SHORT NOTE

Probability Distribution of Number of Collisions Necessary for Slowing Down of Neutrons

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There has been a long-standing interest in obtaining stochastic expressions of the number of collision to neutron slowing down problems, with the dual purpose of underlying physical processes and establishing the reactor physics education. The previous paper has described the exact expression of average number of collisions necessary for the slowing down of neutrons(1). The analytic expression of probability distribution of the number of elastic collisions for $A>1$, $A$ is mass number of the medium, was not presented in it. For $A=1$ this distribution is the Poisson distribution but for $A>1$ slowing down process with the lethargy $u$ is an integer-valued non-Markovian process. In this note we describe the analytic function of the probability distribution.

The distribution

$$p_u(n)=\text{Prob}[N=n]$$

(1)
gives the probability that the observed number of elastic collisions $N$ is an integer $n$ during the slowing down from the initial to a given lethargy $u$. It is related to the density function $w_u(u)$ which is the probability that a neutron with an initial lethargy $u_0=0$ has a lethargy $u$ after exactly $n$ collisions as follows;

$$p_u(n)=\int_0^u [w_{u-n}(u')-w_u(u')]du', \quad (n\geq 2).$$

(2)

For $n=1$

$$p_u(1)=1-\int_0^u w(u')du'$$

(3)

and for $n=0$, $p_u(0)=0$ ($u>0$) and $p_u(0)=1$ ($u=0$).

The function $w_u(u)$ is given by many authors as follows(2):

$$w_u(u)=\frac{e^{-u}}{(1-\alpha)^n(n-1)!} \sum_{k=0}^{M} (-1)^k \alpha^k C_k(u-kX)^{n-1},$$

$$0 \leq u \leq nX, \quad (4)$$

and $w_u(0)=0$ $u>nX$, $M$ is the largest value of $k$ for which the bracket term in the summation is negative; $M=[u/X]$ and $X=-\ln \alpha$ where $\alpha=\frac{(A-1)/(A+1)^2}$ with $A$ the mass number of the medium. (In the previous paper $M$ denoted by $k(1)$.)

When $u \leq X$, the integrals of Eqs. (2) and (3) are directly given by

$$p_u(n)=\frac{u^{n-1}+\alpha(n-1)!\left\{\sum_{k=0}^{n-2} (u^k/k!)-e^u\right\}}{e^u(1-\alpha)^n(n-1)!}, \quad (n\geq 2),$$

$$p_u(n)=\frac{1-\alpha e^u}{e^u(1-\alpha)}, \quad (n=1).$$

(5)

For $X \leq u$, the analytic expression is considerably complicated and has not been reported.

At first we note that Eq. (2) must be integrate in every lethargy interval

$$mX \leq u' < (m+1)X, \quad (m=0,1,2,\ldots,M).$$

Two integral functions are defined by

$$E(n; m)=\int_{mX}^{(m+1)X} w_u(u')du',$$

$$MX \leq u' < (M+1)X, \quad (m=0,1,2,\ldots,M-1),$$

(6)

$$F(n; u, M)=\int_{MX}^{u} w_u(u')du',$$

$$MX \leq u \leq (M+1)X, \quad (M=1,2,\ldots).$$

(7)

Substituting Eq. (4) into Eqs. (6) and (7), and integrating them, we obtain

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Finally the distribution \( p_u(n) \) for the set lethargy \( u \) \([M X \leq u < (M+1)X]\) is given by

\[
\begin{align*}
  \frac{1}{(1-\alpha)^n} & \left[ \sum_{r=0}^{n-1} [(m-k)X]^{n-2-r}e^{-u(u-kX)} \right] \\
  & - \sum_{r=0}^{n-1} [(m-k)X]^{n-1-r}e^{-u(u-kX)} \\
  & + \sum_{r=0}^{n} (-1)^r C_k \left[ (1-k/n) \sum_{r=0}^{n} \alpha^r [(m-k)X]^{n-2-r}e^{-u(u-kX)} \right] \\
  & - \sum_{r=0}^{n-1} \alpha^r [(m-k)X]^{n-1-r}e^{-u(u-kX)} \right] \right], \\
  \\end{align*}
\]

The probability distribution for deuterium obtained from Eq. (10) is plotted in Fig. 1 as a function of random variable, the number of collision. The REDUCE 3.3, an algebraic operation program calculated the average number of collisions necessary for slowing down for several scattering media by Eq. (10). Numerical values coincide with the results of the Table 1 in Ref. (1) from the analytic expression obtained by the moment generating function.

---REFERENCES---

(2) MARSHAK, R.E.: Rev. Mod. Phys., 19, 185 (1947).