Effect of Cold Neutrons on the Neutron Pulse and Wave Propagations in Crystalline Media

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With the view to contributing information on the effect of cold neutrons on neutron pulse propagation and on die-away phenomena and phenomena related to neutron wave propagation, measurements of propagating pulse shapes in graphite and lead prisms were carried out with 300°K water moderated and 77°K ice moderated sources. Corresponding theoretical analyses were also performed, with use made of the successive iteration method, and a coupled two-group theory based on the distributed source diffusion equation was developed.

As a result, it was established that: (a) infiltrating cold neutrons below Bragg-cut-off energy, which has a sharply peaked pulse head, produce peculiarly shaped pulse propagation responses from detector; (b) the resonance phenomena observed in wave propagation can be fairly well explained as due to the interference between the thermal neutron group and the infiltrating cold neutron group, the cold neutrons in question being those below the Bragg cut-off energy in the case of graphite, and in the case of lead those below 0.01 eV; and (c) in the analyses related to resonance phenomena in graphite, the experimental characteristics are more consistently represented in the results of calculations using the BNL-325 data as compared to those using the UNCLE data for the total cross section below the Bragg cut-off.

I. INTRODUCTION

Many studies have in recent years been undertaken on neutron wave propagation in non-multiplying and multiplying media. These studies have been aimed at determining the dispersion relations, i.e., the relation between the complex inverse relaxation length and the wave frequency in the media, and to derive parameters for such factors as diffusion and thermalization.

One of the subjects of propagation experiments in moderating media concerns the interpretation of resonance phenomena reported by Utsuro et al.1(2) and Takahashi et al.3(4) in graphite prisms. These resonance phenomena have proved an impediment to the formulation of clearcut dispersion relations in a wide range of frequency and transverse buckling. It is known that the phenomena are induced by the interference between the thermal neutron wave and the infiltrating cold neutrons below the Bragg cut-off energy, which affect the total cross section, and that they occur in the continuous eigenvalue region in wave propagation problems.

However, in the pulse propagation experiments by Takahashi et al.5(6) using thermalized neutron source with near Maxwellian spectrum, the results left some doubt as to whether the resonance might not have been due to error introduced in the numerical Fourier transformation of the pulse responses. On the other hand the experiment by Utsuro et al.7(8) was carried out using a sinusoidally modulated beam source from the reactor thermal column of graphite. Their source had a sharply separated cold neutron component about 10 times larger than the Maxwellian below the Bragg cut-off energy of graphite. In this case, the suspicion could be attached to the particularity of the source spectrum. No experiments in related phenomena have been reported since the publication of these two papers.

In the theoretical field, the eigenvalue problem of neutron wave propagation in crystalline media have been treated by many authors who made use of the transport equation9-18. From these studies, it can be concluded that the existence of a discrete asymptotic mode is limited to a frequency range under a certain critical frequency, and the

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continuum at $\Sigma_{\text{min}}$, just under the Bragg cutoff for crystalline media, determines the critical frequency.

The resonance phenomena observed in the experiments have been explained qualitatively from the phase difference between the pseudo-mode (extension of the discrete asymptotic mode) and the sub-Bragg-cut continuous mode. However, the calculation of continuous modes involves complications due to difficulties in the inverse transformation of Fourier or Laplace transformed transport equations, and apart from a paper by Yamagishi\(^9\) no attempts have so far been made to estimate quantitatively the magnitude of the continuous mode. Analyses of the eigenvalue spectrum of the Boltzmann transport operator have been undertaken with use made of a simplified degenerated kernel\(^{10}\), but an analytical approach to the transport operator with an actual scattering kernel and coherent scattering cross section should be impossible.

Nishina et al.\(^{11}\) studied the problem by applying the diffusion equation with distributed source and an one-term degenerated kernel\(^2\), but an analytical approach to the transport operator with an actual scattering kernel and coherent scattering cross section should be impossible.

In the present work, we aim: (a) to illustrate experimentally, more clearly than previous studies\(^{3}(4)\), the resonance phenomena on the neutron wave propagation in different kinds of crystalline media under various conditions; and (b) to establish a method of analysis with actual scattering kernels and cross sections. To this end, pulse propagation experiments have been carried out in graphite and lead prisms with two kinds of source (300°K water moderated and 77°K ice moderated sources) with accurate statistics. The data were analyzed with the view to determining (a) the effect of resonance on propagating pulse shapes; and (b) the consistency in the resonance in wave amplitude between the behavior observed by Fourier transformation of the pulse data, and the results of theoretical analyses. For the theoretical analyses, we have developed a successive iteration method for the diffusion equation with distributed source.

Prediction by transport theory applied to eigenvalue spectrum reveals that the continuous eigenvalue $\kappa_c=\left((\Sigma(E)+i\omega/v)/\mu\right)$ occupies the full frequency range in any lead prism\(^{14}\). In an infinite graphite system, the discrete eigenvalue $\kappa_d$ enters the sub-Bragg cold continuum from the thermal inelastic scattering $\mathcal{B}_{\text{te}}$ at about 2000 rad/sec (see Fig. 1). And the existence of the discrete asymptotic mode is limited within the frequency range below about 900 rad/sec. In a finite graphite system with transverse dimensions smaller than about 100 x 100 cm\(^2\) ($B_\Omega=1.87\times10^{-3}$ cm\(^{-2}\)), there is no discrete asymptotic mode for any frequency. These two representative crystalline media have selected in our present study on account of these properties.

**II. EXPERIMENTS**

Measurements of pulse propagation in graphite and lead prisms were carried out with "thermal" as well as with the "cold" source. The principle adopted for obtaining a pulsed thermal neutron source is similar to that presented by Sumita et al.\(^{12}\). The thermal source was obtained in the forms of

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(Continued text)
thermalized neutrons from 14 MeV pulsed neutrons generated by Cockcroft accelerator with 8 cm thick water layer at room temperature. In the same way, the cold source was obtained using a 77°K ice layer (8 cm thick) cooled with liquid nitrogen (Fig. 2). The measured leakage neutron energy spectra have been published elsewhere. The spectrum from the room temperature water layer could be regarded as pure Maxwellian with a neutron temperature of 300°K, while the spectrum from the 77°K ice was not Maxwellian. This latter spectrum could be regarded roughly as 100°K Maxwellian. The result of Ag filter transmission experiments have revealed that the thermalization time necessary for attaining equilibrium in 77°K ice is about 200 μsec.

The propagating media were arranged as shown in Fig. 2. The lead system was obtained by piling 99.99% pure lead blocks each measuring 5 × 10 × 20 cm³. The resulting pile had transverse dimensions of 30 × 30 cm² and a length of 70 cm along the propagating axis (z-axis). For the measurements in graphite (density, 1.7 g/cm³), two prisms were prepared. One was 50 × 50 × 100 cm³ and the other 70 × 70 × 200 cm³.

Fig. 2 Experimental arrangement

A Mitsubishi Electric ND-8523 BF₃ counter (1.4 cm diam., 10 cm active length) was used as neutron detector. Pulse responses from the detector, which was inserted perpendicularly to the z-axis and at the centre, were analyzed with ordinary pulsed neutron circuits. The channel length of the TMC time analyzer was 8 μsec for the lead experiment, and 50 μsec for the graphite experiment. The source pulse repetition rate was from 50 to 100 Hz. Some pulse responses at large distances in the lead prism with the cold neutron source were measured with use made of the KUR linac. Backgrounds induced by the injection of fast and epi-thermal neutrons were eliminated with a 0.5 mm Cd-sheet inserted between the source and the medium.

While the measurements at the larger distances in graphite systems were fairly difficult on account of high backgrounds, particularly in the case of the cold source, reliable data could still be obtained by alternative accumulation: cumulative data were obtained from about 10 successive runs without Cd-filter, followed by the same number of runs with Cd-filter. Measurements in lead prism were easier than in graphite due to lower backgrounds.

Some example of space dependent pulse responses will be shown later in comparison with the theoretical curves. The measured pulse responses at various distances from source, in lead and in graphite prisms were transformed by Fourier code into corresponding values expressed in terms of amplitude and phase and presented as function of wave frequency.

III. THEORETICAL ANALYSES

1. Successive Iteration Method

The phenomena treated in this work contain the effect of continuous eigenvalue spectra of wave propagation, so that rigorous theoretical analysis must be made upon estimating the magnitude of the continuous mode. However, the inverse transform of the transport operator with actual scattering kernel and a complicated scattering cross section is almost impossibly difficult. The successive iteration method developed here can be applied to the integral type transport equation, but it is not a practical method on account of the long computing time required.

From eigenvalue spectra, the continuous mode is known to have the dispersion relation $\kappa_v = \alpha_v + i\beta_v = (\Sigma(E) + i\omega/v)/\mu$, where $\kappa_v$, $\Sigma(E)$, $\omega$, $v$ and $\mu$ represent a complex inverse relaxation length, energy dependent total cross section, wave frequency, neutron velocity and cosine...
angle between position vector and angular vector, respectively. This provides the physical image of the penetrating neutrons between one collision and the next: the factor $e^{-\frac{\alpha}{v}(z_2-z_1)}$ represents the probability that a neutron colliding at $z_1$ reaches $z_2$ with no collision and with a phase shift of $\alpha(z_2-z_1)$.

Considering a practical case in the experiment, the magnitude of the continuous mode, induced by such infiltrating neutrons, will become greatest upon penetration at a point between their emission from the source and their first collision, especially for the sub-Bragg cold neutrons with small $\alpha$. The reason are that in the higher frequencies of the continuous eigenvalue region, the neutron wave will attenuate rapidly after leaving the source, and that the thermal inelastic down-scattering to sub-Bragg-cut energy is small. Then, with increasing distance, the dominant will become that played by the infiltrating neutrons with $\alpha$ smaller than the real part $\alpha_d$ of the complex inverse relaxation length $\alpha_d$ of the pseudo-mode as the extension of a discrete asymptotic mode, which will be constituted of multi-collision neutrons.

In practice, space dependent energy spectra calculation by the “successive collision method” with the integral transport equation in crystalline media shows that, for a Maxwellian source distribution, the spectra below the Bragg cut-off energy are largely determined by the uncollected and the first collision components. From the foregoing considerations, we can use the diffusion approximation, with distributed source equivalent to the “first flight correction”, if angular distribution need not be taken into account.

The distributed source is expressed as follows, assuming $\mu=1$ for a relatively large distance $z$:

$$
Q(z, t, E) = \int_0^\infty \Sigma_{\alpha}(E)S(E)e^{-\alpha z/v}f(\tau)\phi(\tau)\frac{d\phi}{d\tau}d\tau,
$$

where $S(E)$ is the source spectrum, and $f(\tau)$ the time dependence of the source intensity. The basic diffusion equation is

$$
\frac{1}{v} \frac{\partial}{\partial t} \phi(E) + (\Sigma_\alpha(E) + \Sigma_{\text{in}}(E) + D(E)B_E^2)\phi - D(E)\frac{\partial^2 \phi}{\partial z^2} = \int_0^\infty \Sigma_{\text{in}}(E')\phi(z, t', E')dE' + Q(z, t, E),
$$

where $B_E^2$ is the energy dependent transverse buckling, $\Sigma_{\text{in}}(E)$ the total inelastic scattering cross section, $\Sigma_{\text{in}}(E'\rightarrow E)$ the thermal inelastic scattering kernel and the other notations are as commonly used.

Fourier and Laplace transforms of Eq. (2) yield

$$
\mathcal{F}(B, s, E) = g_0(B, s, E)\Phi_0(B, s, E) + g_0(B, s, E)\int_0^\infty \Sigma_{\text{in}}(E')\mathcal{F}(B, s, E')dE',
$$

where

$$
g_0(B, s, E) = \frac{1}{\Sigma_\alpha + \Sigma_{\text{in}} + DB_E^2 + \frac{\Sigma_{\text{in}}}{v} + DB_E^2},
$$

$$
\mathcal{F}(B, s, E) = \int_0^\infty \phi(z, t, E)e^{iBz-st}dt,
$$

$$
q(B, s, E) = \int_0^\infty Q(z, t, E)e^{iBz-st}dt.
$$

Equation (3) is Fredholm’s integral equation of the second kind, and so the solution is expressed in Neumann series $\varphi_n$:

$$
\mathcal{F}(B, s, E) = \sum_{n=0}^{\infty} \varphi_n(B, s, E),
$$

$$
\varphi_n(B, s, E) = g_0(B, s, E)\int_0^\infty \Sigma_{\text{in}}(E'\rightarrow E)\mathcal{F}_{n-1}(B, s, E')dE',
$$

$$
\varphi_0(B, s, E) = g_0(B, s, E).n \geq 1.
$$

If the $m$-th approximate solution when terminating the Neumann series at $n=m$ is written $\mathcal{F}_m$, the recursion formula for successive iteration is obtained with Eqs. (5) and (6):

$$
\mathcal{F}_m(B, s, E) = g_0(B, s, E)\int_0^\infty \Sigma_{\text{in}}(E'\rightarrow E)\mathcal{F}_{m-1}(B, s, E')dE',
$$

$$
\varphi_0(B, s, E) = \varphi_0(B, s, E).
$$

(1) Space-time Energy Dependent Flux Calculation

The Fourier-Laplace inverse transform of Eq. (10) are easily carried out using the convolution theorem of Fourier and Laplace transforms, if we express the inverse transform of $\mathcal{F}_m(B, s, E)$ by $\varphi_m(z, t, E)$:

$$
\varphi_m(z, t, E) = \phi_0(z, t, E) + \int_0^\infty dE' \int_0^\infty dt' G_0(z-z', t-t', E')
\int_0^\infty \Sigma_{\text{in}}(E'\rightarrow E)\phi_{m-1}(z', t', E')dE',
$$

$$
m \geq 1.
$$
\[
\phi_0(z, t, E) = \int_0^z dz' \int_0^t dt' \Phi(z', t', E)
\]
\[
\cdot G_0(z - z', t - t', E) dt', (13)
\]
where \( G_0(z,t,E) \) is the inverse transform of \( g_0(B,s,E) \) in Eq. (4), and already familiar to us in the form
\[
G_0(z,t,E) = v / \sqrt{4 \pi D v t} \exp \left(-\frac{z^2}{4 D v t} - \lambda(E) t \right),
\]
(14)
where \( \lambda(E) = (\Sigma_a(E) + \Sigma_{in}(E) + D(E) B_t^2) v \). (15)

If \( f(t) \) is given, we can calculate \( \Phi(z,t,E) \) by Eq. (1), and then \( \phi_0(z,t,E), \phi_1, \phi_2, \ldots, \phi_m(z,t,E) \) can be successively derived using the recursion formula (12) and (13). Physically, this process means that the initial flux calculation is carried out regarding the inelastic scattering as absorption, then the feed-back component from the first inelastic scattering is corrected and the second, third, \ldots multiple inelastic scattering components are added successively. This is why we call the procedure "successive inelastic scattering method"(17)(19).

In actual calculation, \( f(t) = e^{-\lambda t} \) was assumed, based on experimentally obtained time decay constants \( \lambda \) of the relevant sources (\( \lambda_3 = 9,200 \) and \( 6,900 \) sec\(^{-1} \) for thermal and cold sources, respectively). For the calculation in graphite, the thermal inelastic scattering kernel adopted was that obtained by the UNCLE calculation(10), which is similar to the SUMMIT, but is based on the phonon distribution model of graphite crystal by Yoshimori-Kitano and Parks. For the lead kernel, the heavy Debye crystal kernel(10) was used with 90\(^\circ\)K Debye temperature. The BNL-325 data were adopted for \( \Sigma_t \) and \( \Sigma_a \).

In the numerical computations using Eq. (12), space integration was limited within the real size of the system, and the transverse buckling \( B_t^2 \) was replaced by the three dimensional geometrical buckling \( B_0^2 \). If \( \Sigma_{in} \gg \Sigma_a \), the boundary conditions at the starting and ending surfaces were found approximately fulfilled. Calculations by 3- and 20-group constants were carried out for both graphite and lead media.

For the 3-group calculations in graphite, the energy intervals adopted were 0.0~0.0015 eV (sub-Bragg-cut), 0.0015~0.002 (Bragg-peak) and 0.002~1 eV (thermal), in consideration of the energy dependent structure of total cross section. For lead, 0.0~0.003 eV (sub-Bragg), 0.003~0.01 eV (intermediate cold) and 0.01~0.22 eV (thermal) were used.

Good convergence of the Neumann series were obtained by about 4 iterations for the 3-group and about 10 for the 20-group calculations.

(2) Calculations for Sinusoidal Wave Propagation

If we replace \( s \) by \( i \omega \) in Eqs. (3)~(11), we can treat the wave propagation problem. The Fourier inverse transform of Eq. (10) can be easily derived through the convolution theorem.

\[
\tilde{\Phi}_0(z, \omega, E) = \phi_0(z, \omega, E) + \int_0^z dz' G_0(z-z', \omega, E) \int_0^\infty dE' \tilde{Q}(z', \omega', E') \tilde{\Phi}_m(z', \omega', E') dE',
\]
(16)

\[
\tilde{\Phi}_0(z, \omega, E) = \int_0^z \frac{1}{2D(E) \kappa(E)} e^{-\kappa(E) z} \phi_0(z, \omega, E) dE,
\]
(17)

\[
\kappa(E) = \frac{\Sigma_{in}(E) + \Sigma_a(E) + D(E) B_t^2 + i \omega}{D(E)}
\]
(18)

and the Fourier transform of Eq. (1) is,

\[
\tilde{Q}(z, \omega, E) = \Sigma_a(E) S(E) \tilde{f}(\omega) e^{-\Sigma_a(E) \omega} e^{-(\Sigma_t(E) + \omega) \omega}.
\]
(20)

where \( \tilde{f}(\omega) \) is the Fourier transform of \( f(t) \), and means the frequency response of the source.

From Eqs. (17), (18) and (20),

\[
\tilde{\Phi}_0(z, \omega, E) = \begin{cases} 
\frac{\Sigma_a(E) S(E) \tilde{f}(\omega)}{D(E) \kappa(E) (\kappa^2(E) - \Sigma_a(E))} & \text{for infinitely long prism,} \\
\frac{\Sigma_a(E) S(E) \tilde{f}(\omega)}{D(E) (\kappa^2(E) - \Sigma_a(E))} & \text{for semi-infinite prism}
\end{cases}
\]
(21)

\[
\phi_0(z, \omega, E) = \phi_0(z, \omega, E) e^{-\Sigma_a(E) \omega} e^{-(\Sigma_t(E) + \omega) \omega}.
\]
(22)

In a manner identical to the space-time dependent case, we can calculate the complex flux \( \tilde{\Phi}_m \) by successively adding multiple inelastic scattering components. In a similar way
with use made of two group diffusion model, Matsumoto has studied the exponential experiment problem in a very small graphite prism.

The results of Eqs. (2) and (16) do not contain the uncollided neutron flux $\phi_{uv}(z, \omega, E)$, i.e., the flux created by the neutrons reaching $z$ without any collision. This $\phi_{uv}(z, \omega, E)$ must be added to $\phi(z, \omega, E)$ in order to obtain the total flux $\phi_t(z, \omega, E)$:

$$\phi_t(z, \omega, E) = \phi(z, \omega, E) + \phi_{uv}(z, \omega, E), \quad (24)$$

$$\phi_{uv}(z, \omega, E) = S(E) f(E) e^{-\Sigma_{uv}(E)} \quad (25)$$

2. A Two-group Theory

Most crystalline media, except such case as lead, have sharp Bragg cuts. This permits in general the calculation of cold neutron penetration with good approximation by two-group treatment—sub-Bragg cold and above-Bragg thermal groups. The calculation will be discussed in a later section. Moreover, the analytical solution is easily obtained as follows.

Two-group distributed source diffusion equations are,

$$\left( \Sigma_{si} + \Sigma_{sj} + D_i B_i^2 + \frac{I_o}{\omega_i} \right) \phi_i(z, \omega) - D_i \frac{\partial \phi_i}{\partial z} - \Sigma_{si} \phi_i = Q_i(z, \omega), \quad (26)$$

where the suffixes $i$ and $j$ are $i=1$ and $j=2$ for the first group, and $i=2$ and $j=1$ for the second group, and this will apply to all the ensuing equations.

The Fourier transform of Eq. (26) is

$$\Phi(B, \omega) = \phi_i(B, \omega) \tilde{g}_i(B, \omega) + \phi_j(B, \omega) \tilde{g}_j(B, \omega), \quad (27)$$

where $\Phi$, and $\phi_i$ are Fourier transforms of $\phi_i$ and $Q_i$, respectively, and

$$\tilde{g}_i(B, \omega) = \frac{B_i^2 + \kappa_i^2}{D_i (B_i^2 + \rho_i)} \quad (28)$$

$$\tilde{g}_j(B, \omega) = \frac{\Sigma_i}{D_i (B_i^2 + \rho_i)} \quad (29)$$

where $\Sigma_i = \frac{1}{D_i} \left( \Sigma_{si} + \Sigma_{sj} + D_i B_i^2 + \frac{I_o}{\omega_i} \right), \quad (30)$

$$\rho_i = \frac{1}{2} \left( \kappa_i^2 + \kappa_j^2 \right) - \sqrt{\left( \kappa_i^2 - \kappa_j^2 \right)^2 + 4 \Sigma_i \Sigma_j / D_i D_j}, \quad (31)$$

$$\rho_j = \frac{1}{2} \left( \kappa_i^2 + \kappa_j^2 \right) + \sqrt{\left( \kappa_i^2 - \kappa_j^2 \right)^2 + 4 \Sigma_i \Sigma_j / D_i D_j}, \quad (32)$$

are easily obtained. Using Eq. (20) as the source $Q_i$, we derive the solution from Eq. (27) with the aid of the convolution theorem:

For $z>0$,

$$\tilde{\phi}_i(z, \omega) = \frac{a}{\rho_i^2 - \Sigma_i^2} \left( e^{-\Sigma_i z} - \frac{\Sigma_i}{\rho_i} e^{-\rho_i z} \right)$$

$$+ \frac{b}{\rho_i^2 - \Sigma_j^2} \left( e^{-\Sigma_j z} - \frac{\Sigma_j}{\rho_j} e^{-\rho_j z} \right)$$

$$+ S(E) \Sigma_i \Sigma_j \frac{1}{\rho_i} \left( e^{-\Sigma_i z} - \frac{\Sigma_i}{\rho_i} e^{-\rho_i z} \right)$$

$$- \frac{1}{\rho_j^2 - \Sigma_i^2} \left( e^{-\Sigma_i z} - \frac{\Sigma_i}{\rho_i} e^{-\rho_i z} \right), \quad (33)$$

where

$$a = \frac{\rho_i^2 - \kappa_i^2}{D_i (\rho_i^2 - \rho_j^2)}, \quad (34)$$

$$b = \frac{\rho_i^2 - \kappa_j^2}{D_j (\rho_i^2 - \rho_j^2)}, \quad (35)$$

$$h = \frac{1}{D_i D_j (\rho_i^2 - \rho_j^2)} \quad (36)$$

Adding the uncollided neutron flux,

$$\tilde{\phi}_t(z, \omega) = \tilde{\phi}_i(z, \omega) + S(E) f(E) e^{-\Sigma_i z}. \quad (37)$$

Two-group calculations for graphite were carried out using Eqs. (33) and (37).

IV. RESULTS AND DISCUSSIONS

1. Pulse Propagation and Die-away Phenomena

(1) Graphite

In Figs. 3 and 4 are shown typical pulse responses in graphite systems from experiment and the results of 3-group calculations by the distributed source diffusion theory (DSST) using Eqs. (12), (13) and (14). These curves represent where the distance $z$ to the observation points relatively far removed from the source. Effect of sub-Bragg cold neutron penetration is seen to appear in the measured pulse responses, particularly in Fig. 4 (50 cm$^2$ graphite prism with cold source). The 3-group calculations may be considered to represent fairly well the characteristics of pulse shapes neighboring the pulse peak, which reflect the interference by the sharply peaked response of sub-Bragg cold neutrons.

Responses by the thermal and the Bragg peak neutrons have smooth responses as shown by the broken lines in Figs. 3 and 4. The sharp pulse peak due to the sub-Bragg neutrons has its maximum point appreciably earlier in time than the responses from the thermal and
Bragg peak neutrons. The time elapse to the sub-Bragg neutron peak is nearly equal to the time-of-flight for the distance z. With sinusoidal wave propagation, the differences between the peak timings are replaced by phase differences between sub- and above-Bragg neutrons.

An examination of the responses of the three energy group reveals that the energy spectra will change markedly with time. It will be affected mainly by the penetration of sub-Bragg-cut-off cold neutrons.

(2) Lead

In Figs. 5 and 6 are shown the most typical pulse responses including those from the penetrating cold neutron component. The characteristics of pulse shapes obtained from experiment are clearly explained by 3-group DSDT calculation, especially in the case of Fig. 6.

The sub-Bragg cold neutrons propagate more slowly than the thermal neutrons, which contrasts with the case of graphite systems. In the extreme case of Fig. 6, the sub-Bragg cold neutrons become the predominant component beyond about 1 msec, and the calculated responses of each group are seen to carry stepped subordinate peaks, induced by the inelastic scattering from another group. The time dependent energy spectrum changes markedly with the effect of sub-Bragg and 3-10 meV cold neutrons. This fact shows that the effect of cold neutron penetration will occur in lead up to about 0.01 eV, which distinguishes the role played by cold neutrons in lead from that in graphite.

2. Resonance Phenomena on Sinusoidal Wave Propagation

(1) Graphite

First, some calculations were carried out for determining the nature of the spectra. The results are presented in Figs. 7 and 8, which show examples of complex energy spectra by 20-group DSDT calculation using the
method described in Sec. II-1-(2). The absolute flux is seen to present a clear break at the Bragg cut-off energy (0.0015 eV), separating the sub-Bragg cold neutrons from the thermal neutrons, which latter has a peak at about 0.03 eV (Fig. 7). The manner of energy dependence is also made clear: Above the Bragg cut-off, the flux is relatively stable on account of inelastic scattering. Below the Bragg cut-off, the decline of the flux is sharp and the sub-Bragg cold neutrons are largely determined by the flux just below the Bragg cut-off, at least when z ≥ 30 cm.

From the above observations, it should be valid to use the two-group analysis for graphite. Therefore, the ensuing analyses for graphite will be undertaken with two-group treatment (sub-Bragg and thermal).

The results of another preliminary calculation are shown in Fig. 9, where curves obtained by DSDT are compared with those by the ordinary diffusion theory (ODT) using the δ-function source. The figure demonstrates that DSDT (solid lines) clearly brings out the interference between the thermal and the cold groups, which ODT (broken lines) completely ignores.

The third preliminary calculation covers the coupling effect by inelastic scattering between the groups (Fig. 10). Amplitude oscillations of the thermal group, which do not appear in the uncoupled theory \(^{(1)}\), are induced by the feed-back through inelastic scattering from the sub-Bragg cold neutrons. This oscillation plays only a second order role in the first main resonance of the total amplitude, but dominates in the second resonance.

For the comparison between calculated and experimental results, two different data are used for the total cross section below the
Fig. 7 Calculated neutron energy spectra for $\omega=8,000$ rad/sec in $70\times70\times200$ cm$^3$ graphite prism with 300$^\circ$K Maxwellian source.

Fig. 8 Calculated energy dependent phase for $\omega=8,000$ rad/sec in $70\times70\times200$ cm$^3$ graphite prism with 300$^\circ$K Maxwellian source.

Fig. 9 Comparison between ordinary diffusion (ODT) and distributed source diffusion (DSDT) calculations for wave amplitude attenuation in graphite.

Bragg cut-off energy, which is theoretically the sum of the total inelastic scattering and absorption cross sections. One is the data from the UNCLE kernel, $\Sigma_i=0.022$ cm$^{-1}$. The other is $\Sigma_i=0.05$ cm$^{-1}$ from the BNL-325 data at 300$^\circ$K.

In Fig. 11 are shown the frequency responses of wave amplitude at specific positions, normalized in reference to the source frequency response. The observed data are compared with calculations based on the two data mentioned above. The characteristics of the observed resonance structure are reflected more clearly in the DSDT calculation based on the BNL-325 data (solid lines) than on the UNCLE (broken lines). Similarly, an example of the frequency dependence of the phase lag is shown in Fig. 12, where again a more faithful representation is given with calculations based on BNL-325 than on UNCLE.

In Fig. 13, the spatial attenuation of wave
amplitude, normalized to the spatial decay at zero-frequency, are shown for representative frequencies. The BNL data again provide a better basis for explaining the observed values, particularly at about 60 cm for 8,000 rad/sec in which resonance occurs on account of the interference between the sub-Bragg cold and the thermal neutron wave propagations. A similar interference appears in the curves representing spatial lag between phases (Fig. 14).

The foregoing results and discussions confirm that resonance phenomena due to the interference between the infiltrating sub-Bragg and the thermal neutrons occur on the wave propagation in graphite at specified positions and frequencies. In the present experiment, resonance phenomena were observed in the frequency range from about 6,000 to 30,000 rad/sec and at distances from source ranging

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Fig. 10 Calculation of wave amplitude vs. frequency with 2-group DSDT in graphite prism (50X50 cm²) with 100°K Maxwellian source

Fig. 11 Comparison between measured wave amplitude and results of 2-group DSDT, in 50X50 cm² graphite prism

Fig. 12 Comparison of phase vs. frequency curves between measurement and 2-group calculation in 50X50 cm² graphite prism
Fig. 13 Comparison of spatial attenuation of wave amplitude between measured results and those by DSDT calculations for 70×70 cm² graphite prism from 30 to 60 cm with 50×50×100 cm³ prism and from about 4,000 to 10,000 rad/sec at 50 cm to 100 cm with 70×70×200 cm³ prism.

(2) Lead
In the case of lead, the total cross section below the Bragg cut from the BNL-325 agrees well with the sum of the absorption and the inelastic scattering cross section by the heavy Debye crystal kernel with a Debye temperature of 90°K.

Similarly as for graphite, 20-group calculation of the complex energy spectra was carried out to verify the validity of the few-group calculation. The result is presented in Fig. 15, where space dependent energy spectra are shown for a representative frequency. At large distances, the spectra can be divided into three sections, i.e., the sub-Bragg group below 0.003 eV, the thermal above 0.01 eV and the intermediate cold. In rough approximation, three group treatment for these three sections should be valid.

In Fig. 16, the frequency responses of wave amplitude in the lead prism with the thermal source are shown for representative positions. The two resonances observed in the experiment are, at least qualitatively, well represented in the DSDT calculation. In this calculation, the intermediate cold neutrons immediately above the Bragg cut plays the dominant role in the first resonance (at about 15,000 rad/sec for 52 cm), which is a characteristic that is distinguished from the case of graphite.

In Fig. 17 are shown the spatial attenuation curves of wave amplitude normalized to zero-frequency attenuation. The oscillations of the experimental data are not very well reflected in the 3-group calculation, but somewhat better in the 20-group result, except for the case of 30,000 rad/sec beyond z=40 cm where the error of numerical integration on
Eq. (16) appears to have become large.
From the fair consistency obtained between experimental and calculated results, it may be concluded that resonance phenomena will occur in wave propagation also in lead prisms.

V. CONCLUSION

In order to illustrate resonance phenomena observed in neutron wave propagation in crystalline media, pulse propagation experiments with thermal and cold sources were carried out for two crystalline media, graphite and lead. The experiments were compared with theoretical analyses, also performed, based on the distributed source diffusion equation.

On the results obtained for pulse propagation and die-away phenomena, the infiltrating sub-Bragg cold neutrons created a sharp peak, which was reflected in the pulse shape of the detector response at points relatively far from the source. At such distances, in lead prism, penetrating neutrons below about 0.01 eV showed their effect on the detector responses, as well as the sub-Bragg neutrons.

The effect of resonance phenomena on the sinusoidal wave propagation can be considered to have been reflected in our calculations on both graphite and lead prisms, with fair consistency between experiment and theory, at least within certain limits of frequencies, positions, transverse bucklings and source spectra. The resonance phenomena are the reflection of the interference between the group propagation phenomena related to thermal neutrons and the infiltrating cold neutrons, the cold neutrons being sub-Bragg-cut neutrons in the case of graphite, and in the case of lead neutrons below about 0.01 eV.

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Fig. 17 Comparison of wave amplitude vs. $z$ between measured results and those by 3- and 20-group DSDT calculations, for lead

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