Effect of Resonance Scattering of Sodium on Resonance Absorption of U-238

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The interference effects of the resonance scattering of Na on the resonance absorption of $^{238}$U were investigated. The results for self-shielding factor obtained from exact treatment were compared with those from the usual conventional method, as well as with results based on the assumption that $\phi(u)\sigma_i=\text{const}$, ($\phi(u)$: flux, $\sigma_i$: total cross section).

In the exact treatment, the neutron spectrum used is obtained with accurate treatment of the slowing down by the elastic scattering of light and medium elements. In the $\phi(u)\sigma_i=\text{const.}$ assumption, the energy dependence of the Na resonance scattering cross section is taken into account as in the exact treatment, whereas only the averaged value is used in the conventional method.

Self-shielding factors and their temperature dependence were calculated for several compositions and energy regions. It was found that the conventional method is not satisfactory, while the assumption that $\phi(u)\sigma_i=\text{const.}$ compares well with the exact treatment.

I. INTRODUCTION

In a large fast reactor, the Doppler effect of the neutron cross section of heavy elements is very important in view of its property of counteracting the positive sodium-void reactivity effect, especially in the energy region between several hundreds eV and several keV, which contributes a large part of the total Doppler reactivity effect(1). In this energy region, there is a large resonance scattering of Na near 3 keV where the neutron spectrum is strongly depressed.

There have been published many sets of multigroup cross sections for the fast reactor, but their dependence on the composition of the system has not been fully and precisely considered: The effective cross sections have not been calculated on the basis of the exact spectra, which vary with composition. For example, for preparing a set of group-effective cross sections such as the HR(2)(3) or the Russian set(4)(5) the Fermi or $1/E$ spectrum is used1, with the depression of spectrum due to resonances taken into account by $1/(E\sigma_i)$ assumption. This assumption is only valid when the collision density is weakly energy-dependent:

$$E\sigma_i(E)\phi(E)=\sigma_i\phi(u)=\text{const.}$$  \hspace{1cm} (1)

Besides, the $\sigma_i(E)$ energy dependence is taken into account only on the element or nuclide in question — such as $^{238}$U, and all other cross sections are assumed constant throughout the energy region envisaged. These assumptions are not necessarily valid, particularly in the cases of large resonance such as that of Na. The method based on those assumptions — which we might term conventional — would appear to call for some examination on its accuracy in the energy regions where there is a large resonance of Na.

In the present treatment, the self-shielding factor of $^{238}$U and the Doppler effect thereof, that is, the gradient of the self-shielding factor with respect to temperature, have been studied with consideration given to the effect of Na resonance scattering. The spectrum was obtained with the use of the ESELEM code(6)(7), in which the slowing down by the elastic scattering of the light- and medium-weight elements is treated precisely. The results from this exact treatment are compared with those from the conventional method. The effect of assuming $\phi(u)\sigma_i=\text{const.}$ is examined by comparing the resulting values with those obtained from the exact treatment. Both treatments are the same except for this particular assumption. In both the exact treatment and the method assuming $\phi(u)\sigma_i=\text{const.}$, * Japan Atomic Energy Research Institute, Tokaimura, Ibaraki-ken.
the energy dependence of the Na resonance scattering cross sections is taken into account, whereas only the average value is used in the conventional method, as already mentioned.

The energy region covered was limited to 1.0~3.94 keV, with the view to restricting our consideration to the resolved region.

The system for which fine-spectrum calculations were performed is as follows: the core comprises 30% of fuel(239PuO2-238UO2), 50% of sodium before sodium loss and 20% of iron, and the blanket comprises 50% of fuel (238UO2), 30% of sodium and 20% of iron.

II. NUMERICAL METHOD

Within the energy interval $dE$ extending from $E_1$ to $E_2$, the effective absorption cross section of $^{238}$U is defined by

$$\dot{\sigma} = \frac{\int_{E_1}^{E_2} \sigma(E) \phi(E) dE}{\int_{E_1}^{E_2} \phi(E) dE}, \quad (2)$$

where $\sigma$ is the absorption cross section of $^{238}$U, given by

$$\sigma = \sum \gamma \frac{\Gamma_\gamma}{\Gamma_\gamma} \phi(\gamma, \psi) + \sigma_{\text{non}}. \quad (3)$$

Equation (2) was calculated for three cases with different extents of assumption adopted for neutron spectrum:

1. Case in which the Fine Spectrum obtained from the ESELEM Calculation is used

In this case, it is assumed that

$$\phi(E) = \frac{1}{\Delta \mu} \int_{\Delta \mu} \phi(u) du, \quad \text{for } E \Delta \mu \in [E_{g-1}, E_g], \quad (4)$$

where the suffix $g$ represents a fine group in the ESELEM calculation, that has equal lethargy width 0.00995, $\phi_g$ is the averaged flux of $g$-th group obtained from ESELEM, $\sigma_{\text{tot}}$ the averaged total cross section, given by

$$\sigma_{\text{tot}} = \frac{\int \sum \gamma \frac{\Gamma_\gamma}{\Gamma_\gamma} \phi(\gamma, \psi) + \sigma_{\text{non}}}{\int \phi(E) dE} \quad (5)$$

where $\sigma(1)$ and $\sigma(2)$ are respectively, the averaged total cross section of $^{238}$U and Na of the $g$-th group, quoted from the ESELEM library, while we define

$$R = N_{\text{Na}}/N_x, \quad S = \frac{\sum N_x \sigma_{\text{Na}}}{N_x} \quad (6)$$

and, similarly

$$\sigma_{\text{tot}} = \sum \gamma \frac{\Gamma_\gamma}{\Gamma_\gamma} \phi(\gamma, \psi) + \sigma_{\text{non}} + \sigma + R \sigma_{\text{Na}} + S. \quad (7)$$

Then, the denominator of Eq.(2)

$$\approx \sum \int_{E_{g-1}}^{E_g} \phi(E) dE,$$

and the numerator of Eq.(2)

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3. Infinite Dilution

The effective cross section for infinite dilution is given by

$$\sigma_{\text{eff}} = \frac{1}{T} \sum E_i \ln\left(\frac{E_i}{E_0}\right).$$

(19)

The self-shielding factor

$$f_i = \frac{\sigma_i}{\sigma_{\text{eff}}}$$

by the exact treatment, and

$$f_0 = \frac{\sigma_0}{\sigma_{\text{eff}}}$$

by the conventional method.

III. RESULTS AND DISCUSSIONS

In all the calculations, the resonance parameters of $^{238}\text{U}$ have been quoted from Ref. (8), and cross sections from the library for the ESELEM code. Two energy regions are considered—one from 2.15 to 3.94 keV (Region I) containing the Na resonance, and the other from 1.0 to 2.15 keV (Region II). The upper energy of the former was chosen for the reason that above it there are no resolved parameters in Ref. (8), and the energy interval of the latter corresponds to that of 14th group of the Russian set. The latter region containing no Na resonance was adopted for comparison with the value of the present typical set of group constants.

In Fig. 1 are presented the total cross section of Na and the spectra calculated for full and half sodium cores near the 3 keV Na resonance. The dip of the spectrum due to Na resonance is very large.

The results of calculation of the self-shielding factors $f_i$ and $f_0$ are shown in Figs. 2 and 3, where curves based upon the correspond-
ing Russian values are also shown for reference. In the energy regions of both Figs. 2 and 3, \( f_1 < f_0 \), and the temperature coefficients, that is, the Doppler coefficients, of both \( f_1 \) and \( f_0 \) are smaller than that of the Russian set.

The value of \( f_0 \) in Fig. 3 is in reasonable accord with the Russian set, in reference to the applicable values of \( \sigma_0 \), defined by

\[
\sigma_0 = \sigma_{\text{mono}} + \sigma_r + S + R \sigma_{\text{Na}}
\]  

(21)

It should be noted that the method used in calculating \( f_0 \) is the same as that of the Russian set.

On the other hand, the value of \( f_0 \) in the Fig. 2 deviates considerably from the Russian set, but such direct comparison is not appropriate in view of the difference in applicable energy regions.

In the energy region I, the Doppler coefficient of \( f_1 \) is larger than \( f_0 \), while it is the reverse in Region II, and the differences increase with \( \sigma_0 \). Therefore, if only the effect due to \(^{238}\text{U}\) is considered, using \( f_1 \) should provide the system with a safer temperature coefficient than using \( f_0 \) to obtain the Doppler coefficient, because as explained earlier, Regions I and II contribute predominantly to the total Doppler effect, and the difference between \( f_1 \) and \( f_0 \) is larger in Region I than in Region II, and besides the flux in Region I is higher than in II.

In calculating \( f_0 \), an exact spectrum is necessary, but it is troublesome to obtain it for every case. As a first approximation, the authors have assumed that \( \phi(u)\sigma_1 \) is constant, that is, \( \phi(u)\sigma_1 = \text{const.} \) in Eqs. (4), (8) and (10). However, it should be noted that the energy dependence of the resonance scattering cross section of Na is taken into account. In Tables 1 and 2 are given \( f_1(\text{ESE.}) \) obtained with the ESELEM and \( f_1(\phi\sigma_1=\text{c}) \) with the assumption of constant \( \phi(u)\sigma_1 \).

In Region I (See Table 1), there is almost no difference between the two values of \( f_1 \), and in Region II \( f_1(\phi\sigma_1=\text{c}) \) is slightly larger than \( f_1(\text{ESE.}) \), but the Doppler coefficient is almost the same for both self-shielding factors. The values of \( \phi(u)\sigma_1 \) are plotted against energy in Fig. 4. It is seen that while \( \phi(u)\sigma_1 \) varies appreciably with energy, the assumption \( \phi(u)\sigma_1 = \text{const.} \) is valid.

![Fig. 4 Variations of \( \phi\sigma_1 \)](image)

**Table 1** Comparison of Self-shielding Factors obtained with Accurate Spectrum and with \( \phi(u)\sigma_1 = \text{const.} \) in the Range 2.15–3.94 keV (Region I)

<table>
<thead>
<tr>
<th>Composition</th>
<th>Core (full Na)</th>
<th>Core (half Na)</th>
<th>Blanket</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R(\text{cf. Eq. (6)}) )</td>
<td>1.8846</td>
<td>0.9423</td>
<td>0.5846</td>
</tr>
<tr>
<td>( T(\text{oK}) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>( f_1(\text{ESE.}) )</td>
<td>0.73_2</td>
<td>0.73_2</td>
</tr>
<tr>
<td></td>
<td>( f_1(\phi\sigma_1=\text{c}) )</td>
<td>0.69_4</td>
<td>0.70_1</td>
</tr>
<tr>
<td>900</td>
<td>( f_1(\text{ESE.}) )</td>
<td>0.81_2</td>
<td>0.82_4</td>
</tr>
<tr>
<td></td>
<td>( f_1(\phi\sigma_1=\text{c}) )</td>
<td>0.77_8</td>
<td>0.78_3</td>
</tr>
<tr>
<td>2,100</td>
<td>( f_1(\text{ESE.}) )</td>
<td>0.88_4</td>
<td>0.88_4</td>
</tr>
<tr>
<td></td>
<td>( f_1(\phi\sigma_1=\text{c}) )</td>
<td>0.84_8</td>
<td>0.84_8</td>
</tr>
<tr>
<td>3,94</td>
<td></td>
<td>0.79_9</td>
<td>0.80_0</td>
</tr>
</tbody>
</table>

\( f_1(\text{ESE.}) \): The spectrum obtained from the ESELEM calculation is used.

\( f_1(\phi\sigma_1=\text{c}) \): \( \phi(u)\sigma_1 \) is constant.
Table 2 Comparison of Self-shielding Factors obtained with Accurate Spectrum and with $\phi(u)\sigma_i=$const. in the Range $1.0 \sim 2.15$ keV (Region II)

<table>
<thead>
<tr>
<th>Composition</th>
<th>Core (full Na)</th>
<th>Blanket</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(cf. Eq.(6))$</td>
<td>1.8846</td>
<td>0.5846</td>
</tr>
<tr>
<td>$T(\degree K)$</td>
<td>$f_i(ESE_\cdot)$</td>
<td>$f_i(\phi\sigma_i=\phi)$</td>
</tr>
<tr>
<td>300</td>
<td>0.579</td>
<td>0.585</td>
</tr>
<tr>
<td>900</td>
<td>0.631</td>
<td>0.657</td>
</tr>
<tr>
<td>2,100</td>
<td>0.685</td>
<td>0.718</td>
</tr>
</tbody>
</table>

$f_i(ESE_\cdot)$: The spectrum obtained from the ESELEM calculation is used.
$f_i(\phi\sigma_i=\phi)$: $\phi(u)\sigma_i$ is constant.

The fact that $f_0$ differs considerably from $f_i$. In practice, the $1/E$ spectrum cannot be realized in a fast reactor. Another shortcoming of the conventional method is that the energy dependence of the Na cross section is not considered in calculating $f_0$.

In a region such as I where there is a large and wide resonance, the self-shielding factor and its Doppler coefficient may vary according to the upper or lower limits of energy of a group. To examine this effect, the authors divided Region I at the peak of the Na resonance into two subregions I-1 and I-2. The resulting $f_i(ESE_\cdot)$, $f_i(\phi\sigma_i=\phi)$ and $f_0$ of each region are as given in Tables 3 and 4. In Region I-1, $f_0>f_i$, as it was for the whole region I, and the Doppler coefficient is almost the same for $f_0$ as for $f_i(ESE_\cdot)$. In Region I-2 $f_0<f_i$ but for one exception of the blanket composition at 300$\degree$K, while the Doppler coefficient is considerably smaller for $f_0$ than for $f_i$.

### Table 3 Comparison of Self-shielding Factors obtained with Different Calculations in the Range 2.98~3.94 keV (Region I-1)

<table>
<thead>
<tr>
<th>Composition</th>
<th>Core (full Na)</th>
<th>Blanket</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(cf. Eq.(6))$</td>
<td>1.8846</td>
<td>0.5846</td>
</tr>
<tr>
<td>$T(\degree K)$</td>
<td>$f_i(ESE_\cdot)$</td>
<td>$f_i(\phi\sigma_i=\phi)$</td>
</tr>
<tr>
<td>300</td>
<td>0.784</td>
<td>0.798</td>
</tr>
<tr>
<td>900</td>
<td>0.833</td>
<td>0.844</td>
</tr>
<tr>
<td>2,100</td>
<td>0.867</td>
<td>0.877</td>
</tr>
</tbody>
</table>

$f_i(ESE_\cdot)$: The spectrum obtained from the ESELEM calculation is used.
$f_i(\phi\sigma_i=\phi)$: $\phi(u)\sigma_i$ is constant.
$f_0$: The conventional method. The value in parenthesis is $\sigma_0$ (cf. Eq.(21)).

### Table 4 Comparison of Self-shielding Factors obtained with Different Calculations in the Range 2.15~2.98 keV (Region I-2)

<table>
<thead>
<tr>
<th>Composition</th>
<th>Core (full Na)</th>
<th>Blanket</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(cf. Eq.(6))$</td>
<td>1.8846</td>
<td>0.5846</td>
</tr>
<tr>
<td>$T(\degree K)$</td>
<td>$f_i(ESE_\cdot)$</td>
<td>$f_i(\phi\sigma_i=\phi)$</td>
</tr>
<tr>
<td>300</td>
<td>0.681</td>
<td>0.679</td>
</tr>
<tr>
<td>900</td>
<td>0.792</td>
<td>0.789</td>
</tr>
<tr>
<td>2,100</td>
<td>0.888</td>
<td>0.874</td>
</tr>
</tbody>
</table>

$f_i(ESE_\cdot)$: The spectrum obtained from the ESELEM calculation is used.
$f_i(\phi\sigma_i=\phi)$: $\phi(u)\sigma_i$ is constant.
$f_0$: The conventional method. The value in parenthesis is $\sigma_0$ (cf. Eq.(21)).
(ESE.). On the other hand, \( f_1(\phi \sigma_i = \psi) \) roughly agrees with \( f_1(\text{ESE.}) \) in this Region I-2, while even in Region I-1 the difference between the two is not large compared to the disagreement with \( f_0 \). The difference in Doppler coefficient between \( f_1(\phi \sigma_i = \psi) \) and \( f_1(\text{ESE.}) \) is moreover negligible in both regions.

IV. CONCLUSIONS

The interference effect of the resonance scattering of Na on the resonance absorption of \(^{238}\text{U}\) has been investigated. The results from the exact treatment have been compared with those from the conventional method and from the assumption of \( \phi(\mu) \sigma_i = \text{const.} \). The principal conclusions drawn are as follows:

The values of \( f_0 \) deviate considerably from \( f_1 \), and in the energy region that includes the resonance of Na, the Doppler coefficient also is considerably larger for \( f_1 \) than for \( f_0 \).

On the other hand, the disagreement between \( f_1(\phi \sigma_i = \psi) \) and \( f_1(\text{ESE.}) \) is at most quite tolerable, if not negligible irrespectively of energy region, while the Doppler coefficient is almost the same for \( f_1(\phi \sigma_i = \psi) \) as for \( f_1(\text{ESE.}) \). Thus the approximation obtained with the assumption \( \phi(\mu) \sigma_i = \text{const.} \) can be considered to be better than the conventional method, which means that the energy dependence of the Na resonance cross section should not be ignored.

In so far as only the effect due to \(^{238}\text{U}\) is considered, a system would be designed more safely with the use of \( f_1 \) instead of \( f_0 \).

In this paper, we have treated only the resolved region of \(^{238}\text{U}\). It will be a problem for further study to deal with the unresolved region and with the effect of fissile material.

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[REFERENCE]