Analysis of Transmitted Gamma-Rays
by Multiple Scattering Method, (I)

Gamma-Rays Transmitted through Slabs of One Material

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Dose buildup factors and number spectra of γ-rays transmitted through a homogeneous finite slab have been estimated by the multiple scattering method, taking into account scattering —including back scattering— up to the fourth order.

The calculations were performed for 1.0, 3.0, 6.0, 8.0 and 10.0 MeV γ-rays normally incident on lead, iron and water slabs of thicknesses from 1 to 15 mfp.

The results of the above calculations are in good agreement with those from other calculations, such as by Monte Carlo and response matrix methods, especially for heavy shielding materials of practical importance and γ-rays of high incident energy.

Further, a method is proposed with which the contribution to the total dose buildup factor by the γ-rays of the fifth and higher orders of scattering can be estimated approximately. With this method, good agreement was obtained with the dose buildup factor calculated by the Monte Carlo method, even for light shielding materials and γ-rays of low incident energy.

I. INTRODUCTION

Estimation, already in the design stage, of γ-rays transmitted through a shield is very important from the viewpoint of health physics and radiation protection.

For a very limited variety of simple geometry, there are simplified methods of calculation that are validly applicable. But with these methods do not provide reliable results for more complicated geometry, such as usually encountered in nuclear and radiation facilities. The Monte Carlo method is one of the principal tools for analyzing shielding problems. This however demands extended computer time for obtaining reliable results. For purposes of practical radiation shielding design, a relatively simple method is called for, which should yield acceptably accurate results on heavy shielding materials of practical importance and for γ-rays of high incident energy. The present attempt is to apply the multiple scattering method to the estimation of γ-rays transmitted through a shield of relatively complicated geometry.

The multiple scattering method was first introduced by Peebles & Plesset(1,2), who calculated number buildup and energy buildup factors of γ-rays for single slabs consisting of homogeneous heavy material, such as uranium, lead and iron, taking into account 0, 1, 2 and 3 orders of scattering, and neglecting back scattering. In heavy materials, the probability of photoelectric absorption increases very rapidly with decreasing photon energy. Thus, their approach to the γ-ray transmission problem was based on the notion that one need consider only those transmitted γ-rays which suffer relatively few scatterings so that back scattering could be neglected even for shield thicknesses of approximately 20 mfp. Further, it has been shown by Peebles & Plesset(3) that the probabilities related to γ-ray transmission through a slab with successive orders of scattering can be formulated easily on the basis of the simple recurrence formula. At that time, however no other calculations were known, nor experiments reported on the transmission of γ-rays through finite slabs, with which to compare the results.

Using the multiple scattering method, the

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$\gamma$-rays transmitted through shields have been calculated in the present study, taking into consideration scattering— including back scattering— up to the fourth order. We have also attempted to modify the recurrence formula, so that they could be applied not only to single slabs of homogeneous material, but also to multiple layers of different materials. With this method it is not very difficult to take higher orders of scattering into consideration. In the present case we have limited ourselves to four orders of scattering, since $\gamma$-rays of higher orders have relatively little effect on the estimation of the transmission of $\gamma$-rays of high incident energy through heavy shielding materials. For practical purposes, this should amply suffice in most cases to obtain reliably accurate information for nuclear and radiation facilities.

Moreover to provide a rough estimation of the influence of higher orders, a supplementary method is proposed with which the contribution to the total dose buildup factor by the $\gamma$-rays of fifth and higher orders of scattering can be determined approximately from the calculations covering four orders of scattering. The dose buildup factors estimated by this method are found to agree with results from the Monte Carlo method within estimated error for light materials and for $\gamma$-rays of low incident energy.

Some practical cases of complicated geometry can be idealized and simplified so as to be validly represented by homogeneous or stratified materials with slab geometry of finite thickness and infinite area. For such simplified geometries, the results of various shielding experiments and calculations by other methods have been published, with which our present results can be compared.

In the following chapter we present calculations covering a wide range of energy and thickness for the transmission dose buildup factors* and number spectra of $\gamma$-rays normally incident on finite slabs of a homogeneous material, and the results are compared with the values calculated by other authors with the moment method(3), the Monte Carlo method(4-7) and the response matrix method(8-9).

II. FORMULA FOR MULTIPLE SCATTERING METHOD

The multiple scattering method describes $\gamma$-rays transmitted as a result of successive scatterings of photons. One can expect more precise results by taking higher orders of scattering into account in energy ranges where scattering plays a significant role. Further, a formula for estimating the multiple scattering of $\gamma$-rays can be derived from simple recurrence formulas.

Problems of $\gamma$-ray transmission through slab geometry are often very difficult to solve when the slab is composed of heterogeneous materials. Examples are stratified slabs composed of different materials, slabs with non-homogeneous density, and slabs possessing holes and ducts. In such cases the slab may be divided into sub-layers of homogeneous material. When such representation is valid, the formulas presented in this paper could be applied not only to single slabs of homogeneous material, but also to heterogeneous materials.

As shown in Fig. 1, the $\gamma$-rays are normally incident on the left-hand face of a slab of

![Fig. 1 Schematic illustrations of geometry considered in calculation](image)

* These have been called by Chilton(8) "exposure buildup factors".
infinite area, and the point of detection, by an idealized point isotropic detector is on the right-hand side. Assume a sub-layer of thickness \( t \) within the slab, and that a photon scattered \( k \) times and retaining an energy \( E' \) is incident on this sub-layer at an angle \( \phi_k \) to the direction normal to the face. Such photons scattered \( k \) times may be incident on this sub-layer either from the side of the source or from the opposite side.

The differential number spectrum of the scattered \( \gamma \)-rays emerging from the exit face of the sub-layer is given as a function of the angular number flux \( N(t, \hat{\Omega}, E) \):

\[
N(t, E) = \int N(t, \hat{\Omega}, E) d\Omega,
\]

where the angular number flux \( N(t, \hat{\Omega}, E) \) is defined such that \( N(t, \hat{\Omega}, E) dE d\Omega \) gives the number of photons transmitted in a unit time through a thickness \( t \) of the sub-layer, with energies in the elemental range \( dE \) about \( E \), moving across a unit area in the direction specified by the unit vector \( \hat{\Omega} \) within an elemental solid angle \( d\Omega \). The angular number flux \( N(t, \hat{\Omega}, E) \) may be divided into components \( N_{k,k+1}(t, \hat{\Omega}_{k+1}, E_{k+1}) \) each giving the angular number flux of a photon transmitted through the sub-layer and suffering \( i \) collisions in the sub-layer. Then,

\[
N(t, \hat{\Omega}, E) = \sum_{i=0}^{\infty} N_{k,k+1}(t, \hat{\Omega}_{k+1}, E_{k+1}).
\]  

The angular number flux

\[
N_{k,k+1}(t, \hat{\Omega}_{k+1}, E_{k+1}) = \frac{1}{|a_{k+1}|} \int_0^{|x_{k+1}|} dx_{k+1} \cdot H_{k,k+1}(x_{k+1}, \hat{\Omega}_{k+1}, E_{k+1}) \cdot e^{-\mu_{k+1}|x_{k+1}|} \omega_{k+1}
\]

(3)

Equation (3) shows that the angular number flux \( N_{k,k+1}(t, \hat{\Omega}_{k+1}, E_{k+1}) \) is represented by photons emerging from the source \( H_{k,k+1}(x_{k+1}, \hat{\Omega}_{k+1}, E_{k+1}) \) and traveling the path \( (a-x_{k+1})/\omega_{k+1} \), where

\[
H_{k,k+1}(x_{k+1}, \hat{\Omega}_{k+1}, E_{k+1}) = \int d\Omega_{k+1}
\]

\[
\int dE_{k+1} \cdot \frac{d\Omega_{k+1}}{dE_{k+1}} \cdot \frac{dE_{k+1}}{E_{k+1}} \cdot M
\]

\[
N_{k,k+1}(x_{k+1}, \hat{\Omega}_{k+1}, E_{k+1})
\]

\[
d\Omega_{k+1} dE_{k+1}
\]

Differential cross section for scattering from direction \( \hat{\Omega}_{k+1} \) to \( \hat{\Omega}_{k+1} \) and from energy \( E_{k+1} \) to \( E_{k+1} \) per unit solid angle and energy range

\( \frac{1}{E_{k+1}} - \frac{1}{E_{k+1}+1} \)

The path of the photon after its \( (k+n) \)-th collision is characterized by its azimuthal angle \( \phi_{k+n} \) in addition to the angles \( \theta_{k+n} \) and \( \phi_{k+n} \), these angles are interrelated by the familiar relation

\[
\cos \theta_{k+n} = \cos \phi_{k+n} \sin \phi_{k+n}.
\]

Starting with Eq. (3), this recurrence relation can be used to calculate any desired \( N_{k,k+1} \). Rigorously, this recurrence relation could be solved to yield for \( N_{k,k+1} \) the expression

\[
N_{k,k+1}(t, \hat{\Omega}_{k+1}, E_{k+1}) = \frac{1}{|a_{k+1}|} \int_0^{|x_{k+1}|} dx_{k+1} \cdot \frac{d\Omega_{k+1}}{dE_{k+1}}
\]

\[
\int dE_{k+1} \cdot M \cdot \frac{d\Omega_{k+1}}{dE_{k+1}} \cdot \frac{dE_{k+1}}{E_{k+1}}
\]

\[
N_{k,k+1}(x_{k+1}, \hat{\Omega}_{k+1}, E_{k+1})
\]

\[
d\Omega_{k+1} dE_{k+1}
\]

where

\[
N_{k,k+1}(x_t, \hat{\Omega}_t, E_t) = e^{-\mu_{t+1}|x_{t+1}|} \delta(E_t - E') \delta(\hat{\Omega}_t - \hat{\Omega}'t)
\]

where, in turn the incident photon is assumed to possess the energy \( E' \), traveling in the direction \( \hat{\Omega}'t \).

The relation between the change in photon energy and the angle of scattering is simply

\[
\frac{1}{E_{k+1}} - \frac{1}{E_{k+1}+1} \sim \cos \theta_{k+1}.
\]  

The differential cross section is given by
combining Eq.(7) with Klein-Nishina's cross section:

$$\frac{d\sigma}{d\Omega_{\text{coll}}} dE_{\text{coll}} = \frac{d\sigma(E_{\text{coll}}, \cos \theta_{\text{coll}})}{d\Omega_{\text{coll}}} \left( 1 - \left( \frac{1}{E_{\text{coll}}} - \frac{1}{E_{\text{coll}}} \right) \cdot \frac{mc^2 - \cos \theta_{\text{coll}}}{\sin \theta_{\text{coll}}} \right).$$

(8)

Integration of Eq.(6) with respect to $E_{\text{coll}}$ can be performed by making use of Eq.(8):

$$N_{k+1}(E_{\text{coll}}, E_{m}) = \int \frac{d\sigma(E_{\text{coll}}, E_{m})}{d\Omega_{\text{coll}}} \int \frac{d\sigma(E_{\text{coll}}, E_{m})}{d\Omega_{\text{coll}}} \left( 1 - \left( \frac{1}{E_{\text{coll}}} - \frac{1}{E_{\text{coll}}} \right) \cdot \frac{mc^2 - \cos \theta_{\text{coll}}}{\sin \theta_{\text{coll}}} \right).$$

(13)

Further, if $\alpha_{k+i}$ is introduced, the above two inequalities Eqs.(13) and (15) can be combined to give the simple representation

$$0 \leq s_{k+i} \leq \frac{k+i-1}{n} \sum_{n=0}^{k+i} s_n \omega_n,$$

(15)

must hold. Here

$$b_k = 0, \quad 0 \leq \omega_k \leq 1$$

(16)

Further,

$$\sum_{n=0}^{k+i} s_n \omega_n = 0, \quad \text{for } i = 0.$$  

(17)

If $a_{k+i}$ is introduced, the above two inequalities Eqs.(13) and (15) can be combined to give the simple representation

$$0 \leq s_{k+i} |\omega_{k+i} - |\alpha_{k+i} - \frac{k+i-1}{n} \sum_{n=0}^{k+i} s_n \omega_n|,$$

(18)

where

$$a_{k+i} = t + b_k, \quad 0 \leq \omega_{k+i} \leq 1$$

(19)

Now, we introduce a function

$$F_{k+i, k+i} = \exp \left( - \mu_{k+i} / \omega_{k+i} \right) \sum_{n=0}^{k+i} s_n \omega_n,$$

(20)

where $k+i > k+i$. Then, if $J_{k+i}$ represents the operation of integration with respect to $s_{k+i} |\omega_{k+i} \leq |a_{k+i} - \frac{k+i-1}{n} \sum_{n=0}^{k+i} s_n \omega_n|$, we have

$$J_{k+i}(F_{k+i, k+i}) = \frac{\left( F_{k+i, k+i} - \mu_{k+i} / \omega_{k+i} \right) \left( a_{k+i} - s_{k+i} \omega_n \right)}{\mu_{k+i} / \omega_{k+i} - \mu_{k+i} / \omega_{k+i}}.$$

(21)

The integration with respect to $s_{k+i} |\omega_{k+i} \leq |a_{k+i} - \frac{k+i-1}{n} \sum_{n=0}^{k+i} s_n \omega_n|$ in Eq.(21) can be performed similarly.

The $\pi$-integrations in Eq.(10) can now be readily undertaken by reiteration of Eq.(21). However it is not a simple operation to integrate over the angle variable, and the integration has been performed by numerical integration. In Eq.(10), $N_{k+i}(t, E_{\text{coll}})$ is represented by the sum of contributions of all possible paths that have $n$ refraction points. Therefore the space covered by the integra-
tion in Eq. (10) is divided into \(2^{n+1}\) sub-space (including the direction of incidence on the sub-layer), where each sub-space represents a particular sequence of forward and backward scatterings. Thereupon, \(N_{k,k+n}(t,E_{k+n})\) may be divided into the components \(n_{k,k+n}(a_k-b_k, a_{k+1}-b_k, \ldots, a_{k+n}-b_k)\), each representing one of all possible combinations of forward and backward scatterings in each sub-space, \(a_k-b_k\) being \(t\) for forward and 0 for back scattering. Thus from Eq. (10),

\[
N_{k,k+n}(t,E_{k+n}) = \sum n_{k,k+n}(a_k-b_k, a_{k+1}-b_k, \ldots, a_{k+n}-b_k). \tag{22}
\]

For \(n=1\), we have

\[
N_{k,k+1}(t,E_{k+1}) = M \int \frac{d\phi_{k+1}}{|\omega_{k+1}|} \frac{d\sigma(E_k \cos \theta_{k+1})}{dE_k} \left[ F_{k+1,k-1} - e^{-\mu_{k+1} |a_k-b_k|} F_{k,k-1}(a_k-b_k, a_{k+1}-b_k, \ldots, a_{k+n}-b_k) \right]. \tag{23}
\]

The differential number spectrum representing the probability that a \(k\)-times scattered photon is incident on the left-hand face of the sub-layer and transmits through the sub-layer after one scattering, is given from Eq. (23) as \(n_{k,k+1}(t,t)\), and that of being reflected out through the left-hand face of the sub-layer after one scattering is \(n_{k,k+1}(t,0)\). Similarly, for \(n=2\) we have eight differential number spectra:

\[
\begin{align*}
N_{k,k+2}(t,t) = & M \int \frac{d\phi_{k+2}}{|\omega_{k+2}|} \int d(\cos \theta_{k+2}) \int \frac{d\sigma(E_k \cos \theta_{k+1})}{dE_k} \int \frac{d\sigma(E_{k+1} \cos \theta_{k+2})}{dE_{k+1}} \\
& \cdot T_{k,k+2}(T_{k,k+2}(P_k-P_{k+2}) - T_{k+1,k+2}(P_{k+1}-P_{k+2})) \tag{25}
\end{align*}
\]

The first two differential number spectra have been derived by Peebles & Plesset (1). The latter two are derived here for the first time; they are required for calculations on slabs composed of heterogeneous materials. Similarly, for \(n=2\) we have eight differential number spectra:
Again, the first four spectra are those by Peebles & Plesset(1) and the remaining four derived here for the first time.

And for $n \geq 3$ the spectra will be easily derived by iteration of Eq. (21).

The calculations by Peebles & Plesset(1) neglected back scattering, but this has been included in the present consideration. The component $n_{k+n}$ when $a_k-b_k=0$ corresponds to the case where a $k$-times scattered $\gamma$-ray is incident on the right-hand face of the sub-layer. In the case of a slab of homogeneous material the sub-layer is regarded as the slab itself. Therefore only the components $n_{k+n}$ whose variables are $a_k-b_k=a_{k+n}-b_k=t$ contribute to the differential number spectrum.

III. RESULTS

The transmission dose buildup factors and the differential number spectra of $\gamma$-rays transmitted through finite slabs of water, iron and lead with thicknesses between 1 and 15 mfp were calculated* for $\gamma$-rays incident in a direction normal to the slab with initial energies of 1.0, 3.0, 6.0, 8.0 and 10.0 MeV.

The present calculations have been performed for the case of a single slab of homogeneous material, with the sub-layer considered in the previous section taken to represent the slab itself, and $k=0$. For the linear attenuation coefficients the values of Davison's table(8) have been adopted. The numerical integrations of the differential number spectrum $N_0,i$ ($i=1, 2, 3, 4$) over the angular variables $\theta$ and $\phi$ have been calculated at mesh intervals of 10 degrees.

Dose buildup factors calculated with scattering taken into consideration up to the fourth order are shown in Table 1 and Fig. 2. In Fig. 2 are also shown, for purposes of comparison, results by the Monte Carlo method(10) and by the response matrix method(7). The Monte Carlo results(10) have been estimated to be in error by a maximum of 15%.

Table 1 Transmission dose buildup factors for $\gamma$-rays normally incident on homogeneous slabs (point isotropic detector) as calculated by multiple scattering method

<table>
<thead>
<tr>
<th>Material</th>
<th>Incident energy $E_0$ (MeV)</th>
<th>Thickness of finite slab (mfp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lead</td>
<td>1</td>
<td>1.40</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.36</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>1.12</td>
</tr>
<tr>
<td>Iron</td>
<td>1</td>
<td>1.75</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.54</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>1.34</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>1.21</td>
</tr>
<tr>
<td>Water</td>
<td>1</td>
<td>1.78</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.59</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>1.41</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>1.30</td>
</tr>
</tbody>
</table>

Deviation from Monte Carlo values (Table 10.14 of Ref. (4)) of the dose buildup factors calculated by the multiple scattering method are given in Table 2 in percentage values. The dose buildup factors calculated by the multiple scattering method with scattering considered up to the fourth order are seen to agree with the Monte Carlo values within estimated error for heavy shielding materials of practical importance and for $\gamma$-rays of high incident energy.

The contribution of the number of scatterings to the total dose buildup factors is represented in terms of $D_k/D_0$ in Fig. 3, where

* The calculations have been performed by HITAC 5020 computer.
Open circles: Dose buildup factors by multiple scattering method taking scattering into consideration up to the fourth order; Solid circles: Those by Monte Carlo method\(^\text{(5)}\); Open triangles: Those by the response matrix method\(^\text{(6)(7)}\); Solid lines: Those by Monte Carlo method (Table 10.14 of Ref.\(^\text{(4)}\)); Broken lines: Those by the moment method\(^\text{(3)}\); Chain lines: Those estimated by multiple scattering method taking scattering into consideration up to the fifth or higher orders.

**Fig. 2** Comparison of transmission dose buildup factors calculated by various methods

<table>
<thead>
<tr>
<th>Material</th>
<th>Incident energy (E_0) (MeV)</th>
<th>Dose buildup factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Thickness of finite slab (mfp)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Lead</td>
<td>1</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td>4(^t)</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>1.8</td>
</tr>
<tr>
<td>Iron</td>
<td>1</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>4(^t)</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.0</td>
</tr>
<tr>
<td>Water</td>
<td>1</td>
<td>−3.3</td>
</tr>
<tr>
<td></td>
<td>4(^t)</td>
<td>−1.1</td>
</tr>
</tbody>
</table>

\(^t\) The dose buildup factors for 4 MeV by multiple scattering method were obtained by interpolation.

\(D_0\) is the dose* due to photons transmitted through the slab without any collision, and \(D_k\) that due to photons transmitted through the slab after \(k\) scatterings. The parameter \(X\) is the thickness (mfp) of the slab. It is seen that even for materials as thick as 15 mfp it is the \(\gamma\)-rays of the first and the second order scatterings that contribute dominantly to the total dose buildup factor. The relatively small contribution of the higher orders of scattering in heavy materials and for \(\gamma\)-rays of high incident energy may be attributed to the large photoelectric absorption of the heavy materials and to the sharp

* "Dose" signifies \(\gamma\)-ray flux measured in terms of roentgens per unit time.
forward angular distribution of the high energy photons. Water and iron are seen to possess patterns of \(D_k/D_0\) similar to each other, and this is due to their similarity in attenuation coefficient. Lead has a \(D_k/D_0\) pattern different from the other two materials, and has a smaller \(D_k/D_0\) for 1 MeV incident energy, due to the large photoelectric effect, whereas the corresponding values for incident energies of 6 and 10 MeV are larger for a slab thickness of 15 mfp. This latter effect is probably because the total attenuation coefficient of lead has a minimum value at about 3 MeV.

We have also calculated the differential number spectra of scattered \(\gamma\)-rays based upon the expression

\[
N_0 = \sum_{i=1}^{6} N_0(X,E).
\]

The results for slabs of finite thickness are shown in Figs. 4, 5 and 6, where they are compared with those by the moment method for infinite media. The ordinate is normalized to a source intensity of 1 photon/sec/cm\(^2\). The small difference seen between the two results at energies close to the incident energy might be due to the difference in the total absorption coefficient employed in the calculations. The discrepancies seen in the low energy regions in the cases of high incident photon energies might be attributed to the fact that the results by the moment method were obtained for infinite media, while those by the
multiple scattering method are for a slab of finite thickness.

In the case of high incident energies sufficiently accurate approximation seems to be obtainable with the multiple scattering method, because the fraction of contribution of the fourth order of scattering to the transmission dose buildup factor is very small as shown in Fig. 3. Therefore, in this case, the major source of discrepancy between the two results might be considered to be due to the relatively large contribution of the back scattering of infinite media, which is assumed in the moment method calculation. In contrast, for
low incident energies, the approximation would not appear sufficiently reliable when determined by the multiple scattering method with scattering taken into account only up to the fourth order. In this case, the effects of still higher orders of scattering should have to be considered.

**IV. DISCUSSION AND CONCLUSIONS**

Dose buildup factors and differential number spectra of γ-rays transmitted through a finite slab of lead, iron and water have been calculated with the multiple scattering method, with scattering—including back scattering—taken into consideration up to the fourth order.

As seen from Table 2, the dose buildup factors calculated by the multiple scattering method for lead slabs of thicknesses from 1 to 15 mfp and incident energies from 1 to 10 MeV agree with Monte Carlo calculations within estimated error for γ-rays incident in the direction normal to the slab. For iron and water the present method has been found reliable only for high incident energies and relatively thin slabs.

Figure 7 represents the region in which one can reliably apply the multiple scattering method with scattering considered up to the fourth order. Judging from the values given in Table 2 for discrepancies between the present and Monte Carlo results, the deviation of the dose buildup factors calculated by the present method can be held within 15% when \( \mu(E_0)E_0 \geq 7.5 \), where \( E_0 \) is source energy and \( \mu(E_0) \) the linear attenuation coefficient.

Plots of the \( D/D_0 \) ratio shown in Fig. 3 as function of the number of scatterings align themselves into straight lines if the ordinate is presented in log scale. An example is shown in Fig. 8 for iron with 6 MeV source energy. The linear relation may not necessarily hold with increasing \( X \), for all values of \( k \). In such case, we may neglect the deviations from linearity at small values of \( k \) and replace the plots in this region by linear extrapolation from the range of relatively large \( k \).

Now, we assume the above linear relation to be represented by

\[
\log(D/D_0) \propto -\nu \cdot k. \tag{28}
\]

The logarithm of \( \nu/X \) are shown as function of \( \log X \) in Fig. 9. We find a linear relation also between \( \log(\nu/X) \) and \( \log X \), which can be represented by

\[
\log(\nu/X) \propto -a \log X. \tag{29}
\]

The coefficient \( a \) appears to be constant for each material and almost independent of the incident energy.

From Eqs. (8), (9),

\[
D/D_0 \propto \exp(-X^{-(a-1)} \cdot k). \tag{30}
\]

The values of \( a \) are 1.37 for water, 1.43 for iron and 1.45 for lead. Thus while each material has its inherent value of \( a \), the difference between the largest and the smallest coefficient is not large, and this would indicate that this coefficient \( a \) is also independent of the kind of material.

The contribution to the total dose buildup factor provided by γ-rays of fifth and higher
The calculated values of \( \frac{D_k}{D_0} \) for a given value of \( X \) are joined by solid straight line, while the extrapolated parts are shown dashed.

**Fig. 8** Calculated values of \( \frac{D_k}{D_0} \) vs. number of scatterings, on semi-logarithmic representation—example for iron slab 1 to 15 mfp thick, with 6 MeV \( \gamma \)-rays normally incident on slab incident energy.

The method discussed in this paper will permit the dose buildup factor to be estimated by adopting in the calculation only a few orders of scattering. An example where this would not apply was found in relatively thick water slabs of 4 mfp or above for an incident energy of 1 MeV or below, when the contribution to the total dose buildup factors provided by \( \gamma \)-rays of the fifth and higher orders of scattering appear to be larger than those estimated by the above method.

It may be concluded however that for the heavier shielding materials of practical importance, and for \( \gamma \)-rays of relatively high incident energy, the multiple scattering method may be considered a sufficiently reliable method for practical purposes to estimate the dose buildup factors and differential number spectra. Further, the method may also be considered applicable to the estimation of dose buildup factors for light materials and for the \( \gamma \)-rays of low incident energy, if appropriate corrections are applied in accordance with the method proposed for higher orders of scattering.

If faster computers become available in the future, the values for lighter materials and for \( \gamma \)-rays of relatively low incident energy would also become subject to accurate calculation by taking higher orders of scattering into consideration.

**References**


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