SHORT NOTE

An Improved Correction for Effect of Internal Conversion and Gamma Sensitivity of $\beta$-Detectors for $4\pi\beta-\gamma$ Coincidence Measurements on Activated Gold Foil

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In calibrating thermal neutron flux density in terms of the thermal neutron absorption cross section of gold, a major source of error in the final result may be introduced by uncertainties in the absolute determination of the disintegration rate of the induced activity. If the $4\pi\beta-\gamma$ coincidence method is applied to absolute measurement, the uncertainties arise mainly from the statistical counting error and the corrections for the effect of internal conversion and $\gamma$-sensitivity of the $\beta$-detector.

Several papers have been reported(1)~(3) on the application of the $4\pi\beta-\gamma$ coincidence method to high precision determination of Au-foil activity, and the necessary corrections have been treated systematically. These reports reveal that the accuracy of the corrections diminishes in importance when a high $\beta$-counting efficiency is attainable. The systematic error in the corrections, however, becomes appreciable in cases where the activated Au-foil is not thin enough, causing the $\beta$-counting efficiency to be reduced appreciably below unity on account of the self-absorption.

Particularly in measurements of low level thermal neutron fluxes, the limited induced activity available often obliges us to make a compromise between the low accuracy in the corrections due to thick foil and the better counting statistics obtained thereby.

The corrections for internal conversion and $\gamma$-sensitivity of the $\beta$-detector involve estimates of the counting efficiency of the $4\pi$ proportional counter with respect to $\gamma$-rays interacting with the counter wall and with the Au-foil itself and to the internal conversion electrons. In these estimates, of major importance is the evaluation of the probability of monoenergetic electron escape from the Au-foil, and indeed the uncertainty in this evaluation would suggest a limit to the accuracy obtained by the corrections.

We have recently found that the escape probability of monoenergetic electron could be fairly well described for a foil of high atomic number by simple calculations based on the experimental formula for the absorption curve of monoenergetic electrons(4). An extension of this method of calculation to the corrections in question which should provide improved accuracy, is reported in this paper.

In the $4\pi\beta-\gamma$ coincidence measurement, the disintegration rate $N_0$ of $^{198}$Au induced in a Au-foil is related to the observed $\beta$, $\gamma$ and coincidence counting rates $N_\beta, N_\gamma$ and $N_C$, respectively — each corrected for dead times, accidental coincidence rate and backgrounds.

The relationship, neglecting very small contributions from possible $\gamma$-$\gamma$ coincidences, and assuming a simple decay scheme for $^{198}$Au with only the main $\beta$-branch followed by a single $\gamma$-ray in cascade, may be written(5)

$$N_0(1+K) = N_\beta N_\gamma / N_C$$

where

$$K = \frac{1 - \varepsilon_\beta}{1 + \alpha \varepsilon_\beta}$$

while $\varepsilon_\beta$: Efficiency of the $\beta$-detector to $\beta$-ray
$\varepsilon_\gamma$: Efficiency of the $\beta$-detector to $\gamma$-ray
$\varepsilon_\text{ece}$: Efficiency of the $\beta$-detector to internal conversion electrons
$\alpha$: Total internal conversion coefficient of the 0.412 MeV transition

(A correction for the actual complex decay scheme can be separately made, and this can be shown to be negligibly small for $^{198}$Au, if the relevant gates in the $\gamma$-channel are set only to accept the photo-peak(5)).

In Eq.(2), since $\varepsilon_\beta$ can be readily determined with sufficient accuracy from the ratio of $N_\beta$ to $N_C$, while $\alpha$ can be found in references, we are here concerned only with evaluating $\varepsilon_\gamma$ and $\varepsilon_\text{ece}$. In the following calculations, we assume that the induced activity is distributed uniformly in a Au-foil, which is

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further taken to be an infinite slab, and that all the electrons escaping from the foil surface are counted.*

1. Efficiency of the β-detector with Respect to the Internal Conversion Electrons

For the calculation of the escape probability of monoenergetic electrons from a thin foil, we have derived an expression based on an experimental formula for the absorption curve of monoenergetic electrons. The method of calculation has proved to represent with good approximation the β-counting efficiency of a 4π proportional counter to 198Au β-rays in a Au-foil of various thicknesses.

For monoenergetic electrons of energy $E$ emitted from sources distributed uniformly in a thin foil of thickness $t$, the expression for the electron escape probability $P(E, t)$ is written

$$P(E, t) = \frac{1}{2t} \int_0^t dx \left[ T\left(\frac{x}{\cos \theta}\right) \sin \theta d\theta + \int_{\pi/2}^{\pi} \left\{ 1 - \int_{\pi/2}^{\pi/2} T\left(\frac{x}{\cos \theta}\right) \sin \theta d\theta \right\} \cdot T\left(\frac{x}{\cos \theta}\right) \sin \theta d\theta + \int_0^{\pi/2} \left\{ 1 - \int_0^{\pi/2} T\left(\frac{x}{\cos \theta}\right) \sin \theta d\theta \right\} \cdot T\left(\frac{x}{\cos \theta}\right) \sin \theta d\theta \right].$$

Here, the function $T(x)$ represents the fraction of a normally incident, monoenergetic electron beam transmitted by an absorber of thickness $x$, and may be expressed, as proposed by Subba Rao, by

$$T(x) = \frac{1 + \exp(-\mu x)}{1 + \exp(-\mu x_0)}.$$

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2. Efficiency of the β-detector with Respect to γ-rays

The γ-efficiency of the β-detector can be obtained by considering separately the interactions of the γ-rays with counter wall, the counting gas and the Au-foil itself. The contribution from the counting gas, however, is very small and can be safely neglected. Experiments on the γ-efficiency in respect of the counter wall of a typical 4π proportional counter have been discussed in several reports. More recent results obtained by Urquhart would appear to be the most reliable, and which indicate that the wall effect is much smaller than reported previously. For this reason, it was judged that only a rough estimate was necessary for the wall effect in the present case. It then follows from the results for 60Co γ-rays by Urquhart that $\varepsilon_{bg}$(wall) = 0.0005 ± 0.0005 for brass 4π proportional counter and 0.412 MeV γ-rays.

In order to obtain the γ-efficiency $\varepsilon_{bg}$(foil) in respect of interaction with the foil, we first calculate the γ-ray flux with the primary energy in the foil. Then the number of secondary electrons produced can be determined from the cross sections of the photoelectric effect and the Compton scattering. The probability of escape of the secondary electrons from the foil may be evaluated in the same way as described for the internal conversion electrons.

Let $\mu$ be the attenuation coefficient of the 0.412 MeV γ-rays in gold. At a depth $z$ from the foil surface, the γ-ray flux

$$\phi(z) = \frac{c(1 - e^{-\mu z})}{\mu} \left[ -E_l(-\mu z) + E_l(-\mu(t-z)) \right]$$

where $-E_l(-\mu z) = \int_z^{\infty} e^{-\mu s} ds$,

while $t$ is the foil thickness, and $c$ the source

* When the source-supporting film in the 4π proportional counter is thin enough (~100 µg/cm² or less), this assumption may be justified.
intensity per unit volume. Since the Au-foils under consideration are thin enough compared with the mean free path of the $\gamma$-rays in gold, the effect of scattered $\gamma$-rays on the calculated flux is considered to be small enough to be neglected.

The energy distribution $f(E)$ of the secondary electrons is determined by considering the first interactions of the $\gamma$-rays:

$$f(E) = k_1 C(E) + k_2 \delta(E - 0.412).$$  \hfill (7)

Here, $E$ is the energy of the secondary electrons in MeV. The first term represents the energy distribution of the Compton electrons and the second term that of the photoelectrons, expressed by the Dirac delta function. The coefficients $k_1$ and $k_2$ are the normalization constants, so that

$$\int_0^{E_{max}} f(E) dE = 1.$$  \hfill (8)

The function $C(E)$ is dependent upon the relationship between the energy of recoil electrons and the scattering angle $\phi$ of the $\gamma$-rays, as well as upon the Klein-Nishina differential cross section. Then,

$$C(E) = \frac{1}{E_\gamma} \left\{ 1 + \frac{a^2(1 - \cos \phi)}{1 + a(1 - \cos \phi)} \right\},$$  \hfill (9)

and $E = E_\gamma \frac{a(1 - \cos \phi)}{1 + a(1 - \cos \phi)}$, \hfill (10)

where $a = E_\gamma / m c^2$, and $E_\gamma$ is the $\gamma$-ray energy in MeV.

Although the $\gamma$-ray flux calculated from Eq. (6) is not isotropic in angular distribution, we assume an isotropic angular distribution of the secondary electrons on the whole. This assumption will not cause serious error in the final result, since the multiple scattering of electrons undergone before they reach the foil surface is favorable to the assumption.

From the above considerations, the $\gamma$-efficiency of the $\beta$-detector in respect of the foil itself reduces to

$$\varepsilon_{\beta\gamma}(\text{foil}) = \frac{m_{\gamma}}{t_{\gamma} c} \int_0^{E_{max}} \int_0^{\pi/2} \phi(\theta) f(E) dE d\theta,$$  \hfill (11)

where $t_{\gamma}$ is the total macroscopic cross section of gold for the 0.412 MeV $\gamma$-rays. The escape probability $q(E, z)$ of electrons of energy $E$ produced at a depth $z$ can be expressed in the same way as Eq. (3):

$$q(E, z) = \frac{1}{2} \int_0^{\pi/2} T(\frac{z}{\cos \theta}) \sin \theta d\theta$$
$$+ \int_{\pi/2}^{\pi} \left\{ 1 - \int_0^{\pi/2} T(\frac{z}{\cos \theta}) \sin \theta d\theta \right\}$$
$$\cdot T(\frac{z}{\cos \theta}) \sin \theta d\theta + \int_0^{\pi/2} \left\{ 1 - \int_{\pi/2}^{\pi} T(\frac{z}{\cos \theta}) \sin \theta d\theta \right\}.$$

Numerical integration of Eqs. (5) and (11) has been carried out for Au-foils of various thicknesses. Substituting the calculated results of $\varepsilon_{\beta\gamma}$ and $\varepsilon_{\beta\tau}$ and the experimentally determined values of $\varepsilon_{\beta\tau}$ into Eq. (2), the correction $K$ has been obtained for various foil thicknesses. The final results are shown in Fig. 1.
γ-efficiency with respect to interaction with the foil still remained within ±5%. The overall error in calculating the correction $K$ is also shown in Fig. 1.

The results shown above should be subject to easy modification to permit their application to the absolute determination of Au-foil activity by means of $4\pi\beta$ counting where the same corrections are necessary to obtain the true $\beta$-counts.

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**REFERENCES**


**New Method of Temperature Control in Capsule Irradiation Vacuum Control Method**

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In irradiating fuel or material specimens, it is often required to keep the temperature at a given level, and with small deviations therefrom, despite variation in flux or in other core parameters.

A new method is proposed based on an original concept, for the control of temperature during irradiation. Both out-of-pile and in-pile experiments have been conducted, with good results.

The generally practiced method of specimen temperature control is to vary the thermal resistance against the heat flow to the capsule surface. To provide the thermal resistance, a gas gap is provided between the outer and inner cans in which the specimens are contained. To vary thermal resistance, a mixed gas system has hitherto been used, in which two kinds of gases with different thermal conductivity are used, such as helium and nitrogen. The ratio of helium gas—which has the greater conductivity—to the total gas mixture determines the thermal resistance.

The proposed new system is to attain the same objective by controlling the degree of vacuum in the gas gap, using only one kind of gas. The gas gap of the capsule is filled with a gas of large thermal conductivity, such as helium. The pressure of the gas is reduced to the extent of bringing about a low value of heat transfer, and a correspondingly small effective thermal conductivity. On reducing the pressure further, when the mean free path of the gas molecule approaches a value comparable to the gas gap distance, the heat transfer of the gas is thought to become proportional to the pressure. Kundsen, using the accommodation factor $\alpha$, has derived the expression for heat transfer between surfaces at different temperatures:

$$E_{\text{t}} = \frac{\Lambda_2}{\Lambda_1} \sqrt{\frac{273.2}{T_1}} \left(T_2 - T_1\right),$$

where $E_{\text{t}}$: Heat transferred (W/cm²)

$\Lambda$: Free molecule heat conductivity (W/cm²·K·μg)

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