Vibrational Characteristics of Support Post Structure in HTR

Basic Characteristic

Masatoshi FUTAKAWA, Kenji KIKUCHI and Yasushi MUTO

Department of High Temperature Engineering, Tokai Research Establishment, Japan Atomic Energy Research Institute*

Received February 25, 1987
Revised April 23, 1987

In order to evaluate the effects of the support post length, the mass supported by post and the hemispherical radii of both post and seat on the vibrational characteristics of the support post structure in HTR, vibration tests were carried out by using a vertical two-dimensional vibration model. Additionally the experimental results were compared with the analytical ones obtained from the Lagrange's equation.

The followings are the conclusions derived:

1. The resonance frequency of support post structure is dependent on the length of post as well as the radii of post and seat, but almost independent of the mass supported by post.
2. The analytical results agree well with the experimental ones.

KEYWORDS: HTR reactor, support post, post seat, post length, mass supported by posts, radius, seat, vibration test, vibrational characteristics, resonance frequency, Lagrange's equation

I. INTRODUCTION

The fuel blocks and the replaceable reflector blocks, which constitute a core of high temperature gas-cooled reactor (HTR) developed by Japan Atomic Energy Research Institute, are supported by the support posts as illustrated in Fig. 1 (1). Each support post is in contact with a hot plenum block at the top end and a lower plenum block at the bottom end through hemispherical seats, because the small inclination or the rotation of support posts can absorb a relative displacement to the lateral direction between the core component blocks, which is caused by the seismic force and the difference of thermal expansion, as shown in Fig. 2.

Ikushima et al. studied the vibrational behavior by using the core-element models of HTR and confirmed the calculation model for an earthquake excitation (2)~(4). These models adopted the simple rolling motion as a characteristic of support post. However, it seems that the vibrational motion of the support post structure exhibits the singular behavior caused by both a rolling and a sliding motion between the post and the seat, which differs fairly from the simple rolling motion. The vibrational characteristics might have an effect on both the integrity of bottom-core and core structure, and the feasibility of control-rod insertion during an earthquake excitation. Therefore, in order to appreciate the dynamic response of both bottom-core and core structure, it is necessary to consider the vibrational behavior of the support post structures on these models. Furthermore, it is significant to estimate the inclination of support post due to a seismic disturbance because the inclination have an effect on the fracture strength (5). Accordingly, it is important to investigate the vibrational characteristics of support post structure in order to establish the calculation model being able to

* Tokai-mura, Ibaraki-ken 319-11.
represent the characteristics from the viewpoint of both the earthquake-resistance and the safety design of HTR.

Two types of vibration test were performed with a two-dimensional model for the support post structure; one is a free damped vibration test and the other is a forced vibration test. The experimental results were compared with the analytical ones obtained from the Lagrange's equation.

Additionally, the static-loading test was carried out to evaluate the relationship between the restoring force against the horizontal load and the inclined angle of post as a function of a variable spring constant.

II. EXPERIMENTAL MODEL AND METHOD

1. Experimental Model

Figure 3 illustrates the vertical two-dimensional vibration model for the support post structure. This model, which consists of two support posts and two seat-blocks, is manufactured to evaluate mainly the effect of the post length $l$, the hemispherical radii of post $r_p$ and seat $r_s$, and the mass supported by posts $m_u$ on the vibrational characteristics of support post structure in the horizontal direction. The material of these components is the candidate graphite IG.
11 in HTR, where apparent density is \(1.78 \times 10^3\) kg/m\(^3\) and Young's modulus \(9.9 \times 10^3\) MN/m\(^2\), respectively.

The constants of the standard experimental model, which is the 1/2.5 small scale for the design of HTR support post structure, are \(l = 240\) mm, \(r_p = 32.0\) mm and \(r_s = 32.6\) mm. The dimensions and mass were varied throughout the experiments, as shown in Table 1.

Table 1 Range of geometrical and mass parameters varied throughout experiment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of post (mm)</td>
<td>120~300</td>
</tr>
<tr>
<td>Radius of post (mm)</td>
<td>32.0~64.0</td>
</tr>
<tr>
<td>Radius of seat (mm)</td>
<td>32.6~65.3</td>
</tr>
<tr>
<td>Mass of upper block (kg)</td>
<td>4.6~15.4</td>
</tr>
</tbody>
</table>

2. Experimental Method

Figure 4 illustrates the block diagram of measuring system. Two types of vibration test were performed. One is the free damped vibration test. After the posts supporting the upper seat-block were perpendicularly set up on the lower seat-block fixed to the horizontal table, the upper seat-block was slightly shifted in the horizontal direction to incline the posts. The free damped vibration was generated by means of the rapid release of the inclining post from the unstable state. An output signal measured by the non-contacting displacement transducer was recorded on a data recorder. It was filtered and converted to frequency responses on a fast Fourier transform (FFT) analyzer. Then the frequencies and the damping ratios of the free damped vibration of the post were calculated by curve fitting method on the FFT analyzer. The displacement-time responses were filtered below 50 Hz, stored on a wave memory and plotted on an X-Y recorder.

The other is the forced vibration test using a horizontal vibration table (1 m x 1 m). The model was excited by a sinusoidal table motion from 1.5 to 10.0 Hz with the constant displacement. The absolute displacement of the input signal for the vibration table was controlled, and the relative displacement between upper seat-block and support rig of the output signal was accurately measured by the non-contacting displacement transducers.

In relation to the above vibration tests the static-loading test was carried out in order to clarify the relationship between the inclination of post, the displacement of upper seat-block in horizontal direction and the restoring force against the horizontal load, and to interpret the singular vibrational behavior of support post structure. The relationship indicates the characteristic of support post structure as a function of a spring constant.

III. CALCULATION METHOD AND FORMULAE

1. Calculation Model

The following two-dimensional model is considered as shown in Fig. 5:

(1) Each of blocks and support posts is treated as a rigid body.

(2) The degrees of freedom are three; the rotation around the center of gravity and the displacements in the plane.

(3) The post can roll or slide on the seat.

2. Restoring Force

The coordinates and parameters are chosen as shown in Fig. 5. Initially, the support posts are perpendicularly set up through the seats. The restoring force \(F_R\), acting on the upper seat-block in the \(x\) direction, is generated as a result of the increased potential energy due to the inclination of support posts.
The moment of force around $P_i$ is represented by the equation

$$M_{FR} = M_{in} + M_{inu}, \quad (1)$$

where

$$M_{FR} = 2F_R(r_p \cos \theta_r + (l_p - r_p) \cos \theta_i)$$
$$M_{in} = mn_p g(r_p \sin \theta_r - (l_p - r_p) \sin \theta_i)$$
$$M_{inu} = 2m_u g(r_p \sin \theta_r - (l_p - r_p) \sin \theta_i).$$

Accordingly, the restoring force is expressed as follows:

$$F_R = \frac{(nm_p + 2m_u) g(r_p \sin \theta_r - (l_p - r_p) \sin \theta_i)}{2(l_p \cos \theta_r + (l_p - r_p) \cos \theta_i)} \quad (2)$$

The relationship between $\theta_r$ and $\theta_i$ in Eq. (2) depends on the motion of rolling or of sliding.

The support post can roll to incline without sliding when $\theta_r$ is smaller than $\theta_f$, which is a friction angle of graphite against itself. The arc $P_iQ_p(P_iQ_{pH})$ is equal to the arc $P_iQ_{sl}(P_iQ_{sh})$. Then $\theta_r$ is able to be related to $\theta_i$ by the next equation

$$\theta_r = R_i \theta_i \quad (3)$$

where $R_i = r_p / (r_s - r_p)$.

The restoring force $F_R$ in Eq. (2) is represented as a function of the inclined angle of post $\theta_i$ with using Eq. (3).

Assuming that a sliding-friction force is equal to the maximum static-friction force, $\theta_r$ is represented by the next equation while a sliding motion is occurring between post and seat

$$\theta_r = \theta_f (\theta_i > \theta_i / R_i). \quad (4)$$

The rolling-friction force acting between post and seat has less effects on the restoring force than $M_{in}$ and $M_{inu}$ do, because a coefficient of rolling-friction is generally much less than $10^{-2}$ (6). Therefore, we neglect the rolling-friction force.

3. Vibrational Equation

The vibrational equation of the support post structure is derived from the Lagrange’s equation

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \ddot{q}} \right) - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} = Q. \quad (5)$$

The total kinetic energy $T$ for support post structure consists of $T_{tp}, T_{tu}$ and $T_{rp}$, where $T_{tp}$ is the kinetic energy of translation for the support posts, $T_{tu}$ the kinetic energy of translation for the upper seat-block and $T_{rp}$ the kinetic energy of rotation for the posts, respectively.

The total potential energy $U$ is the sum of the potential energy of posts $U_p$ and the potential energy of upper seat-block $U_u$. The $Q$ is the non-conservative force. The inclined angle of support post $\theta_i$ is available for the generalized coordinate $q$.

Then $T$ and $U$ are given by the next equations

$$T = T_{tp} + T_{tu} + T_{rp} + T_{tu}$$
$$= n I_{dp} / 2 \dot{\theta}^2 + n m_p / 2 (\dot{X}_{dp} + \dot{Y}_{dp}), \quad (6)$$
$$U = U_p + U_u$$
$$= n m_p g Y_{op} + m_u g Y_{ou}, \quad (7)$$

where $X_{dp} = (r_s - r_p) \sin \theta_r + (l_p - r_p) \sin \theta_i$, $Y_{op} = (r_s - r_p) (1 - \cos \theta_r)$, $Y_{ou} = (l_p - r_p) (1 - \cos \theta_i)$.

When the support post rolls on the seat, we get

$$\theta_r = \theta_{rp} + R_i (\theta_i - \theta_{ri}) \quad (8)$$

But when it slides on the seat, we get

$$\theta_r = \theta_f \quad (\theta_i - \theta_{si}) > |\theta_f / R_i|. \quad (9)$$

On the other hand, assuming that $\theta_i$ is small
and \( Q = 0 \), Eq. (5) results in

\[
[n l_{\alpha p} + (n m_p + 4 m_u) l_p^2] \ddot{\theta}_i + [(n m_p + 2 m_u) (R_s - l_p)] g \theta_i = 0, \tag{10}
\]

where \( R_s = r_s (r_s - r_p) \).

Then, if a damping motion is able to be represented by an equivalent viscosity and external force \( f(t) \) is acting on the support post structure, we can obtain the following vibrational equation:

\[
[n l_{\alpha p} + (n m_p + 4 m_u) l_p^2] \ddot{\theta}_i + C \dot{\theta}_i + [(n m_p + 2 m_u) (R_s - l_p)] g \theta_i = f(t). \tag{11}
\]

From Eq. (10) the natural frequency \( F(Hz) \) is approximately expressed as follows:

\[
F = \frac{1}{2\pi} \sqrt{\frac{g (n m_p + 2 m_u) (R_s - l_p)}{n l_{\alpha p} + (n m_p + 4 m_u) l_p^2}}. \tag{12}
\]

The dynamic responses obtained from the strict equation derived from Eq. (5) and also from the simplified Eq. (11) were calculated by using the Runge-Kutta integration method. The time step increment was taken \( 10^{-2} \) s in calculation.

### IV. RESULTS AND CONSIDERATION

#### 1. Restoring Force

**Figure 6** shows \( F_R \) as a function of the displacement of upper block \( X_{Gu} \) or the inclined angle of post \( \theta_i \) for the standard experimental model. The \( F_R \) increases rapidly and then decreases gradually with increasing \( \theta_i \). It should be noticed that the characteristic of restoring force of support post structure indicates the bilinear type which consists of two springs; one is the spring possessing a positive constant for \( \theta_i \leq \theta_{im} \) and the other the spring possessing a negative constant for \( \theta_i > \theta_{im} \). The turning inclined-angle at the maximum \( F_R \) and the returning inclined-angle to \( F_R = 0 \) were independent of the mass of upper seat-block \( m_u \), although the maximum \( F_R \) depends on \( m_u \).

The slender lines represent the analytical results obtained from Eq. (2). The angle \( \theta_f \) of friction in Eq. (2), measured by the inclining test(7), is 13\(^\circ\). Despite the change of the mass of upper seat-block, the analytical results are in good agreement with the experimental ones in the range of \( 0 \leq \theta_i \leq \theta_{im} \), but they do not agree with each other in the range of \( \theta_i > \theta_{im} \). The difference between analytical results and experimental ones gradually increase with increasing \( \theta_i \) when \( \theta_i > \theta_{im} \). A kinetic friction force acting during sliding is smaller than a static friction one, as measured by the inclining test(7). The reason why the discrepancy in analytical and experimental values increases with increasing \( \theta_i \) when \( \theta_i > \theta_{im} \) seems to be attributed to the difference of friction mechanism. That is, the support post inclines to roll on the seat for \( 0 \leq \theta_i \leq \theta_{im} \) while it slides for \( \theta_i > \theta_{im} \).

The comparison between experimental and analytical hysteretic curves for restoring force as a function of displacements is shown in Fig. 7. The experimental result is well describable by the analytical one with using \( \theta_f = 13^\circ \) in Eq. (2), in the case of hysteresis curve with small area. The rolling motion occurs in the regions AB, CD and EF, and the sliding motion in the regions BC, DE and FB. The spring constants for the rolling motion in AB, CD and EF agree almost with each other.

#### 2. Free Damped Vibration

**Figure 8** shows the examples of free damped vibration for the standard experimental model with \( m_u \) of 4.6 kg, where the initial horizontal displacement (or the initial inclined angle) is changed. The periods of vibration are almost independent of the initial displacements and
indicate about 0.22 s. The amplitude of vibration increases with the initial displacement where the initial displacement is lower than about 1 mm. When it becomes more than 1 mm, the amplitude is almost unchangeable regardless of the initial displacement but the displacement does not damp to zero. This is because the support post begins to slide on the seat during the initial input of displacement. Additionally, as the spring constant for each rolling motion is identical as shown in Fig. 7, the period of vibration is almost independent of the initial displacement.

An example of frequency response which was measured by FFT analyzer for the standard model is shown in Fig. 9. The resonance frequency at the maximum value of magnitude hardly varied with the initial displacement in each model. Figure 10 shows the relation between initial displacement and damping ratio in the standard model. It is noticed that the damping ratio is almost independent of the initial displacement and decreases with increasing $m_w$.

Figure 11 shows the comparison of free damped vibration between the experimental result and the analytical one. The analytical result was
obtained from Eq. (11), using the measured value for a damping ratio. The analytical result shows a favorable correlation with the experimental one, but it has a little longer period than the experimental one. Besides, the analytical result agrees almost with the calculated ones using the strict equation derived from Eq. (5)(6).

Figure 12 shows the effect of \( m_u \) on the natural frequency for the support post structure. It is noted that \( m_u \) hardly influences the natural frequency. The analytical result obtained from Eq. (12) agrees with the experimental one.

Figures 13 and 14 represent the effects of \( l_p \) and \( R_2 \) on the natural frequencies, respectively. The frequencies depend on \( l_p \) as well as \( R_2 \). The analytical results can describe the change of frequencies adequately. Here, the asterisked values indicate the case of the standard model with \( m_u \) of 4.6 kg. The above characteristics for natural frequencies were applicable to the other test models.

It is predictable from Eq. (12) that the natural frequency of the support post structure installed in HTR(1) is about 2 Hz. Accordingly, the resonance motion may occur in the support post structure because the dominant frequency of an earthquake excitation is below 10 Hz. It may influence not only the vibrational behavior of core whose resonance frequency is estimated about 1 \( \sim \) 3 Hz(4) but also the performance of control-rod insertion.

3. Forced Vibration

The magnification factor in terms of displacement is plotted against the frequency in Fig. 15. The measured average values for damping ratios as indicated in Fig. 10 are adopted to calculate the forced vibration response with using Eq. (11). The experimental magnification factor and frequency at resonance agree adequately with the
analytical ones, respectively. Additionally, the post stands perpendicularly on the seat even after excited from 2 to 10 Hz. In this case, the maximum value of vibrational angle of post $\theta_t$ is much smaller than $\theta_t/R_1$, so that the post seems to vibrate only to roll on the seat.

The vibrational amplitude became suddenly large as soon as the sliding motion might occur because of the increasing input-amplitude. And then the vibrating post began to incline and eventually almost fell down. This is because the spring constant of the support post structure changes abruptly from the positive constant to the negative constant, as shown in Figs. 6 and 7. **Figure 16 (a)** shows the example of displacement-time response in 0.2 mm input-amplitude which indicates both the rolling and the sliding motions. Just after the inclination due to sliding at resonance, the post continues to vibrate with rolling motion again and does not fall down. The behavior is able to be adequately described by the analytical result using Eq. (5), as shown in **Fig. 16(b)***. The angle $\theta_f$ of friction used in the analysis is about half as large as the static-friction angle. It can be deduced that the post inclines easily during the vibration of itself because of a small kinetic friction against the seat. The inclination of the post due to vibration might affect not only the fracture strength of itself but also the alignment of core blocks which is essential to the reactor coolant flow distribution(9).

**V. CONCLUSION**

The basic experiments both on the vibrational characteristics and on the restoring force for support post structure were carried out by using the vertical two-dimensional vibration model. The experimental results were compared with the analytical ones.
The followings are the conclusions derived:

1. The restoring force characteristic of the support post structure is able to be represented by a bilinear spring which possesses both a positive constant for rolling motion and a negative constant for sliding motion.

2. The mass supported by post has little effect on the resonance frequency of support post structure, and has a significant effect on the damping ratio.

3. The resonance frequency depends on the length of post and the radii of both post and seat.

4. The resonance frequency is practically predictable by the analysis using the Lagrange's equation.

5. The forced vibration behavior which includes both the rolling and sliding motions can be predicted by using both the above equation and the experimentally obtained damping ratio.

6. Future work is required to develop the calculation model which is able to represent the vibrational behavior of both support post structure and core element, in order to appreciate the earthquake-resistance and the feasibility of control-rod insertion in HTR.

[NOMENCLATURE]

- \( C \): Damping coefficient, \( F \): Frequency
- \( F_R \): Restoring force, \( f \): Force
- \( G \): Center of gravity
- \( g \): Acceleration of gravity
- \( I_o \): Moment of inertia
- \( l \): Length of post
- \( l_p \): Half length of post
- \( M_{FR} \): Restoring moment
- \( M_m \): Moment due to mass, \( m \): Mass
- \( n \): Number of posts
- \( O_s \): Center of hemispherical end of post
- \( O_s \): Center of rotation of post
- \( P \): Contacting point of support post with seat after inclination of support post
- \( Q \): Non-conservative force
- \( Q_s \): Contacting point of support post with seat before inclination of support post
- \( Q_t \): Contacting point of seat with support post before inclination of support post
- \( q \): Generalized coordinates
- \( r_p \): Hemispherical radius of end of post
- \( r_s \): Hemispherical radius of seat
- \( T \): Total kinetic energy
- \( T_r \): Kinetic energy of rotation
- \( T_t \): Kinetic energy of translation
- \( t \): Time, \( U \): Potential energy
- \( x, y \): Cartesian coordinates
- \( X_o \): Displacement of center of gravity in \( x \)-direction
- \( Y_o \): Displacement of center of gravity in \( y \)-direction
- \( \theta_i \): Inclined angle of post
- \( \theta_{io} \): Initial inclined angle of post
- \( \theta_{is} \): \( \theta_i \) at the maximum value of \( F_R \)
- \( \theta_r \): Rotated angle of post around point \( O_s \)
- \( \theta_{ro} \): Initial rotated angle of post around point \( O_s \)
- \( \theta_f \): Friction angle of graphite against itself

(Suffix)
- \( h \): Top, \( l \): Bottom
- \( p \): Support post, \( u \): Upper seat-block

ACKNOWLEDGMENT

The authors wish to thank Dr. K. Sanokawa and Mr. T. Ikushima for their helpful suggestions and comments. Thanks are also extended to Mr. K. Tachibana for his help in conducting the experiments.

REFERENCES