phi_{22}(f)}
\]  

is calculated.

An example of the results is shown in Fig. 8. According to the results, the frequency which gives the upper bound of the strong correlation is at most 25 Hz as in Fig. 8 and changes with the flow rate. The typical coherence values are in the range of 0.9~0.95.

The constant \( k \) in Eq. (8) can be obtained from the measured coherence function as follows: Describe the upper bound frequency giving strong correlation on the coherence function with \( f(p_0) \), then \( k \) is given by \( 2Lf(p_0)/u(p_0) \) from Eq. (8) where \( p_0 \) can be determined using Eq. (16). If we use 0.7 of \( \Gamma_{12}(f) \) as the lower limit of strong correlation, then \( f(p_0)=23.8 \) Hz and \( k=1.45 \) for the case as shown in Fig. 8. In the same manner, for \( \Gamma_{12}(f)=0.8 \) and 0.6; \( f(p_0)=22.0 \) and 25.5 Hz, and \( k=1.34 \) and 1.58 were obtained respectively.

2. Propagation Time Measurement

As mentioned, the cross-correlation function of the fluctuations is computed to obtain the passing time of the fluid. Some examples of the measured cross-correlation function are shown in Fig. 9. In this measurement, the sampling interval and sample size are 1 ms and 4,096 per channel respectively. It is seen from the figure that the slope of the correlation function becomes less steep as the flow rate decreases. This is due to the fact that the higher harmonic components of the fluctuation are relatively weaker as the flow rate decreases, as described in the power spectrum analysis.

In the propagation time measurement, to obtain a statistical error smaller than 1%,
the fluctuation signal was processed based on the results of Ref. (11) as follows: the sampling interval and sample size were 0.5 ms and 24,576 per channel respectively, and the correlation function was obtained from the average of three independent measurements. The calculation is made in the time domain.

The result of the propagation time measurement is indicated in Table 2, where $Q_r$ is the reference flow rate measured by the electromagnetic method which is generally used, $\tau_m$ is the measured propagation time and $Q_c$ is calculated by $\pi a^2 L / \tau_m$, i.e. flow rate without correction.

<table>
<thead>
<tr>
<th>Reference flow rate $Q_r$ (m³/h)</th>
<th>Propagation time $\tau_m$ (ms)</th>
<th>Flow rate $Q_c$ (m³/h)</th>
<th>$Q_r/Q_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,298.5</td>
<td>28.27 ± 0.26</td>
<td>1,400.0 ± 13.1</td>
<td>0.928</td>
</tr>
<tr>
<td>1,132.3</td>
<td>32.75 ± 0.29</td>
<td>1,208.6 ± 10.8</td>
<td>0.937</td>
</tr>
<tr>
<td>1,001.8</td>
<td>37.05 ± 0.42</td>
<td>1,068.5 ± 12.1</td>
<td>0.938</td>
</tr>
<tr>
<td>874.2</td>
<td>42.50 ± 0.45</td>
<td>931.4 ± 9.9</td>
<td>0.939</td>
</tr>
<tr>
<td>749.0</td>
<td>49.36 ± 0.50</td>
<td>801.9 ± 8.2</td>
<td>0.934</td>
</tr>
<tr>
<td>618.8</td>
<td>59.82 ± 0.56</td>
<td>661.7 ± 6.3</td>
<td>0.935</td>
</tr>
<tr>
<td>494.4</td>
<td>75.00 ± 0.45</td>
<td>527.8 ± 3.2</td>
<td>0.937</td>
</tr>
<tr>
<td>362.1</td>
<td>103.1 ± 1.10</td>
<td>383.9 ± 4.2</td>
<td>0.943</td>
</tr>
</tbody>
</table>

The values following the sign ± in $\tau_m$ are the measured standard error in 11 time measurements of the correlation function obtained as above mentioned. Figure 10 shows the comparison of $Q_c$ with $Q_r$.

It appears from the table or figure that $Q_c$ can be corrected by multiplying by a factor. The factor is in the range of 0.928~0.943 and the mean value is 0.936. The measured statistical error of the propagation time is about ±1% and the reproducibility of the measurement is good. Correction of the systematic error is described in the subsequent section in detail.

### IV. DISCUSSION

As confirmed experimentally, the propagation velocity of the lower frequency components of the fluctuation is larger than that of the higher components. This is due to the fact that the flow fluctuation propagates with each local mean flow velocity. Therefore, in order to obtain the true flow rate, the flow velocity distribution in a pipe must be taken into account.

To obtain the correction factor in Eq. (23), the values $\lambda$, $p_0$ and $I(p_0)$ are required, and $\lambda$ can be calculated by Eq. (20) provided that $Re$ is known. The following formulae can be used to obtain $p$ and $\mu$ for calculating $Re^{(12)}$:

\[
\rho = 954.150 - 0.12735 t - 4.60045 \times 10^{-6} t^2 + 9.66704 \times 10^{-10} t^3, \tag{27}
\]

\[
\log_{10} \mu = -2.4892 + 220.65 / (t+273.15) - 0.4925 \log_{10} (t+273.15), \tag{28}
\]

where $t$ represents the sodium temperature in °C. To obtain $p_0$, we have to define the criterion of strong correlation. Here we temporarily assume $\Gamma'_H(f) = 0.7$ as the criterion.
Then, $p_0$ is obtained from $f(p_0)$ which establishes $I_{p_0}(f(p_0)) = 0.7$ using Eq. (16) in the same manner as in the previous chapter. The $I(p_0)$ can be calculated using the formula derived in APPENDIX. Thus, $Re, \lambda, f(p_0), p_0, I(p_0)$ and $K$ are obtained as shown in Table 3 for the experimental conditions.

Table 3 Corrected flow rate and parameters used in correction

<table>
<thead>
<tr>
<th>Reference flow rate $Qr (m^3/h)$</th>
<th>Corrected flow rate $Qc (m^3/h)$</th>
<th>Correction factor $K$</th>
<th>Friction coeff. $\lambda(\times 10^{-2})$</th>
<th>$I(p_0)$</th>
<th>$p_0$</th>
<th>$f(p_0)$</th>
<th>Reynolds number $Re(\times 10^4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,298.5</td>
<td>1,311.8</td>
<td>0.935</td>
<td>0.952</td>
<td>2.05</td>
<td>0.759</td>
<td>23.8</td>
<td>3.44</td>
</tr>
<tr>
<td>1,132.3</td>
<td>1,132.4</td>
<td>0.938</td>
<td>0.973</td>
<td>2.16</td>
<td>0.781</td>
<td>22.9</td>
<td>2.99</td>
</tr>
<tr>
<td>1,001.8</td>
<td>1,001.2</td>
<td>0.938</td>
<td>0.993</td>
<td>2.21</td>
<td>0.791</td>
<td>21.3</td>
<td>2.63</td>
</tr>
<tr>
<td>874.2</td>
<td>872.7</td>
<td>0.938</td>
<td>1.015</td>
<td>2.23</td>
<td>0.793</td>
<td>19.0</td>
<td>2.29</td>
</tr>
<tr>
<td>749.0</td>
<td>751.4</td>
<td>0.936</td>
<td>1.041</td>
<td>2.16</td>
<td>0.781</td>
<td>15.6</td>
<td>1.96</td>
</tr>
<tr>
<td>618.8</td>
<td>620.0</td>
<td>0.935</td>
<td>1.074</td>
<td>2.18</td>
<td>0.785</td>
<td>13.4</td>
<td>1.62</td>
</tr>
<tr>
<td>494.4</td>
<td>494.5</td>
<td>0.936</td>
<td>1.115</td>
<td>2.24</td>
<td>0.795</td>
<td>11.3</td>
<td>1.29</td>
</tr>
<tr>
<td>362.1</td>
<td>359.7</td>
<td>0.937</td>
<td>1.175</td>
<td>2.32</td>
<td>0.810</td>
<td>9.1</td>
<td>0.95</td>
</tr>
</tbody>
</table>

The correction factor $K$ is in the range of 0.935~0.938; while $Qr/Qc$, which is considered to be a measured factor, takes values in the range of 0.928~0.943. The mean value of $Qr/Qc$ and of $K$ are 0.936 and 0.937 respectively. Broadly speaking, $p_0$ equal to 0.79 gives $K = 0.937$ which means the flow velocity obtained by the correlation method is cross-sectional mean flow velocity in the region of 0.79a from the pipe center. The corrected flow rate $Q$ is obtained by multiplying $Qc$ in Table 2 by the average value of $K$, i.e. 0.937. Figure 10 shows the comparison of $Q$ with $Qr$. It appears from Table 3 that $Q$ agrees within ±1% in the full range of the measurement.

In the derivation process of $K$, we assumed $I_{p_0}(f) = 0.7$ as the criterion of the strong correlation so far. However, the correction factor $K$ does not vary so much with the value of $I_{p_0}(f)$ as the criterion. For example, in the case of 1,298.5 m$^3$/h, $K$ takes the values in the range of 0.932~0.938 for $0.8 \leq I_{p_0}(f) \geq 0.6$; and the variation of $K$ for the range of $I_{p_0}(f)$ is at most 0.6%. The value around 0.7 of $I_{p_0}(f)$ is appropriate as the criterion of the strong correlation.

The total error which contains the statistical one is within ±2%. The accuracy of 2% is comparable to that of the initial calibration of the EMF which was done in a sodium test loop before installation. Thus, we can determine the correction factor from the measured coherence function and the cross-correlation method is successful for the calibration of the EMF.

V. CONCLUSION

The followings were concluded from the study:

1. The lower and higher frequency components of the voltage fluctuations are caused by the flow fluctuations generated near the center of and near the wall of the pipe, respectively. Since the flow velocity near the center is larger than that near the wall, the propagation velocity of the lower frequency components is larger than the velocity of the higher frequency components.

2. The nominal propagation time obtained by the correlation method is equal to the average value of those of the frequency components weighted by the magnitude of
the crosspower spectrum of the fluctuation. Then, the propagation velocity obtained from the correlation method does not directly indicate the mean flow velocity of the fluid.

(3) However, in the case where the Reynolds number is larger than $10^6$ and the distance between the electrodes is appropriate to sense the correlation of the flow fluctuation around the pipe center, where the flow velocity distribution shows moderate variation with the distance from the pipe center, the propagation time obtained by the correlation method indicates that of the fluid near the pipe center. As the ratio of the average flow velocity in the range near the center to the mean flow velocity is determined when the flow velocity distribution in the pipe is known, the flow rate can be obtained.

(4) The correction factor which transforms the flow velocity by the correlation method to the true one can be determined from the coherence function by taking into account the flow profile and the frequency distribution of the velocity fluctuation in a pipe. In the case of 12B EMF of “JOYO” primary cooling system, the calculated correction factor was 0.937 and fairly agreed with measured data. In this connection, the flow velocity obtained by the correlation represents roughly the average one in the region from the center to 0.79 times the inner radius of the pipe.

(5) The calibration error of the correlation method does not exceed $\pm 2\%$. The accuracy of 2% is comparable with that of the initial calibration in a sodium test loop before installation.

It is judged from these results that the correlation method is successful for checking the degradation of the EMF characteristic and in-service calibration of “JOYO” primary EMF using the correlation method is pursued.

[NOMENCLATURE]

- $a$: Inner radius of pipe
- $B$: Magnetic flux
- $f(p)$: Frequency of flow velocity fluctuation at point $(p, \theta)$
- $F(p)$: Function defined by Eq. (11)
- $I(p')$: Integral defined by Eq. (13)
- $k$: Correction factor for maximum frequency giving strong correlation
- $K$: Correction factor for flow rate
- $l$: Mixing length
- $L$: Distance between electrodes
- $p = r/a$
- $q = (3/7)^{1/4} \cdot \sqrt{p}$
- $Q$: Flow rate corrected from $Q_c$
- $Q_p$: Flow rate without correction obtained from correlation method
- $Q_r$: Reference flow rate
- $r$: Variable representing distance to pipe center
- $Re$: Reynolds number
- $R_L$: Cross-correlation function of flow velocity fluctuation
- $s$: Distance to pipe wall
- $t$: Time variable
- $T$: Time duration for averaging
- $u$: Instantaneous local flow velocity
- $u'$: Flow velocity fluctuation in direction $z$-axis
- $\bar{u}$: Local mean flow velocity in direction $z$-axis
- $u^*$: Shear velocity
- $U$: Whole cross-sectional mean flow velocity
- $u_f$: Cross-sectional mean flow velocity from pipe center to $p_0$
- $v(t)_i$: Voltage fluctuation observed at electrode $i$
- $W(p, \theta)$: Weighting function for calculating output voltage of EMF
- $z$: Coordinate representing position on pipe center line
- $z_i$: $z$ coordinate of electrode $i$
- $\Gamma_{22}(f)$: Coherence function
- $\theta$: Angle of polar coordinate on pipe cross section
- $\lambda$: Friction coefficient
- $\mu$: Viscosity of sodium
- $\nu(p)$: Maximum frequency showing strong correlation at $(p, \theta)$
- $\rho$: Mass density of sodium
\( \tau, \tau_0: \) Time delay
\( \tau_m: \) Time delay giving maximum value of \( \varphi_{12}(\tau) \)
\( \phi(f): \) Phase angle of \( \Phi_{12}(f) \)
\( \varphi_{ij}(\tau): \) Correlation function
\( \Phi_{ij}(f): \) Power spectrum density

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**APPENDIX**

1. Derivation of \( \bar{u}(\rho) \)

The indefinite integral of Eq. (9) is obtained from the Ref. (9) in the text as follows:

\[
\int d\bar{u} = -u^*F(\rho) + \text{const.}
\]

From the above equation, the following equation is derived:

\[
\bar{u}(\rho) = \bar{u}_0 - u^*F(\rho), \tag{A1}
\]

where \( \bar{u}_0 = \bar{u}(\rho) \big|_{\rho = 0}. \)

Integrating Eq. (A1) over the whole cross section, we obtain

\[
U = \bar{u}_0 - 2u^* \int_{\rho} F(\rho) d\rho = \bar{u}_0 - u^*I(\rho') \big|_{\rho' = 1}. \tag{A2}
\]

Eliminating \( \bar{u}_0 \) from Eqs. (A1) and (A2) leads to Eq. (10) in the text.

2. Derivation of \( I(\rho_0) \)

The indefinite integral of each term in Eq. (11) in the text is obtained as follows:

\[
I_1(\rho) = \int \rho \ln \left( \frac{1 - \sqrt{\rho}}{1 + \sqrt{\rho}} \right) d\rho = \left( \frac{\alpha^2}{2} - \alpha \right) \ln \alpha - \frac{\alpha^2}{4} + \alpha - (\beta - 1)^4 \ln \beta + \frac{\beta^4}{4} - \frac{4}{3} \beta^3 + 3\beta^2 - 4\beta + \ln \beta
\]

\[
I_2(\rho) = \int \rho \tan^{-1} \sqrt{\rho} d\rho = \frac{1}{2} \left( (\rho^2 - 1) \tan^{-1} \sqrt{\rho} - \frac{\rho^{3/2}}{3} + \sqrt{\rho} \right)
\]

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\[ I_\gamma (p) = \int \rho \ln \left( \frac{q^2 - \sqrt{2} q + 1}{q^2 + \sqrt{2} q + 1} \right) d\rho \]

\[ = \frac{7}{6} (\gamma \ln \gamma - \gamma) - \frac{7}{3} \left\{ \delta^2 + 2\delta - (2\delta - 1)^{3/2} - (2\delta - 1)^{1/2} \right\} \ln \delta + \frac{7}{3} \left\{ \frac{\delta^2}{2} + 2\delta - \frac{2}{3} (2\delta - 1)^{3/2} \right\} \]

\[ I_\delta (p) = \int \rho \tan^{-1} \left( \frac{\sqrt{2} q}{1 - q^2} \right) d\rho = \frac{7}{6} \left\{ (q^4 + 1) \tan^{-1} \left( \frac{\sqrt{2} q}{1 - q^2} \right) - \frac{\sqrt{2}}{3} q^3 - \sqrt{2} q \right\} \]

where \( \alpha = 1 - p, \quad \beta = 1 + \sqrt{p}, \quad \gamma = q^4 + 1, \quad \delta = q^8 + \sqrt{2} q + 1, \) for \( q = (3/7)^{1/4}, \sqrt{b}. \)

Then, we have

\[ I(p_o) = \frac{2}{p_0^8} \left\{ -2.5I_\gamma (p_o) - 5I_\delta (p_o) + 5 \frac{(3/7)^{1/4}}{\sqrt{8}} I_\delta (p_o) + 10 \frac{(3/7)^{1/4}}{\sqrt{8}} I_\gamma (p_o) - 7.783 \right\}. \]