Reactivity Worth of Fuel Elements  
Measurement and Perturbation Analysis

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Reactivity worths of fuel elements were measured in the Ozenji Critical Facility (OCF) and analyzed with three group perturbation method. The result shows that the worth of one single fuel pin can be well predicted by calculation over a very wide range of the core spectrum, namely from a lattice of 2.5% enrichment and 0.43 volume ratio to that of 1.5% enrichment and 3.5 volume ratio.

The analysis indicates the importance of thermal neutron flux peaking remaining after the removal of a fuel pin. Only by incorporating this effect can the reactivity worth of a fuel pin be correctly evaluated. In the present study, the neutron spectrum in the water hole where the peaking occurred was assumed to be the same as in the reflector. The reflector spectrum seems to provide better agreement with experiment than the core spectrum. Validity of the analysis was extended to a bundle of sixteen fuel pins by measuring the reactivity worths of bundles of fuel pins as well as the thermal neutron flux distributions. One dimensional diffusion calculations were employed throughout the analysis.

I. INTRODUCTION

It is of practical importance and of analytical interest to understand the physical behavior of the reactivity effects of fuel elements. The present series of experiments and analyses provide information in this domain as well as material to serve in basic considerations on perturbation analyses applied to such problems.

In the present work, spatial distribution of the reactivity worth of a fuel pin was first of all measured by the period method for various single and two region lattices. This was followed by measurements of flux perturbation and reactivity effect for groups of fuel pins in typical cores, to ascertain the limitations to the validity of perturbation analysis.

The Ozenji Critical Facility (OCF), located at Ozenji, Kawasaki-shi, was utilized for the experiment. This is a slightly enriched UO$_2$-H$_2$O critical assembly. The fuel consists of UO$_2$ pellets, 10 mm diameter and of density 10.4. The effective length of the fuel pins is 840 mm. The 2S aluminum sheath has 12.2 mm outer and 10.6 mm inner diameters. Two different enrichments are available: 1.49% and 2.49% (1.5% and 2.5% nominal). Square lattices were used in all cases except one where a hexagonal lattice of 0.43 water to fuel volume ratio was utilized.

Analysis was made by three group perturbation method, and resulted in a discrepancy with experiment of less than 1%. The lattices investigated ranged from a combination of 2.5% enrichment and 0.43 water to fuel volume ratio to that of 1.5% enrichment and 3.5 volume ratio (which we will call "2.5%-0.43 R" and "1.5%-3.5 R" lattices, respectively). Correctness of the perturbation analysis was verified by measuring the reactivity effect caused by changing the enrichment through the use of different fuel. This last experiment was the one having the smallest perturbation with respect to flux peaking and spectrum change. The limit of applicability of the analysis was further extended by analyzing the reactivity worths of bundles of fuel pins in typical lattices.

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II. EXPERIMENTAL METHOD

The spatial distribution of a fuel pin were first of all measured in the radial direction in the case of cylindrical cores and in the direction perpendicular to the region-interface in the case of two region rectangular cores. This was followed by measurements on bundles of fuel pins in typical core configurations.

The period method was mostly employed for reactivity measurement: With all the control rods withdrawn, and with an effectively infinite thickness of water reflector in all directions, the excess reactivities of a standard core* and of a reference core (one fuel pin at the position of interest removed from the standard core) were measured. The difference between the two excess reactivities gives the reactivity worth of the fuel pin. The fuel pin withdrawn was reinstated in the same position after each measurement. Once in about every three measurements or even more frequently, the excess reactivity of the standard core was measured to verify cumulative drift of reactivity that might be caused by the successive withdrawals and insertions of fuel pins.

Period was measured with compensated ionization chambers by observing the $e$-fold time of the reactor power. A waiting time of more than three period intervals was strictly observed to make sure that the asymptotic period was obtained. The period was measured at least twice for each observation. The experimental error in reactivity measurement was estimated to be about $\pm 0.3\varepsilon$ or less, including coverage for differences between fuel pins of the same nominal composition, and the effect of deviations from straightness in the aluminum sheathing. The correction necessary to cover the moderator temperature drift during measurements was less than $\pm 0.1\varepsilon$. This correction was made with measured or calculated temperature coefficients$^{(a)}$. At certain critical positions such as near the two region boundary of the M-3 core (described later), where the neutron flux changes very sharply from one point to the next, the experimental reproducibility was as poor as $\pm 1\varepsilon$ or so, despite extremely careful measurements, with the reactivity drift of the standard core verified once in every two measurements.

In the M-3 core, a BF$_3$ counter was also installed, and the period was obtained by making a least-square fit of the counting rates to an exponential function of time. The periods obtained with CIC and BF$_3$ counters agree, on the average, within about 1 sec, which corresponds to a difference of about 0.2 $\varepsilon$ at a period of around 50 sec. The agreement is illustrated in Fig.1.

For measurement on a fuel pin in a single region 2.5%-2.5 R core and on bundles of fuel pins in typical core configurations, calibrated rods were used to measure the reactivity worth.

III. ANALYTICAL METHOD

The three group perturbation theory$^1$ was applied in analyzing the experiment. We start from the stationary equations$^{**}$

$$
\begin{align*}
-D_1\nabla^2 \phi_1 + (\Sigma_{a1} + \Sigma_{m1}) \phi_1 &= \sum_{j=1}^{3} \nu \Sigma_{j1} \phi_j \\
-D_1\nabla^2 \phi_2 + (\Sigma_{a2} + \Sigma_{m2}) \phi_2 &= \Sigma_{a1} \phi_1 \\
-D_1\nabla^2 \phi_3 + \Sigma_{50} \phi_1 &= \sum_{j=1}^{3} \nu \Sigma_{j1} \phi_j
\end{align*}
$$

(1)

* The standard core is one that has symmetric and the simplest geometry, and with an experimentally adequate amount of excess reactivity.

** With Deutsch's formalism, $\Sigma_{a1}=0$ and

$$
\sum_{j=1}^{3} \nu \Sigma_{j1} \phi_j = \delta \nu \sum_{j=1}^{3} \phi_j.
$$

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or, using matrix notations,
\[(A - \nu F)\phi = 0,\] (2)
where
\[A = \begin{pmatrix}
-D_1v^2 + \Sigma_{st} + \Sigma_{st}, & 0, & 0 \\
-D_2v^2 + \Sigma_{st} + \Sigma_{st}, & 0, & 0 \\
-\Sigma_{st}, & -D_3v^2 + \Sigma_{st} & 0
\end{pmatrix}, \]
\[F = \begin{pmatrix}
\Sigma_f, & \Sigma_f, & \Sigma_f, \\
0, & 0, & 0 \\
0, & 0, & 0
\end{pmatrix}, \]
\[\phi = \begin{pmatrix}
\phi_1 \\
\phi_2 \\
\phi_3
\end{pmatrix}. \] (3)

According to the usual perturbation method, the static reactivity
\[\delta C = -\frac{\delta \nu}{\nu} = -\frac{\int \int \int \int \phi^*(\delta A - \nu \delta F)\phi \, d\tau d\theta d\theta d\phi}{\int \int \int \int \phi^* (\nu F) \phi \, d\tau d\theta d\theta d\phi}, \] (4)
where \(\phi^*\) is the perturbed flux. For small perturbations, \(\phi^*\) is almost identical with \(\phi\), so that
\[\delta C = -\frac{\int \int \int \int \phi^*(\delta A - \nu \delta F)\phi \, d\tau d\theta d\theta d\phi}{\int \int \int \int \phi^* (\nu F) \phi \, d\tau d\theta d\theta d\phi}. \] (5)

Going back to the usual notations and with Green's theorem, Eq. (5) now becomes
\[\delta C = -\frac{1}{L_y} \int \left[ \int \int \left( \Sigma_{f1} \nu \delta \Sigma_{f1} (\phi_j^* \phi_i) - \frac{\Sigma_{f1}}{\Sigma_{f1}} \delta \Sigma_{f1} (\phi_j^* \phi_i) - \Sigma_{f1} D_{f1} \nu \phi_j^* \phi_i - \frac{\Sigma_{f1}}{\Sigma_{f1}} \Sigma_{f1} (\phi_j^* \phi_i) \right) \right] d\tau d\theta d\phi. \] (6)

In order to calculate the reactivity worth of the fuel pins, the volume integral of Eq. (6) must be performed. Since the integral along the \(z\)-axis is cancelled out for the present type of experiment, the following equations are used for the actual one-dimensional calculations.

(1) Rectangular core:
\[\delta C = S_{2,11} \times \text{value of } \left( \phi^* (\delta A - \nu \delta F) \phi \right) \text{ where interested} \]
\[\sum_{i=1}^{3} \sum_{j=1}^{3} \left[ \Sigma_{f1} \phi_j^*(r_i) \phi_i (r_j) \cdot (r_i + r_j) \right] (\Delta \tau \cdot \Sigma_{f1}) K, \]
where \(i, j, k\): Numbers of mesh, energy group, and region, respectively.

(2) Cylindrical core:
\[\delta C = \sum_{k=1}^{3} \sum_{l=1}^{3} \left[ \sum_{i=1}^{3} \phi_j^*(r_i) \phi_i (r_j) \cdot (r_i + r_j) \right] (\Delta \tau \cdot \Sigma_{f1}) K. \]
analysis. The first is that of flux peaking caused by the water gap left after the removal of fuel pins. This is too prominent to be covered by the approximation of Eq. (5). The effect was therefore incorporated in the analysis as follows. First, the thermal neutron flux distribution was calculated for a critical cylindrical core. Then, its central portion was replaced by the perturbed composition and the flux distribution was recalculated. Finally, by normalizing these distributions in the unperturbed regions, the average thermal neutron flux peaking factor was calculated for the central perturbed area. This value was utilized for the perturbations in all other parts of the core. Calculations were performed with the SUNRISE 4-1 code mentioned before. The effect of the flux peaking was then incorporated by multiplying the perturbation of the group constants concerned — instead of multiplying the flux itself — by this thermal flux peaking factor.

The second problem concerns the neutron spectrum in the water gap. When a fuel pin is withdrawn, a water hole of between 14mm and 22mm equivalent diameter is introduced. Maintaining the philosophy of adopting the simplest acceptable model, it was assumed that the neutron spectrum in the water hole was the same as in either core or reflector. The actual spectrum should be somewhere between these two extreme cases. Also, strictly speaking, the first and the second problems are closely related to each other, both in space and in energy, and should be simultaneously solved. This is, itself, an interesting and complicated problem but shall not be discussed here in further detail, since it is beyond the scope of the present perturbation analysis.

IV. EXPERIMENTAL AND ANALYTICAL RESULTS

1. Single-region Cores

2.5%-1.5 R, 2.5%-2.5 R and 1.5%-2.5 R lattices were studied. The cores are all cylindrical in geometry. Table 1 provides on outline of the cores, and core loading maps are shown in Fig. 2. The experimental and analytical procedures are described in Sections 2 and 3 respectively. Results obtained are presented in summary form in Fig. 3. Experimental errors are estimated to be about ±0.3% on the average. It is to be noted that the fuel has negative reactivity worth only in the hardest core, the 2.5%-1.5 R lattice. The broken lines in Fig. 3 represent contributions from individual factors of the perturbation (fission term, absorption term, etc.). The solid lines are the sums of the individual contributions, and represent the reactivity worth to be compared with experiment. It is seen that the reactivity effect is mainly determined by the competition between the $\Sigma_{oa}$ and $\nu\Sigma_{r}$ terms.

<table>
<thead>
<tr>
<th>Table 1 Descriptions of Single-region Cores Studied</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core No.</td>
</tr>
<tr>
<td>Enrichment (%)</td>
</tr>
<tr>
<td>W/\text{UO}_2</td>
</tr>
<tr>
<td>Core shape</td>
</tr>
<tr>
<td>Active height(cm)</td>
</tr>
<tr>
<td>Dimensions (plane)</td>
</tr>
<tr>
<td>Number of fuel pins</td>
</tr>
<tr>
<td>Method of reactivity measurement</td>
</tr>
<tr>
<td>Water temp. (°C)</td>
</tr>
</tbody>
</table>

* Direction of measurement

Fig. 2 Core Loading Map (Single cores)
and that the gradient terms are negligible compared with the other terms. Here, the neutron spectrum in the water hole is assumed to be the same as in the core. The water peaking due to the water hole is incorporated as described in Section 3.

In order to obtain an indication of the magnitude of the effect of the water peaking, reactivity worths neglecting the peaking were calculated, the results being as shown by the triangles in Fig. 3. The importance of the effect of peaking is clearly visible, and it is seen that neglect of the effect invariably results in a value erring on the negative side by an amount ranging from 2 to 5%. On the other hand, the effect of the neutron spectrum undulations on reactivity is only a fraction of the peaking effect. This is illustrated in Fig. 3 and Table 2. The table shows the analyses for a pin at the center of cores with different spectra for cases of peaking both considered and neglected. The core spectrum for 2.5%-1.5R lattice and the reflector spectrum for 2.5%-2.5R and 1.5%-2.5R lattices give values closest to experiment for centrally positioned fuel pins. Analyses for the peripheral pins, especially in the 2.5%-1.5R core, on the other hand indicate that the reflector spectrum provides better agreement with experiment. It may be concluded that the effect of water peaking is too important to be neglected, and that the reflector spectrum for the water hole gives on the average better agreement.

To summarize the analyses for single region cores:

1. Three group perturbation analysis with Deutsch group constants, taking into account the peaking factor and modification brought to the neutron spectrum due to

<p>| Table 2 Reactivity Worths of a Fuel Pin at Core Center |
|---------------------------------|-----------------|</p>
<table>
<thead>
<tr>
<th>Lattice</th>
<th>Measured worth</th>
<th>Calculated worth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Core spectrum††</td>
</tr>
<tr>
<td></td>
<td></td>
<td>peaking considered</td>
</tr>
<tr>
<td>2.5%-1.5R</td>
<td>-8.5 ± 0.3</td>
<td>-8.5</td>
</tr>
<tr>
<td>2.5%-2.5R</td>
<td>3.3 ± 0.3</td>
<td>2.5</td>
</tr>
<tr>
<td>1.5%-2.5R</td>
<td>4.2 ± 0.3</td>
<td>2.6</td>
</tr>
<tr>
<td>1.5%-2.5R</td>
<td>-5.8 ± 0.3</td>
<td>-5.7</td>
</tr>
</tbody>
</table>

† Exchange of fuel pins of different enrichments, namely from 1.5 to 2.5%  
†† Spectrum in the water hole left after the removal of a fuel pin
the water hole being left after the removal of a fuel pin, agreed with experiment with an average discrepancy of less than 1\%.

(2) The effect of water peaking is too important to be neglected. Adoption of the average thermal flux peaking factor incorporating the perturbation of thermal group constants, combined with the use of the reflector spectrum for the water hole, gives the best results.

(3) The effect on reactivity caused by difference in enrichment between fuel pins could be very well predicted by the analysis. This could be expected since the flux and spectrum perturbations are quite small. This also affords an essentially positive justification of the analytical models employed.

2. Two-region Cores

The examination of the validity of the analytical method is now extended to two-region lattices of much harder and softer spectra. The features of the cores studied, M-3, M-11 and M-12, are outlined in Table 3. Core loading maps indicating the positions of measurement are shown in Fig.4. The lattice composition now ranges from 2.5\%-0.43R to 1.5\%-3.5R. The experimental and analytical procedures were exactly the same as described in Section IV.1 except that the group constants were now calculated with JUPITER and TEMPEST codes. In addition to the reactivity worth of single fuel pins, the reactivity effect due to difference in fuel enrichment was also measured as in the previous case, in order to ascertain the correctness of the analytical models.

The results of experiment and analyses are summarized in Figs. 5 and 6. The accuracy of measurement varies from place to place, but on the average, the errors are estimated to be about \( \pm 0.5\% \). Where the neutron flux sharply changed, straightness of the fuel pins sensitively affected the reactivity worth and caused scattering of the data. This effect amounted to differences reaching 1\% or so in the worst case. The period was measured with compensated

\[\text{Table 3 Descriptions of Two-region Cores Studied}\]

<table>
<thead>
<tr>
<th>Core No.</th>
<th>M-3</th>
<th>M-11</th>
<th>M-12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regions</td>
<td>A (left)</td>
<td>B (right)</td>
<td>A (outer)</td>
</tr>
<tr>
<td>Enrichment(%)</td>
<td>1.5</td>
<td>2.5</td>
<td>1.5</td>
</tr>
<tr>
<td>W/UO(_2)</td>
<td>3.5</td>
<td>1.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Number of fuel pins</td>
<td>432</td>
<td>450</td>
<td>856</td>
</tr>
<tr>
<td>Active height (cm)</td>
<td>84</td>
<td>84</td>
<td>84</td>
</tr>
<tr>
<td>Dimensions (mm) (plane)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Method of reactivity measurement</td>
<td>Period</td>
<td>Period</td>
<td>Period</td>
</tr>
<tr>
<td>Water temp. (°C)</td>
<td>(\sim 10)</td>
<td>(\sim 24.5)</td>
<td>(\sim 25)</td>
</tr>
</tbody>
</table>

\* Direction of measurement
\dagger Hexagonal lattice

(a) M-3 core (half)  (b) M-11 core (quarter)  (c) M-12 core (quarter)

\(\odot\) Measured point  (All rods fully withdrawn  Core fully reflected)

Fig.4 Core Loading Map (Two-region cores)

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In addition, for the M-3 core a BF$_3$ counter was added as mentioned in Sec. 2. A cycle of 9 sec counting and 1 sec waiting was adopted. The crystal-timer used was accurate to about $10^{-5}$ sec. In Fig. 5 is shown the result of BF$_3$ data for the M-3 core, and the indicated error is the larger of internal and external standard deviations obtained from exponential fitting. When both are smaller than 0.5 $\epsilon$, an error of 0.5 $\epsilon$ is assigned. For M-11 and M-12 cores, CIC data were employed and the error was assumed to be 0.5 $\epsilon$. The systematic discrepancy between BF$_3$ and CIC data is about 0.3 $\epsilon$, which corresponds to differences of 0.15 and 0.03 $\epsilon$ respectively at periods of 30 and 80 sec, as shown in Fig. 1. This is of negligible order compared with 0.5 $\epsilon$.

Experimental results indicate that, as a whole:

1. The reactivity worth of a fuel pin is positive in the A-region (1.5% enriched, softer spectrum) and negative in the B-region (2.5% enriched, harder spectrum), except around the two-region boundaries.
The worth of a fuel pin changes very sharply near the two-region boundary where there occurs a marked change in the spatial distribution of the neutron flux. The measured values there scatter by about 1σ or more.

In Fig. 7 is shown the contributions of the individual perturbation effects; the terms such as for fission and absorption are separately represented. The three group constants derived from JUPITER (Hetero)-TEMPEST (Amouyal) codes were utilized, and the code used for calculation was the modified SUNRISE 4-1. The following statements can be made on the basis of this analysis:

1. Fission and absorption terms, especially of the thermal group, are predominant. This is especially so for lattices with softer spectra.
2. Diffusion terms are so small as to be negligible.
3. When the spectrum gets harder, the slowing down term becomes important, and in the hardest lattice exceeds the magnitude of the absorption term.
4. Depending on whether the water gap peaking is taken into consideration or not, the calculated worths of a fuel pin differs to a considerable extent. The worth with this effect ignored is always appreciably more negative than with the effect considered. The difference is clearly beyond experimental error.
5. As shown in Fig. 6, change in enrichment of the fuel did not occasion a large correction for flux perturbation, and the effect of flux peaking was quite small, all of which was expected. The effect is almost negligible compared with experimental error. The agreement between experiment and analysis attests to the correctness of the analytical models employed.
Most of these results have already been observed in the previous analyses related to the single region experiment, and are here-with reconfirmed and extended to a wider range of lattice spectra. In Fig. 8 the values of the peaking factors used in the foregoing calculations are summarized as a function of the water to fuel volume ratio.

V. DISCUSSIONS

It has been shown that the worth of a fuel pin can be well predicted by the three group perturbation method, and that consideration for the water hole peaking of the thermal neutron flux cannot be neglected. The result is, however, limited to the worth of one single fuel pin and gives no information on the worth of a group of fuel pins.

In order to ascertain the limits of validity of the analytical method and examine its applicability to a multiplicity of fuel pins, the thermal neutron flux peaking and reactivity worth were measured and analyzed on a bundle of fuel pins. The lattices investigated were 2.5%-2.5 R single region core and the 2.5%-1.5 R region of the (two region) M-3 core.

The thermal neutron flux was measured only for the former lattice, with 4 pins and 16 pins withdrawn. A Dy(4 %)-Al alloy wire was used, and β flow counting performed with an automatic wire scanner20, which automatically corrects activation decay. The Cd ratio is near 50 for this core and bare activation can give the thermal neutron flux within 2 % error. The measured thermal neutron flux peaking for the water holes produced by removing the 2x2 and 4x4 fuel pins were 1.76 and 2.66, respectively. These correspond to calculated values of 1.86 and 2.51. The reflector spectrum in the water hole is assumed. Comparisons of these values should provide the means of obtaining satisfactory results from the peaking calculations.

The reactivity worth was measured with a calibrated shim rod. The results are shown together with analyses in Figs. 9 and 10. The calculation of reactivity was made with reference to a critical core. Consideration for flux peaking is absolutely and undoubtedly necessary and without this the worth can not be predicted with any
certainty by the analyses. The accuracy with which the water hole peaking is calculated governs the reliability of the final reactivity predictions. Besides, since for these lattices the excess reactivities have been compensated by the insertion of a control rod, the comparison between experiment and calculation is not as simple as in the case of a single fuel pin where cores are always free from the introduction of neutron absorbers. The present analysis however should serve in explaining the essential physical features of the reactivity effect of fuel pins. In order to further improve the accuracy of the analysis, it should be necessary to calculate the peaking factor more accurately, and also to take into account the epithermal and fast neutron flux perturbations. It is also be requisite to correctly evaluate the neutron spectrum in the water hole.

VI. CONCLUSIONS

It has been shown that the reactivity effect of fuel pins can well be analyzed by three group perturbation method over a wide range of core spectrum. The thermal neutron flux peaking in the water hole left after the removal of fuel pins turns out to be very significant. The importance of taking this effect into consideration is strongly emphasized. The reflector spectrum in the water hole was assumed in the present analysis. The analysis indicates that reactivity effect is largely determined by the competition between the fission and absorption terms of the thermal group, and that the gradient term is negligible. When the core spectrum becomes harder, the slowing down term competes with the thermal absorption term and can even exceed the absorption term.

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