Accurate Estimation of Subcriticality Using Indirect Bias Estimation Method, (II) Applications

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Subcriticalities were estimated by applying the Indirect Bias Estimation Method to subcritical experiments on a light-water moderated/reflected low-enriched UO₂ lattice cores. Two measurable values, prompt neutron time-decay constant and spatial-decay constant were calculated using MCNP 4A and JENDL-3.2. With these calculation errors, the biases in calculated reactivity were derived from the Indirect Bias Estimation Method. The differences between the calculated and measured spatial-decay constants were more or less at the same extent of experimental errors. These results show that the accuracy of subcriticality estimation of MCNP 4A and JENDL-3.2 ranges within the uncertainty which can be achieved by the exponential experiment. The differences between calculated and measured prompt neutron decay constants derive significant biases in calculated reactivity. The subcriticalities were estimated by using the effective multiplication factors adjusted based on these biases in calculated reactivity.

KEYWORDS: subcriticality, indirect bias estimation method, experimental data, calculation errors, TCA reactor, exponential experiment, pulse neutrons, spatial-decay constants, prompt neutron time-decay constants, accuracy, MCNP 4A, JENDL-3.2

I. INTRODUCTION

In the accompanying paper(1), the Indirect Bias Estimation Method was proposed to estimate a bias in calculated reactivity. This method derives a bias in an unmeasurable value such as reactivity from calculation error of a measurable value. In Ref.(1), as measurable values, neutron count rate distribution, prompt neutron time-decay constant, and spatial-decay constant were chosen. Corresponding to those three measurable values, three formalisms for estimating biases in calculated reactivity were given. A preliminary applications using neutron count rate distribution were presented in Ref.(2).

To ascertain the applicability of the Indirect Bias Estimation Method, the authors performed subcritical experiments at light-water moderated/reflected low-enriched UO₂ lattice core, TCA (Tank-type Critical Assembly)(3) of Japan Atomic Energy Research Institute. The neutron count rate distributions and vertical neutron decay constants were measured for several subcritical cores with an external neutron source placed. Since many critical and subcritical experiments have been conducted at TCA thus far, the authors utilized prompt neutron time-decay constants measured in the past experiment(4) as measured values to be compared with calculation.

As a computer code and a cross section library to be validated against TCA experiment, continuous energy Monte Carlo code, MCNP 4A(5) and JENDL-3.2 cross section library(6) were chosen. MCNP 4A is now being widely used not only for shielding analysis but also for criticality safety analysis because of its versatility. MCNP has a wide array of capability, for example, detector tally calculation, geometry and source description. MCNP can construct a calculation model of subcritical experiment of TCA with less approximations. Fuel rods, measurement devices can almost exactly be modeled. Therefore, the direct comparison between the calculated and measured values such as neutron count rate distributions and vertical-decay constants can easily be made. Furthermore, MCNP has a capability of time-dependent tally calculation, giving the time-decay behavior which follows a pulse generation. Due to this capability, a simulation calculation of pulse neutron experiment is possible.

As an example of the Indirect Bias Estimation Method, the biases in reactivity calculated by MCNP 4A is given in this paper. It is certain that the Indirect Bias Estimation Method can be applied to deterministic codes which calculate vertical decay constant and time-decay constant as eigenvalues. However, deterministic codes usually entail processes of cell-homogenization and group collapsing, which precludes a direct comparison between
measured and calculated values. Thus, an application of the *Indirect Bias Estimation Method* is more relevant for Monte Carlo calculations than for deterministic calculations.

We first show the experiment method at TCA subcritical core assembly in which the vertical decay constants and prompt neutron time-decay constants were measured. The calculation methods and results for MCNP 4A are then shown. The biases in calculated reactivity are obtained using the *Indirect Bias Estimation Method* and the accuracy of MCNP 4A's calculation is discussed.

II. EXPERIMENTS

The Tank-Type Critical Assembly (TCA) is a light-water-moderated critical assembly in which the core is composed of a fuel rod array supported by upper and lower grid plate. The reactor is operated by raising the water level from the bottom of the core tank by a feed water pump. A vertical cross section of the TCA is shown in Fig. 1. The experimental core lattice is constructed in an open-top cylindrical core tank having a diameter of 1.832 m and a height of 2.1 m by holding the fuel rods vertically with a set of grid plates. The experimental core has 1.956 cm center-to-center spacings in square lattice, which corresponds to the water-to-fuel volume ratio, 1.83, and the temperature was 20.3 ± 0.4 °C. The 1.25-cm-diameter UO₂ pellets are clad in an aluminum tube and its enrichment is 2.6 wt. %.

1. Exponential Experiment

The authors conducted exponential experiment[4] for four subcritical configurations of TCA. The horizontal fuel rod patterns are shown in Figs. 2(a) through (d). They are referred to as 17×17−7×7, 17×17−12×12, 17×17−α and 17×17−β, respectively. The 17×17−α and 17×17−β core consist of arraying 141 and 80 fuel rods at random, respectively. Two aluminum void tubes, along which 235U micro-fission counters were scanned, were inserted within and without the fuel rod array. A neutron source of ²⁵²Cf was placed in a perforated aluminum tube at the elevation of the bottom of the fuel pellets. The light-water level was 100 cm from the bottom of the fuel pellets throughout the experiments. Vertical neutron count rate distributions were measured at every 5 cm elevation by scanning ²³⁵U micro fission counter in the aluminum void tube A (see Fig. 2). The vertical decay constants were obtained by fitting the measured count rates to the exponential function.

2. Pulse Neutron Experiment

Suzaki[4] measured prompt neutron time-decay constants with the n×n (n=7,9,11,13,15,17) square lattices of TCA. The arrangement of the pulse neutron experiment is shown in Fig. 3. The light-water level was 122.5 cm. The 17×17 core was critical configuration and the remainders were subcritical. Neutrons generated by the D-T reaction were supplied to the core with a pulse width of about 3.5 μs, and a repetition rate of about 1 to 3 Hz, and a ²³⁵U micro fission counter located in
the reflector region measured the time-decay of neutrons. To obtain the fundamental-mode time-decay constant of prompt neutrons \( \alpha \) from the observed time-behavior of neutron counts, the measured neutron counts were fitted to the exponential function.

III. CALCULATIONS

Two subcritical experimental methods (pulse neutron and exponential method) described in the previous chapter were simulated using the continuous energy Monte Carlo code MCNP 4A. The continuous energy cross section library for MCNP, FSXLIB-J3R2(6), which was generated from JENDL-3.2 were used for these calculations.

1. Exponential Experiment

Using MCNP 4A, criticality calculations and fixed-source calculations were performed for four core configurations as shown in Figs. 2(a) through (d). The neutron count rate distributions were calculated using the fixed-source option of MCNP 4A. The actual microfission counter used for the experiment had a diameter of 0.62 cm and a active length of 2.5 cm. However, the objective of these calculations is not to obtain the absolute value of neutron flux, but to obtain the vertical decay constant of the neutron count rate. The detector cell for calculation was defined as cylinder having a diameter of 1.27 cm and a length of 5 cm, and was located inside the aluminum void tube A. This diameter is the same as the inner diameter of the aluminum void tube. The volume of this detector cell is approximately eight times larger than that of the actual fission counter so that the statistical uncertainty of calculated count rate can be reduced. Such an enlarged detector cell for calculation does not affect the accuracy of calculated vertical decay constants. The calculated neutron count rate was obtained by taking the inner product of the cross section for fission reaction of \(^{235}\text{U}\), \( \sigma_f \), and the neutron spectrum in the detector cell, \( \phi(E) \):

\[
\int \phi(E)\sigma_f(E)\,dE.
\]  

The neutron flux in the cell detector was obtained using the track length estimator. It took approximately one day as the elapsed computation time on Sun SPARC Station 20 to reduce the standard deviations of calculated count rates less than 2%.

2. Pulse Neutron Experiment

MCNP 4A can obtain a time-dependent behavior of neutron counts by dividing the tally by the time bin width. In a pulse neutron experiment, a decay of prompt neutrons are measured. MCNP 4A also produces the prompt fission neutrons only unless the criticality calculation option is used. Since the pulse neutron method measures the decay constant of fundamental mode flux, its simulation calculation depends on neither the location nor the efficiency of a neutron counter. Therefore, a whole core region can be taken as the neutron counter, and the source emitting point can be located at the core center regardless of an actual experimental arrangement. Since such a counter region is symmetric with respect to the source point, the asymmetric higher-order mode flux which obscures the fundamental mode flux does not appear in the calculated neutron counts. Furthermore, due to the large volume of the counter region, a larger number of neutron counts can be accumulated within the counter and the statistical uncertainty can be reduced.

In the simulations of the pulse neutron experiment, 14 MeV neutrons were emitted isotropically and the subsequent neutron decay up to 3 ms within the whole fuel rod array were observed by accumulating the \(^{235}\text{U}\) fission reaction rates defined by Eq. (1) into time-bin of 20 or 25 \( \mu \)s width each for \( n \times n \) \((n=7,9,11,13,15,17)\) fuel rod arrays. The particle weighting and biasing in MCNP were not used in order to have a strictly analog Monte Carlo calculation that follows physically the actual particle random walks. The elapsed computation time on SUN SPARC Station 20 ranged from 5 to 8 days, depending on the subcriticality. Since neutron counts in a large subcritical core decay faster, a large subcritical core requires longer computation time to maintain the accuracy of the neutron counts in each time-bin.

IV. RESULTS AND DISCUSSION

1. Exponential Experiment

The measured and calculated vertical neutron count distributions for \( 17 \times 17 - 12 \times 12 \) are shown in Fig. 4.
Fitting the measured and calculated count rate distributions to the exponential function, the vertical decay constants were obtained. The fittings were made between 15 and 60 cm from the bottom of the cores to remove the "end-effect" near water surface and the transient effects near the neutron source. The measured vertical decay constant \( \gamma_{z,m}^2 \) and calculated vertical decay constant \( \gamma_{z,c}^2 \) are shown in Table 1 with their standard deviations. The effective multiplication factor \( k_e \) obtained by MCNP criticality calculation are shown in Table 1. In the last column of Table 1, \( k_m \) shows the effective multiplication factor obtained by

\[
\rho_m = 1 - \frac{1}{k_m} = -K(\gamma_{z,m}^2)(\gamma_{z,m}^2 + B_z^2),
\]

where \( K(\gamma_{z,m}^2) \): Buckling-reactivity conversion factor as a function of \( \gamma_{z,m}^2 \) (defined in Eq.(33) in Ref.(1))

\[
B_z^2 = \left( \frac{\pi}{H + \lambda_z} \right)^2
\]

\( H \): Water level (100 cm)

\( \lambda_z \): Vertical extrapolation length (12.2±0.3 cm(3)).

The biases and their uncertainties \( \sigma(\rho_m-\rho_c) \) are shown in Table 2. The definition of \( \sigma(\rho_m-\rho_c) \) is given by Eq.(37) in Ref.(1). The \( \sigma_m(\rho_m-\rho_c) \) and \( \sigma_c(\rho_m-\rho_c) \) arise from the uncertainties of measured and calculated \( \gamma_{z,m}^2 \), respectively. As seen in Table 2, the biases \( \rho_m-\rho_c \) is more or less at the same extent of experimental uncertainties \( \sigma_m(\rho_m-\rho_c) \). This shows that the accuracy of MCNP calculation ranges within the precision which can be achieved by the exponential experiment.

### 2. Pulse Neutron Experiment

The calculated time-decay of \( ^{235}\text{U} \) fission reaction rate for \( n=9, 15, 17 \) are shown in Figs. 6(a), (b) and (c), respectively. As shown in Fig. 6(a), the influence of the higher-order modes can be observed until 0.4 ms. Naturally, as time elapses, the statistical uncertainty of the neutron counts in each time-bin becomes larger. On the other hand, the influence of higher-order modes is not significant for \( n=15 \). Although time elapses, the statistical uncertainties of count rates for \( n=15 \) is kept lower than a large subcritical core such as \( n=7,9 \). Therefore if the \( \gamma_{z,m}^2 \) value of a fuel rod array is identical with that of another fuel rod array, \( K \) values are also identical with each other. That is, the function \( K(\gamma_{z,m}^2) \) obtained for right square configurations can be applied to non-right square ones in this experiment(17). The dependence of \( K \) value on \( \gamma_{z,m}^2 \) is shown in Fig. 5, and \( K \) values of four fuel rod patterns are listed in Table 1.

\[
\rho_m - \rho_c = -K(\gamma_{z,m}^2)\gamma_{z,m}^2 + K(\gamma_{z,c}^2)\gamma_{z,c}^2 + B_z^2(K(\gamma_{z,c}^2) - K(\gamma_{z,m}^2)).
\]

The biases and their uncertainties \( \sigma(\rho_m-\rho_c) \) are shown in Table 2. The definition of \( \sigma(\rho_m-\rho_c) \) is given by Eq.(37) in Ref.(1). The \( \sigma_m(\rho_m-\rho_c) \) and \( \sigma_c(\rho_m-\rho_c) \) arise from the uncertainties of measured and calculated \( \gamma_{z,m}^2 \), respectively. As seen in Table 2, the biases \( \rho_m-\rho_c \) is more or less at the same extent of experimental uncertainties \( \sigma_m(\rho_m-\rho_c) \). This shows that the accuracy of MCNP calculation ranges within the precision which can be achieved by the exponential experiment.
fore, an accurate calculation for fundamental-mode decay constant becomes difficult, as the subcriticality becomes larger. The fundamental-mode decay constants for each core were obtained by fitting the count rates during the time intervals which do not include higher-order mode and unacceptable statistical uncertainties.

Since the neutron source emitting point is almost symmetric with respect to the active region of the fuel rod array, the asymmetric higher-order mode is not supposed to be included in the count rate. The higher-order mode effect observed in Fig. 6(a) is due to the symmetric mode as shown in Fig. 7. To examine the higher-order mode in the core of \( n = 9 \), two horizontal counters having a thickness of 5 cm across the fuel rod array were located at the point of the maximum and minimum flux. The calculated time-decay in Counter A and B are shown in Figs. 8(a) and (b). Obviously, the time-decays are expressed by adding the higher-order mode to the fundamental mode.

The measured prompt neutron decay constants \( \alpha_m \)'s taken from Ref.(4) and calculated decay constants \( \alpha_c \)'s are shown in Table 3. Since the error in \( \alpha_m \) is unknown except \( n = 17 \), the error in \( \alpha_m \) was assumed to be 0.5% of \( \alpha_m \). The biases in calculated reactivity \( (\rho_m - \rho_c) \) are shown in Table 4 and these values are obtained by the following equation (Eq.(29) in Ref.(1)):

\[
\rho_m - \rho_c = -\alpha_m A_c(\alpha_m) + \alpha_c A_c(\alpha_c) + \beta_{\text{eff},c}(\alpha_m) - \beta_{\text{eff},c}(\alpha_c),
\]

Table 2 Biases in calculated reactivity and their uncertainties for exponential experiment

<table>
<thead>
<tr>
<th>Rod pattern</th>
<th>( \rho_m - \rho_c )</th>
<th>( \sigma_m (\rho_m - \rho_c) )</th>
<th>( \sigma_c (\rho_m - \rho_c) )</th>
<th>( \sigma (\rho_m - \rho_c) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>17×17‐7×7</td>
<td>2.61×10⁻³</td>
<td>9.3×10⁻⁴</td>
<td>1.22×10⁻³</td>
<td>1.53×10⁻³</td>
</tr>
<tr>
<td>17×17‐12×12</td>
<td>1.91×10⁻²</td>
<td>1.02×10⁻²</td>
<td>5.6×10⁻³</td>
<td>1.16×10⁻²</td>
</tr>
<tr>
<td>17×17‐α</td>
<td>2.29×10⁻³</td>
<td>1.51×10⁻³</td>
<td>1.66×10⁻³</td>
<td>2.24×10⁻³</td>
</tr>
<tr>
<td>17×17‐β</td>
<td>−1.03×10⁻²</td>
<td>1.57×10⁻²</td>
<td>8.3×10⁻³</td>
<td>1.78×10⁻²</td>
</tr>
</tbody>
</table>

Fig. 6 Calculated time-decay behaviors

Fig. 7 Schematic of counter location

Counter A and B are located at the point of the maximum and minimum flux, respectively.
Accurate Estimation of Subcriticality Using Indirect Bias Estimation Method, (II)

In Ref. (4), $L_c(a)$ and $\beta_{eff,c}(\alpha)$ values were calculated for several $n \times n$ ($n=3$ to 17) fuel rod arrays of TCA using CITATION and 4-group constants. Then, these $L_c(a)$ and $\beta_{eff,c}(\alpha)$ values were interpolated or extrapolated by polynomials as a function of $\alpha$. The function $L_c(a)$ is shown in Fig. 9. The uncertainties of $(\rho_m-\rho_c)$ defined in Eq. (36) in Ref. (1) are also shown in Table 4.

The effective multiplication factor $k_e$ obtained by MCNP criticality calculation is shown in Table 4. The adjusted effective multiplication factor $k_{adj}$ is defined as

$$1 - \frac{1}{k_{adj}} = (\rho_m - \rho_c) + 1 - \frac{1}{k_e}. \quad (5)$$

The adjusted $k_{adj}$ for each fuel rod array is shown in Table 4. Whereas a true $k_{eff}$ is never known for subcritical system, the true $k_{eff}$ for a critical system is always unity. The validation of the Indirect Bias Estimation Method can be made by determining whether an

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**Table 3** Measured and calculated prompt neutron decay constant and prompt neutron generation time

<table>
<thead>
<tr>
<th>Rod pattern</th>
<th>$\alpha_m$ (s$^{-1}$)</th>
<th>$\alpha_e$ (s$^{-1}$)</th>
<th>$A_+$ ($\alpha_m$) (s)</th>
<th>$A_+$ ($\alpha_e$) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7x7</td>
<td>4.156±21</td>
<td>4.303±32</td>
<td>1.23×10$^{-4}$</td>
<td>1.50×10$^{-4}$</td>
</tr>
<tr>
<td>9x9</td>
<td>3.827±19</td>
<td>3.923±8</td>
<td>9.43×10$^{-5}$</td>
<td>1.01×10$^{-5}$</td>
</tr>
<tr>
<td>11x11</td>
<td>3.336±17</td>
<td>3.313±7</td>
<td>7.45×10$^{-5}$</td>
<td>7.39×10$^{-5}$</td>
</tr>
<tr>
<td>13x13</td>
<td>2.550±13</td>
<td>2.350±4</td>
<td>5.64×10$^{-5}$</td>
<td>5.31×10$^{-5}$</td>
</tr>
<tr>
<td>15x15</td>
<td>1.375±7</td>
<td>1.224±2</td>
<td>4.59×10$^{-5}$</td>
<td>4.50×10$^{-5}$</td>
</tr>
<tr>
<td>17x17</td>
<td>197±1</td>
<td>109±1</td>
<td>3.84×10$^{-5}$</td>
<td>3.82×10$^{-5}$</td>
</tr>
</tbody>
</table>

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**Table 4** Biases in calculated reactivity and adjusted $k_{eff}$ for pulse neutron experiment

<table>
<thead>
<tr>
<th>Rod pattern</th>
<th>$\rho_m-\rho_c$</th>
<th>$\sigma$ ($\rho_m-\rho_c$)</th>
<th>$k_e$</th>
<th>$k_{adj}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7x7</td>
<td>1.23×10$^{-1}$</td>
<td>3.0×10$^{-3}$</td>
<td>0.609±0.002</td>
<td>0.658±0.013</td>
</tr>
<tr>
<td>9x9</td>
<td>3.53×10$^{-2}$</td>
<td>5.5×10$^{-3}$</td>
<td>0.727±0.002</td>
<td>0.746±0.003</td>
</tr>
<tr>
<td>11x11</td>
<td>-3.80×10$^{-3}$</td>
<td>2.16×10$^{-3}$</td>
<td>0.820±0.002</td>
<td>0.818±0.003</td>
</tr>
<tr>
<td>13x13</td>
<td>-1.92×10$^{-2}$</td>
<td>1.0×10$^{-3}$</td>
<td>0.892±0.002</td>
<td>0.877±0.003</td>
</tr>
<tr>
<td>15x15</td>
<td>-8.05×10$^{-3}$</td>
<td>2.1×10$^{-4}$</td>
<td>0.954±0.002</td>
<td>0.947±0.002</td>
</tr>
<tr>
<td>17x17</td>
<td>-3.40×10$^{-3}$</td>
<td>5×10$^{-5}$</td>
<td>1.003±0.002</td>
<td>1.000±0.002</td>
</tr>
</tbody>
</table>

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Fig. 8 Calculated time-decay behaviors at Counter A and B

Fig. 9 $A$ vs. $\alpha$
adjusted $k_{adj}$ for a critical system becomes closer to unity. It is shown in Table 4 that the adjusted $k_{adj}$ for critical system ($n=17$) becomes almost unity. This suggests that the Indirect Bias Estimation Method is capable of estimating a $k_{eff}$ which is closer to a true value. The bias in calculated reactivity ($\rho_m - \rho_c$) is relatively large for large subcritical systems such as $n=7,9$. This is partly because $A_c(\alpha)$ is sensitive to $\alpha$ for a large $\alpha$ value. The results in Table 4 indicate that the combination of MCNP 4A and JENDL-3.2 overestimates $k_{eff}$ for $n=11,13,15,17$ and, on the other hand, underestimates for $n=7,9$. That is, the bias in calculated reactivity of MCNP 4A and JENDL-3.2 depends on the subcriticality.

V. CONCLUSIONS

The Indirect Bias Estimation Method has been applied to the exponential experiment and pulse neutron experiment. These experiments conducted at TCA were simulated using continuous energy Monte Carlo code MCNP 4A. The biases in calculated reactivity obtained from the Indirect Bias Estimation Method were presented for both experimental methods. The bias obtained from the exponential experiment was more or less at the same extent of the experimental error. It is concluded that the accuracy of MCNP calculation falls within the uncertainty which can be achieved by the exponential experiment. However, significant differences between the calculated and measured prompt neutron decay constants were observed. Thus, the adjusted effective multiplication factors were obtained from the biases of the calculated reactivity and the calculated effective multiplication factors. The sensitivity of a prompt neutron decay constant $\alpha$ to prompt neutron generation time becomes large, as $\alpha$ becomes large, i.e., the subcriticality becomes large. That is, a calculated bias or adjusted effective multiplication factor for a large subcritical core considerably depends on the accuracy of prompt neutron generation time. Nevertheless, the effect of the uncertainty of generation time on the bias is relatively small, if $\alpha_c$ is close to $\alpha_m$. The adjusted effective multiplication factor for a critical core becomes almost unity, which shows the relevance of the Indirect Bias Estimation Method.

---REFERENCES---