Temperature Coefficient of Reactivity in Light-Water Moderated and Reflected Cores Loaded with Highly-Enriched-Uranium Fuel

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Both experimental and analytical studies have been performed on the temperature coefficient of reactivity in a light-water moderated and reflected core loaded with highly-enriched-uranium fuel, which was constructed in the Kyoto University Critical Assembly (KUCA). The temperature effect on reactivity was measured for the range of 20~70°C to investigate separately the effects of (1) the fuel pitch (H/235U atomic ratio) and (2) the core shape on this physical quantity. The experimental data were analyzed with use of the SRAC code system. The calculated eigenvalue $k_{\text{eff}}$ agreed with the measured one within 0.5% in the $C/E$ ratio for both the 2- and 3-dimensional diffusion calculations. The experimental data were approximately reproduced by both the eigenvalue and perturbation calculations. It was found that the contribution of the core region was negative to the temperature coefficient of reactivity, whereas that of the reflector region was positive. The synthesis of these contributions made the temperature coefficient negative in total. The degradation of moderation was the main contributor in the core region, whereas the decrease in the neutron absorption in the reflector region. The positive contribution of the reflector region became larger as the H/235U atomic ratio became smaller and the core shape became more slender.

KEYWORDS: computer codes, critical experiment, diffusion theory, eigenvalue calculation, few-group constants, H/235U atomic ratio, KUCA, perturbation theory, reactivity, SRAC, temperature coefficient

I. INTRODUCTION

The temperature coefficient of reactivity is one of the most important physical quantities to assess the inherent stability of a reactor. It is difficult, however, to obtain an accurate value of this quantity by the neutronics calculation. The reason is that this quantity is determined through complicated competitions of positive effects on reactivity with negative ones, both of which are caused by the increase in reactor temperature. Although this problem is well known\(^1\)\(^2\), it is still current to study on this subject. An accurate neutronics calculation would be further required to reveal the complicated mechanism of the temperature effect on reactivity not only qualitatively but also quantitatively.

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To investigate the temperature effects on reactivity in research reactors loaded with highly-enriched-uranium fuel, light-water moderated and reflected rectangular-parallelepiped cores were constructed in the C-core(3)(4) of a multi-core type critical assembly, Kyoto University Critical Assembly (KUCA). These cores had simple configurations suitable to provide the benchmark data for the assessment of the neutronics calculation. The present investigation was focused on the observation how the temperature coefficient of reactivity depends upon (1) the H/235U atomic ratio (i.e., fuel pitch or moderator-to-fuel ratio) and (2) the core shape (i.e., core configuration).

The experimental data were analyzed using a code system SRAC(5) developed at Japan Atomic Energy Research Institute (JAERI). The validity of a few computational methods based on diffusion theory was examined in the present analysis. On this basis, further studies were carried out to investigate (1) the contribution of the core region to this physical quantity, separately from that of the reflector region, (2) the contribution of each nuclear feature (diffusion, moderation, absorption etc.) to this quantity, and (3) each effect of various physical processes (thermal expansion, thermal neutron spectral shift etc.) on this quantity.

II. EXPERIMENTAL

1. Description of Cores

Light-water moderated and reflected cores were constructed in the Al C-core tank of 1.8 m in depth and 2 m in diameter at the KUCA. The core was equipped with three control rods (C1, C2 and C3) and three safety rods (S4, S5 and S6), all of which were located at the boundary of the core and reflector regions. Cadmium was employed as the neutron absorber for either control or safety rod.

An illustration of a fuel plate is shown in Fig. 1. The fuel plate has a flat shape (ETR-type) and contains uranium-aluminum (U-A1) alloy in Al clad. One fuel plate contains 8.89 g of 235U and 9.55 g of U, namely the enrichment is 93.10%. The U content in U-A1 alloy is 20\%, i.e., 0.59 g/cm³ U. One by one, each fuel plate can be inserted between two Al side plates to form a fuel element.

Two types of side plates shown in Fig. 2 were used to vary the H/235U atomic ratio in the fuel region. These side plates have grooves for insertion of fuel plates in a 4.54 mm or a 3.49 mm pitch, which is employed to construct the so-called C45 or C35 core, respectively.

A view of a fuel element is shown in Fig. 3. The fuel elements were installed on a grid plate in the C-core tank with a 71 mm pitch in one direction and a 142 mm pitch in the other direction to form a core. Thirty-one or 40 fuel plates in maximum can be loaded in the C45 or the C35 fuel element, where the H/235U atomic ratio is 315 or 212, respectively.

The temperature effects on reactivity were measured for the three cores shown in Fig. 4. For identification, these cores were designated as the C45G0 (5 Rows), C35G0 (5 Row) and C35G0 (3 Rows) cores. In the above notation, G0 means that there was no light-water gap in the fuel region.
In the cores (a) and (b) shown in Fig. 4, the fuel elements were assembled in 5-row configurations. From this figure, it is clear that the longitudinal sizes of these two cores were exactly the same, and the lateral sizes of them were approximately equal with each other. Consequently, the differences in the temperature coefficients between these two cores would be mainly attributed to the change in the fuel pitch. The core (c) in Fig. 4 is a 3-row core constructed with the same fuel pitch of 3.49 mm as the core (b). Since this core was long in the lateral direction and narrow in the longitudinal direction, the dependence of the temperature coefficient upon the core configuration could be investigated through a comparison with the core (b), whose shape was nearly right square.

2. Procedure of Experiment

The temperature effect on reactivity was measured by the following procedure:

(1) Seven thermocouples and two quartz-type thermometers were settled at various positions in the system as shown in Fig. 5.

(2) Light-water was fed to the C-core tank, then the stroke of one control rod was adjusted to attain criticality, under the condition that all other rods were fully withdrawn.

(3) After the core reached criticality, the excess reactivity was measured by the positive period method. When the core was subcritical even under the condition that all control rods were fully withdrawn, the subcriticality was measured by the source multiplication method.

(4) After the measurement of the excess reactivity or subcriticality, light-water was dumped into the dump tank of the C-core, where heaters and a stirrer were installed.

(5) When light-water was heated up to an appropriate temperature, sometimes with the aid of the auxiliary core tank loop having a heater, Steps (2) and (3) were carried out.

(6) Steps (4) and (5) described above were repeated at other temperatures to measure the temperature effects on reactivity.

Table 1 shows the measured excess reactivities obtained through the above procedure and the temperature coefficients obtained from the measured data of excess reactivities. Here, the temperature coefficient was calculated as follows: Let \( \rho(T) \) be the excess reactivity at a temperature \( T \) (°C) and \( \rho(T + \Delta T) \) be the excess reactivity at a temperature \( T + \Delta T \) (°C). Then, the temperature coefficient \( \alpha(T + \Delta T/2) \) was obtained as

![Fig. 3 View of fuel element](image-url)

![Fig. 4 Designation of cores](image-url)
\[
\alpha(T + DT/2) = \left[ \rho(T + DT) - \rho(T) \right] / DT.
\]

To compare with the calculated results, the measured excess reactivities were fitted to a quadratic curve using the method of least squares.

![Diagram of thermometers in cores]

**Fig. 5** Typical positions of thermometers in cores

**Table 1** Measured temperature effects on reactivity

<table>
<thead>
<tr>
<th>Designation of core*</th>
<th>C45G0(5 Rows)</th>
<th>C35G0(5 Rows)</th>
<th>C35G0(3 Rows)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of fuel plates</td>
<td>263</td>
<td>256</td>
<td>286</td>
</tr>
<tr>
<td>Fuel pitch (mm)</td>
<td>4.54</td>
<td>3.49</td>
<td></td>
</tr>
<tr>
<td>Measured excess reactivities (%(\Delta k/k))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.394 at 27.4°C</td>
<td>0.255 at 25.9°C</td>
<td>0.358 at 22.1°C</td>
<td></td>
</tr>
<tr>
<td>0.321 at 35.8°C</td>
<td>0.225 at 33.5°C</td>
<td>0.363 at 30.3°C</td>
<td></td>
</tr>
<tr>
<td>0.239 at 43.8°C</td>
<td>0.172 at 40.8°C</td>
<td>0.343 at 38.5°C</td>
<td></td>
</tr>
<tr>
<td>0.120 at 52.9°C</td>
<td>0.114 at 47.4°C</td>
<td>0.306 at 46.5°C</td>
<td></td>
</tr>
<tr>
<td>0.0** at 61.0°C</td>
<td>0.0** at 56.6°C</td>
<td>0.247 at 54.2°C</td>
<td></td>
</tr>
<tr>
<td>—0.209** at 70.6°C</td>
<td></td>
<td>0.148 at 64.1°C</td>
<td></td>
</tr>
<tr>
<td>—0.070 at 70.0°C</td>
<td></td>
<td>0.070 at 70.0°C</td>
<td></td>
</tr>
</tbody>
</table>

Results of least square fitting:
- \(a: -1.26x10^{-4}\)
- \(b: -5.54x10^{-4}\)
- \(c: 5.04x10^{-1}\)

Temperature coefficients:
- \(a: -0.87, \text{ at } 31.6°C\)
- \(b: -0.99, \text{ at } 39.8°C\)
- \(c: -1.31, \text{ at } 48.4°C\)

of reactivity (10⁻²\(\Delta k/k\)/°C):
- \(a: -0.39, \text{ at } 29.7°C\)
- \(b: -0.72, \text{ at } 37.2°C\)
- \(c: -0.87, \text{ at } 44.1°C\)

\(\beta_n = \text{Effective delayed neutron fraction}\)
- C45G0: 0.00754, C35G0: 0.00761.

\(\tau = \text{Prompt neutron lifetime}\)
- C45G0: 55.2 μsec, C35G0: 49.2 μsec.

* Refer to Fig. 4.
** The core was just critical.
*** The subcriticality was measured by the source multiplication method, whereas the excess reactivities were measured by the positive period method.

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\[ \rho(T) = aT^2 + bT + c, \quad (2) \]

where \( a, b, c: \text{const.} \)

These results are also tabulated in Table 1. These data were processed using temperatures measured by TC#11 among the nine thermometers shown in Fig. 5. Note here that the maximum difference in measured temperatures was less than 1°C, which indicated the uniformity of temperature in the reactor system. The maximum experimental error was estimated to be ±0.005%\( \Delta k/k \) for the excess reactivity measurement in the C-core from experience.

The temperature coefficients of all the three cores are negative, except for the C35G0 (3 Rows) core in the low temperature region. The experimental data will be compared later with the calculated results.

### III. CALCULATIONS

The calculation was performed using SRAC(5), which was developed as a code system for the analysis and design of a thermal nuclear reactor.

The calculation can be divided into the following two procedures: Namely, (1) the generation of 4-group constants and (2) the analysis of the temperature effect on reactivity. The flow chart of calculation is shown in Fig. 6.

#### 1. Generation of 4-group Constants

The 4-group constants for the core calculation were generated at three temperatures 300 K (27°C), 325 K (52°C) and 350 K (77°C), for which scattering kernels are available in the neutron cross section library of SRAC. In this procedure, the following serial two steps were performed using SRAC to take the heterogeneity of core into account:

- Step 1: Cell calculations
- Step 2: Super cell calculations

The fundamental cross section library of SRAC is produced mainly from the ENDF/B-IV file with 107 energy groups. The transport cross sections for the \( P_0 \) transport calculation were obtained by the \( B_1 \) approximation, and the diffusion coefficients were generated as \( D = 1/3 \Sigma_t \). The resonance absorption for heavy nuclides was calculated by two methods: The table look-up method for the neutron energy \( E \geq 130.07 \text{ eV} \), and the collision probability method using the ultrafine energy points of 4,600 for \( 130.07 \text{ eV} \geq E \geq 0.68256 \text{ eV} \) (thermal cut-off). Thus, the user library was generated with a structure of 107 energy groups (70 fast and 37 thermal groups) as shown in Table 2. At this step, the following three physical processes were taken into account:

1. Doppler broadening
2. Thermal expansion (i.e. decrease in atomic number density of light-water)
3. Thermal neutron spectral shift (i.e. change of scattering kernel in thermal energy region).

Note that the thermal expansion effects of fuel plates, side plates, and other core structures were neglected, since these effects were negligibly small compared with those of light-water.

#### Step 1: Cell Calculations

The cell calculations were performed to obtain the 19-group collapsed and homogenized constants for a fuel region by employing models shown in Fig. 7. This step was further divided

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**Fig. 6 Flow chart of calculation**

- **Primary Cell**
  - \( 107 \)-Group Cross Sections
  - Public Library
  - \( 107 \)-Group Cross Sections
  - User Library

- **Secondary Cell**
  - Collision Probability
  - Method (107 Groups)
  - \( 107 \)-Group Homogenized
  - Constants (fuel element)

- **Super Cell I**
  - \( 19 \)-Group Homogenized
  - Constants (moderate, structure)

- **Super Cell II**
  - \( 4 \)-Group Homogenized
  - Constants (for axial structures)

- **Core Calculation**
  - \( \text{CITATION (4 Groups)} \)
  - Three-dimensional X-Y-Z
  - Eigenvalue Calculation

- **Core Calculation**
  - \( \text{CITATION (4 Groups)} \)
  - Two-dimensional X-Y
  - Perturbation Calculation

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Table 2  Energy group structure employed in calculation

<table>
<thead>
<tr>
<th>Group number</th>
<th>Fast neutron</th>
<th>Thermal neutron</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Energy (eV)</td>
<td>Energy (eV)</td>
</tr>
<tr>
<td>Fine Coarse</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1 1</td>
<td>0.10000E+8</td>
</tr>
<tr>
<td>2</td>
<td>77.7800E+7</td>
<td>0.12341E+4</td>
</tr>
<tr>
<td>3</td>
<td>0.60653E+7</td>
<td>0.74852E+3</td>
</tr>
<tr>
<td>4</td>
<td>0.47237E+7</td>
<td>0.58295E+3</td>
</tr>
<tr>
<td>5</td>
<td>0.36788E+7</td>
<td>0.41399E+0</td>
</tr>
<tr>
<td>6</td>
<td>0.28651E+7</td>
<td>0.45400E+3</td>
</tr>
<tr>
<td>7</td>
<td>0.22313E+7</td>
<td>0.35357E+3</td>
</tr>
<tr>
<td>8</td>
<td>0.17377E+7</td>
<td>0.27536E+3</td>
</tr>
<tr>
<td>9</td>
<td>0.13534E+7</td>
<td>0.21445E+3</td>
</tr>
<tr>
<td>10</td>
<td>0.10540E+7</td>
<td>0.16702E+3</td>
</tr>
<tr>
<td>11</td>
<td>0.82085E+6</td>
<td>0.13007E+3</td>
</tr>
<tr>
<td>12</td>
<td>0.63928E+6</td>
<td>0.10130E+3</td>
</tr>
<tr>
<td>13</td>
<td>0.49787E+6</td>
<td>0.78893E+2</td>
</tr>
<tr>
<td>14</td>
<td>0.38774E+6</td>
<td>0.61442E+2</td>
</tr>
<tr>
<td>15</td>
<td>0.30197E+6</td>
<td>0.47851E+2</td>
</tr>
<tr>
<td>16</td>
<td>0.23518E+6</td>
<td>0.37266E+2</td>
</tr>
<tr>
<td>17</td>
<td>0.18316E+6</td>
<td>0.29023E+2</td>
</tr>
<tr>
<td>18</td>
<td>0.14264E+6</td>
<td>0.22603E+2</td>
</tr>
<tr>
<td>19</td>
<td>0.11109E+6</td>
<td>0.17604E+2</td>
</tr>
<tr>
<td>20</td>
<td>0.86517E+5</td>
<td>0.13710E+2</td>
</tr>
<tr>
<td>21</td>
<td>0.67380E+5</td>
<td>0.10677E+2</td>
</tr>
<tr>
<td>22</td>
<td>0.52475E+5</td>
<td>0.83153E+1</td>
</tr>
<tr>
<td>23</td>
<td>0.40868E+5</td>
<td>0.64760E+1</td>
</tr>
<tr>
<td>24</td>
<td>0.31828E+5</td>
<td>0.50435E+1</td>
</tr>
<tr>
<td>25 5</td>
<td>0.24788E+5</td>
<td>0.39279E+1</td>
</tr>
<tr>
<td>26</td>
<td>0.19305E+5</td>
<td>0.30590E+1</td>
</tr>
<tr>
<td>27</td>
<td>0.15034E+5</td>
<td>0.23824E+1</td>
</tr>
<tr>
<td>28</td>
<td>0.11109E+5</td>
<td>0.18554E+1</td>
</tr>
<tr>
<td>29</td>
<td>0.91188E+4</td>
<td>0.16374E+1</td>
</tr>
<tr>
<td>30</td>
<td>0.71017E+4</td>
<td>0.14450E+1</td>
</tr>
<tr>
<td>31 3</td>
<td>0.55308E+4</td>
<td>0.12752E+1</td>
</tr>
<tr>
<td>32</td>
<td>0.43074E+4</td>
<td>0.11253E+1</td>
</tr>
<tr>
<td>33</td>
<td>0.33546E+4</td>
<td>0.99312E+0</td>
</tr>
<tr>
<td>34</td>
<td>0.26126E+4</td>
<td>0.87643E+0</td>
</tr>
<tr>
<td>35</td>
<td>0.20347E+4</td>
<td>0.77344E+0</td>
</tr>
</tbody>
</table>

† Read as $0.10000 \times 10^8$. 

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into two steps; the primary and secondary cell calculations.

(a) Super Cell I Calculation

The super cell I calculation was performed using a one-dimensional (1-D) cylindrical geometry shown in Fig. 8 to generate the 4-group (3 fast and 1 thermal groups) constants of the core and reflector regions. Here, the effective radius of core $R_c$ was introduced, since the actual core had a rectangular-parallelepiped geometry.

\[
(2.405/R_c)^2 = (\pi/a)^2 + (\pi/b)^2, \quad (3)
\]

where $a$ and $b$ are the longitudinal and lateral dimensions of the actual core, respectively. Through survey calculations, however, it was found that the results were rather insensitive
to the little change in $R_e$. Note here that the axial buckling obtained experimentally was used for the vertical direction, as shown in Fig. 8.

(b) Super Cell II Calculation

The super cell II calculation was performed in order to obtain the 4-group constants of axial structures for 3-dimensional (3-D) calculations and the group-dependent axial bucklings (4 groups) for 2-dimensional (2-D) calculations. The 1-D slab geometry option of CITATION was employed for this purpose. In this step, the transverse buckling was calculated using the value of reflector saving shown in Fig. 8 under an assumption that the horizontal reflector saving is equal with the vertical one. Then, the calculated axial neutron flux distributions were fitted by cosine curves to obtain the group-dependent axial bucklings at each temperature as listed in Table 3.

<table>
<thead>
<tr>
<th>Designation of core</th>
<th>Energy group No.</th>
<th>$27^\circ$C</th>
<th>$52^\circ$C</th>
<th>$77^\circ$C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$B^2$ ($10^{-2}$ cm$^{-1}$)</td>
<td>Reflector saving(cm)</td>
<td>$B^2$ ($10^{-2}$ cm$^{-1}$)</td>
<td>Reflector saving(cm)</td>
</tr>
<tr>
<td>C45G0</td>
<td>1</td>
<td>1.9088</td>
<td>7.45</td>
<td>1.8992</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.9114</td>
<td>7.43</td>
<td>1.9018</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.9141</td>
<td>7.40</td>
<td>1.9053</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.9123</td>
<td>7.42</td>
<td>1.9036</td>
</tr>
<tr>
<td>C35G0</td>
<td>1</td>
<td>1.8131</td>
<td>8.39</td>
<td>1.8020</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.8182</td>
<td>8.34</td>
<td>1.8071</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.8224</td>
<td>8.30</td>
<td>1.8122</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.8207</td>
<td>8.31</td>
<td>1.8097</td>
</tr>
</tbody>
</table>

2. Analysis on Temperature Coefficient

Using the 4-group constants and group-dependent axial bucklings obtained through the previous procedure, the temperature coefficients were calculated with use of CITATION. In this procedure, two methods were employed; the eigenvalue and perturbation calculations.

Method 1: Eigenvalue Calculation

The eigenvalue calculations were performed for the three temperatures ($27$, $52$ and $77^\circ$C) to obtain the temperature coefficients. Both the 3-D X-Y-Z and 2-D X-Y geometry options were employed for this method. The X-Y cross sections of 3-D calculation models are shown in Fig. 9 and the X-Z cross section of a 3-D core calculation model in Fig. 10. The 2-D calculation models correspond to Fig. 9. The axial bucklings obtained in the super cell II calculation were used in the 2-D calculations.

Method 2: Perturbation Calculation

The 2-D perturbation calculations were
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Application factors $k_{\text{eff}}$ at $27^\circ\text{C}$ in comparison with the experimental data. The experimental data are obtained by substituting $27^\circ\text{C}$ into Eq. (2), whose coefficients have been determined as listed in Table 1. The calculated $k_{\text{eff}}$ values agree with the experimental data within 0.5% in the $C/E$ ratio both for the 2-D and 3-D calculations at the three temperatures 27, 52 and 77$^\circ\text{C}$.

Figure 11 shows the temperature effects on excess reactivity in the individual cores. Note

Fig. 10 $X$-$Z$ cross section of 3-D core calculation model performed using the same calculation models and axial bucklings as those employed in the 2-D calculations by Method 1. Here, both the exact and the first order perturbation methods were employed.

IV. RESULTS AND DISCUSSIONS

1. Assessment of Calculations

Table 4 shows the calculated effective multiplication factors $k_{\text{eff}}$ at $27^\circ\text{C}$ in comparison with the experimental data. The experimental data are obtained by substituting $27^\circ\text{C}$ into Eq. (2), whose coefficients have been determined as listed in Table 1. The calculated $k_{\text{eff}}$ values agree with the experimental data within 0.5% in the $C/E$ ratio both for the 2-D and 3-D calculations at the three temperatures 27, 52 and 77$^\circ\text{C}$.

Figure 11 shows the temperature effects on excess reactivity in the individual cores. Note

![Fig. 10 X-Z cross section of 3-D core calculation model](image)

![Fig. 11 Calculated and measured temperature effects on excess reactivity](image)

Table 4 Comparison of calculated effective multiplication factor and experimental data

<table>
<thead>
<tr>
<th>Designation of core</th>
<th>Measured excess reactivity at $27^\circ\text{C}$ ($%\Delta k/k$)$^\dagger$</th>
<th>Effective multiplication factor $k_{\text{eff}}$</th>
<th>$C/E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C45G0 (5 Rows)</td>
<td>0.397</td>
<td>2-D$^{\ddagger}$ 1.0040</td>
<td>0.9964</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3-D$^{\ddagger\ddagger}$ 1.0023</td>
<td>0.9983</td>
</tr>
<tr>
<td>C35G0 (5 Rows)</td>
<td>0.253</td>
<td>2-D</td>
<td>0.9979</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3-D</td>
<td>0.9972</td>
</tr>
<tr>
<td>C35G0 (3 Rows)</td>
<td>0.363</td>
<td>2-D</td>
<td>0.9972</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3-D</td>
<td>0.9988</td>
</tr>
</tbody>
</table>

$^\dagger$ The value was obtained by substituting $27^\circ\text{C}$ into Eq. (2), whose coefficients had been determined by the method of least squares (Refer to Table 1).

$^{\ddagger}$ Two-dimensional calculation, $^{\ddagger\ddagger}$ Three-dimensional calculation.
that the calculated values with 3-D models are normalized to the experimental data at 27°C.

Figure 12 shows the temperature coefficients

\[ \alpha(T) = 2aT + b. \]  

The magnitude of the negative temperature coefficient is greatest in the C45G0 (5 Rows) core. The next greatest magnitude is found in the C35G0 (5 Rows) core, and the smallest magnitude in the C35G0 (3 Rows) core. The negative gradients of temperature coefficients in the C35 cores are slightly steeper than the gradient in the C45 core.

Calculated temperature coefficients have slightly larger negative values than the experimental data. The gradients of temperature coefficients calculated for the 5-row cores agree well with those of the experimental data, whereas the gradient of calculated results for the 3-row core slightly differs from that of the experimental data.

Table 5 shows the comparison of temperature coefficients by the eigenvalue and perturbation calculations. This table also includes the experimental data obtained by Eq. (4). All calculated temperature coefficients have slightly larger negative values than the experi-

<table>
<thead>
<tr>
<th>Method of calculation</th>
<th>Temperature coefficient (×10⁻⁴/k/k°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C45G0(5 Rows)</td>
</tr>
<tr>
<td></td>
<td>39.5°C</td>
</tr>
<tr>
<td>Eigenvalue calculation</td>
<td></td>
</tr>
<tr>
<td>3-D¹</td>
<td>-1.22</td>
</tr>
<tr>
<td>2-D²</td>
<td>-1.23</td>
</tr>
<tr>
<td>Exact perturbation method²</td>
<td></td>
</tr>
<tr>
<td>Core</td>
<td>-2.04</td>
</tr>
<tr>
<td>Reflector</td>
<td>0.77</td>
</tr>
<tr>
<td>Total</td>
<td>-1.27</td>
</tr>
<tr>
<td>First order perturbation method³</td>
<td></td>
</tr>
<tr>
<td>Core</td>
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</tr>
<tr>
<td>Reflector</td>
<td>0.73</td>
</tr>
<tr>
<td>Total</td>
<td>-1.27</td>
</tr>
<tr>
<td>Experiment²</td>
<td>-1.05</td>
</tr>
</tbody>
</table>

5): The neutron flux distribution in the perturbed system was used in the calculation.
6): The neutron flux distribution in the initial system was used in the calculation.
7): The values were obtained by substituting the temperature into Eq. (4), whose coefficients have been determined by the method of least squares (Refer to Table 1).
mental data. There is no remarkable difference between the results of the eigenvalue and perturbation calculations. This indicates that the both methods can be applied to calculate the temperature coefficient, however, the results of the perturbation calculation have slightly larger negative values than those of the eigenvalue calculation.

The results of the eigenvalue calculation using the 2-D and 3-D models are approximately equal with each other. This fact assures the validity of the 2-D calculation model employed in the present study. Note that the result of the preliminary 2-D calculation using a temperature independent axial buckling did not reproduce the experimental data, since the objective core under investigation was small in size where the change of axial buckling might cause a rather large effect on the temperature coefficient. The temperature coefficients by the exact and the first order perturbation methods are approximately equal with each other. The contribution of each region calculated by the exact perturbation method is slightly greater (namely, the contribution of the core region has a larger negative value and that of the reflector region has a larger positive value) than that by the first order perturbation method.

Table 6 shows the breakdown of temperature coefficients in the C35G0 (5 Rows) core by the perturbation calculation. The difference between the exact and the first order perturbation methods is found mainly in the fission and absorption terms of the core region and in the absorption term of the reflector region. These terms are closely related to the thermal neutron flux distributions.

<table>
<thead>
<tr>
<th>Region</th>
<th>Component of nuclear features</th>
<th>Contribution of each component (%Δk/k)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exact perturbation method(1)</td>
<td>First order perturbation method(2)</td>
</tr>
<tr>
<td></td>
<td>52°C</td>
<td>77°C</td>
</tr>
<tr>
<td>Core</td>
<td>F</td>
<td>-2.33674</td>
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<tr>
<td></td>
<td>A</td>
<td>2.24857</td>
</tr>
<tr>
<td></td>
<td>F+A</td>
<td>-0.08817</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>-0.34308</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>-0.05447</td>
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<td></td>
<td>DB(2)</td>
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<tr>
<td>Total</td>
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<td>Reflector</td>
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<td>0.32936</td>
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<tr>
<td></td>
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<tr>
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<td>D</td>
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<td></td>
<td>DB(3)</td>
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<tr>
<td>Total</td>
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<td>0.24744</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>-0.24720</td>
</tr>
</tbody>
</table>

1): F: Perturbed reactivity by change in fission rate, A: Perturbed reactivity by change in neutron absorption rate, A+F: Perturbed reactivity by change in fission and neutron absorption rates, D: Perturbed reactivity by change in degradation of moderation effect, DB: Perturbed reactivity by change in vertical leakage.
2): The neutron flux distribution in the perturbed system was used in the calculation.
3): The neutron flux distribution in the initial system was used in the calculation.
Figure 13 shows the calculated flux distributions of thermal neutrons in the C35G0 (5 Rows) core at 27 and 77°C. Note that the neutron flux is normalized to unity at the core center. The calculated neutron flux at 77°C is larger than that at 27°C in the reflector region and in the core region adjacent to the reflector region. This indicates that the number of neutrons moderated in the reflector region becomes larger with the increase in temperature. Although the change in the reactor condition occurs over the whole system with the increase in temperature, the first order perturbation theory is reasonably applicable in this case, since the change is rather small in the forward neutron flux.

Fig. 13 Calculated thermal neutron flux distributions in C35G0 (5 Rows) core at 27 and 77°C (The neutron flux distributions are normalized to unity at the core center.)

2. Region-dependent Temperature Coefficient of Reactivity

Figure 14 shows the temperature coefficients calculated by the exact perturbation method. For all cores under investigation, the contributions of the core region are negative to the temperature coefficients and have negative gradients, whereas the reflector region gives approximately constant positive contributions. The sum of these two contributions makes the negative temperature coefficient having a negative gradient in total.

The negative contributions of core regions are approximately equal with each other in magnitude. As for the positive contributions of reflector regions, the largest magnitude is found in the C35G0 (3 Rows) core, the second is in the C35G0 (5 Rows) core and the smallest in the C45G0 (5 Rows) core. This results in the tendency of the magnitudes of total temperature coefficients in the above three cores.

Figure 15 shows the calculated thermal neutron flux distribution in each core, which is considered to be closely related to the positive temperature coefficient in the reflector region. Note that these flux distributions are normalized to unity at the core center.

Firstly, comparing the C45 and C35 5-row cores, it is found that the flux peaking in the reflector region of the C35 core with a smaller H/235U atomic ratio is larger than that of the C45 core. Secondly, comparing the C35 5-row and 3-row cores, the flux peaking in the slender 3-row core is larger than that in the nearly square 5-row core. The reason is that the number of neutrons moderated in the reflector region becomes larger when the H/235U atomic ratio (in this case, water-to-metal-volume ratio) becomes smaller and the shape of core region becomes more slender. This indicates that, if the efficiency of neutron reflection grows larger...
with the increase in temperature, a larger positive temperature effect on reactivity will be caused in a reactor system which has a larger flux peaking in the reflector region.

3. Contributions of Nuclear Features

Figure 16 shows the contribution of each nuclear feature (diffusion, moderation, absorption etc.) to the temperature coefficient in the C35G0 (5 Rows) calculated by the exact perturbation method; (a) corresponds to the core region and (b) to the reflector region. In the core region, the change in the neutron absorption rate also causes the change in the fission rate, therefore, the sum of these two contributions is plotted in Fig. 16(a).

In the core region, the main contribution to the temperature coefficient comes from the degradation of moderation as shown in Fig. 16(a). The negative gradient of temperature coefficient in the core region is mainly attributed to this term. The degradation of moderation is caused by the decrease in macroscopic scattering cross sections due to the decrease in the atomic number density of light-water.
From the consideration that the coefficient of thermal expansion for light-water is an increasing function with temperature, the tendency of this moderation term is understandable.

In the reflector region, the absorption term is the main contributor to the temperature coefficient as shown in Fig. 16(b). The change in this term is attributed to the decrease in absorption cross sections of light-water, which is caused by both the thermal expansion and the thermal neutron spectral shift.

4. Examination on Physical Processes

Figure 17 shows the results of the eigenvalue calculation considering two main physical processes (i.e., thermal expansion and thermal neutron spectral shift), separately. From this figure, it is found that the thermal expansion causes the negative effect on reactivity for all the cores, whereas the thermal neutron spectral shift does the positive one, and the overall temperature coefficient is negative. Note that the Doppler broadening causes negligibly small effect on reactivity, since highly-enriched-uranium fuel was loaded in the core.

The negative temperature coefficient caused by the thermal expansion of light-water is mainly attributed to the negative one in the core region. The thermal expansion causes the decrease in the atomic number density of light-water with the increase in temperature. This gives the negative effect on reactivity mainly by the decrease in the H/\(^{235}\)U atomic ratio. In other words, the leakage probability of fast neutrons increases mainly with the decrease in macroscopic scattering cross sections of light-water. The increase in the leakage probability of thermal neutrons causes rather small effect on reactivity compared with that of fast neutrons.

The positive temperature coefficient caused by the thermal neutron spectral shift is mainly attributed to the positive one in the reflector region. The thermal neutron spectral shift causes the decrease in microscopic absorption cross sections for thermal neutrons. This causes the increase in the efficiency of neutron reflection by light-water reflector, which introduces the positive effect on reactivity in the reactor.

V. CONCLUSIONS

The results obtained through the present study are summarized as follows:

1. In the temperature range of 20～70°C, the temperature coefficients of reactivity were negative for all the cores under investigation. The magnitude of negative temperature coefficient was greatest in the C45G0 (5 Rows) core. The next magnitude was found in the C35G0 (5 Rows) core, and the lowest was in the C35G0 (3 Rows) core.

2. The calculated eigenvalue \( k_{\text{eff}} \) agreed with the measured one within 0.5% in the C/E ratio for both the 2-D and 3-D calculations. The measured temperature effects on reactivity were approximately reproduced by both the eigenvalue and perturbation calculations.

3. In each of the three cores, the contribution of the core region was negative to the temperature coefficient and had a negative gradient, whereas the reflector region gave an approximately constant positive contributions. The sum of these contributions made the negative temperature coefficient having a negative gradient in total.
(4) The contribution of the reflector region became larger, as the flux peaking of thermal neutrons in reflector became larger. The flux peaking became larger, as the H/$^{235}$U atomic ratio became smaller and the core shape became more slender.

(5) In the core region, the main contribution to the temperature coefficient of reactivity was attributed to the degradation of moderation, which was caused by the decrease in macroscopic scattering cross sections due to the decrease in the atomic number density of light-water.

(6) In the reflector region, the main contribution to the temperature coefficient was attributed to the decrease in microscopic absorption cross sections, which was caused by the increase in neutron temperature.

(7) The results of 2-D calculations agreed well with those of 3-D calculations. This indicates that one could successfully calculate the temperature coefficient by the 2-D model, if the change in axial buckling due to the increase in reactor temperature were adequately taken into consideration.

(8) The calculated temperature coefficients by the exact and the first order perturbation methods agreed well with each other. This indicates that, although the change in the reactor condition occurred over the whole system with the increase in reactor temperature, the first order perturbation theory was applicable to calculate the temperature coefficient. The reason is found in the fact that, for the present core under investigation, the neutron flux distribution changed gently over the whole system and the negative effect on reactivity in the core region competed with the positive one in the reflector region.

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The calculations were mainly performed at the Data Processing Center of Kyoto University and partly at the computer center of JAERI.

REFERENCES


