Theories of Miscible Injection in Iranian Fractured Reservoirs

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Abstract: Fractured reservoirs (e.g. Asmari reservoirs in Iran) provide over 20 % of the world oil reserves in our nation. Iran is one of the world's leading energy producing countries. Almost 90% of Iranian reservoirs are carbonate and oil production is due to natural fractures. The declining oil production from Iranian fractured reservoirs after several decades of exploitation and the significant amount of oil still remaining in place are of great concern to the Iranian oil company and fully justify its interest in EOR processes. One of the important mechanisms in EOR from fractured reservoirs is miscible fluid injection. Unfortunately, our knowledge about miscible fluid displacement in fractured porous media is limited and there is very little work in the literature on this method. Modeling of miscible displacement in fractured porous media is the subject of this study. In this modeling, mass transfer between matrix and fracture due to diffusion, and crossflow from matrix to fracture and from fracture to matrix are considered. From theoretical aspects, it is concluded that miscible injection in fractured reservoirs can be very efficient. Thus, it is proposed for increasing oil recovery from fractured reservoirs, miscible fluid injection be done in some suitable reservoirs.

Key words: fractured reservoirs, miscible displacement, crossflow equilibrium, Iranian carbonate reservoirs

1 Introduction

Iran is one of the world’s leading energy producing countries with an estimated 9% of the world’s remaining recoverable oil reserves and 17% of its natural gas reserves. Iran’s total oil in place is estimated to more than 500 billion barrels, and the recoverable oil reserves are estimated at 120 billion barrels. The reserves can be increased through different measure to increase recovery from existing fields, and through new field discoveries.

The vast resources of Iran mean that even a very modest increase in the recovery factor of one percent will add more than 5 billion barrels to its reserves. The ultimate benefits of improving oil recovery from existing fields may prove to be even higher than one percent, by further implementing new technologies and revising production plans.

More than ninety percent of Iranian petroleum reservoirs are carbonate. Carbonate reservoirs provide a large share of the world oil reserves. From a simple reservoir engineering classification, one may divide carbonate reservoirs into layered and fractured reservoirs. Carbonate reservoirs formations are generally tight, and...
the flow conditions for oil in the formation matrix are poor. The time needed to produce the oil will therefore be longer than the high permeable sandstone reservoirs. However, the carbonates are fractured to various degrees, i.e. the reservoirs consist of tight matrix blocks with fractures in between.

Generally, fractured hydrocarbon reservoirs provide over 20% of the world oil reserves and production. One of the prolific fractured reservoirs is the Asmari limestone reservoirs in Iran. In the Asmari reservoirs, most of the hydrocarbon is in In the matrix, but fracture PV could be as high as 1 to 2%. In these reservoirs, the average matrix porosity is around 20% and the matrix permeability is often low. The ultimate recovery from Asmari reservoirs is estimated to be around 22%.

In the case of fractured reservoirs, a major concern for miscible fluid injection is the early breakthrough and production of large quantities of the injected fluid. Capillary pressure contrast of the fracture and the matrix is a major parameter which causes low recovery efficiency of fractured reservoirs. The reduction of re-infiltration (through elimination of capillary pressure) is one positive element of miscible displacement in fractured porous media. Miscibility also eliminates the contrast in capillary pressure between matrix and fracture. A third positive element of miscible displacement in fractured porous media is that it provides a significant crossflow between the fracture and matrix.

There is very little work in the literature on miscible displacement in fractured porous media. Theoretical work is limited to a few papers published in the Russian literature. These papers center around the same formulation but do not address the basic issues of miscible displacement in fractured porous media. These papers include the effect of gravity on crossflow between the fracture and matrix. However, the assumptions which are required to proceed with the solution of the flow equations in a 3-D space are many and some unjustified. In 1969, Thompson and Mungan mainly studied the effect of displacement rate on recovery efficiency. They didn’t present any mathematical model for their work.

The purpose of this work is to provide a theoretical analysis of the miscible displacement in fractured porous media. Physical concepts related to the process will be emphasized in this work. In this study, a one dimensional model will be used. As will be seen, the one dimensional model can capture the main features of miscible displacement in fractured porous media.

### 2 Mathematical Formulation

We assume that (a) All fluids are incompressible and first contact miscible, (b) Each media has uniform properties, (c) dispersion is negligible compared with the convection effect by crossflow, (d) viscous fingering is negligible, (e) the displacement is strictly one dimensional in two media, and (f) there is no volume change of mixing. Fractional flow theory can be extended to two layers problems with the inclusion of viscous crossflow in the limiting case of vertical equilibrium (maximum crossflow). Using the volume element in Fig. 1 mass balance on a layer 1 (fracture medium) yields:

\[
\text{Mass In-Mass Out = Mass Accumulated}
\]

\[
\Delta t \left( q_{1|x} + a.W.\Delta x(C_2 - C_1) - q_{1|\Delta x} - Q_{XF1}C_{1|\Delta x} \right) = \phi_r h_r W.\Delta x(C_{1|\Delta t} - C_{1|t}) \]

\[\text{For the adjacent media 2 (matrix), the solvent cross-flowing out of layer 1 must necessarily end up on layer 2. Therefore, a similar mass balance yields:}\]

\[
\Delta t \left( q_{2|x} + q_{2|\Delta x} - a.W.\Delta x(C_1 - C_2) + Q_{XF2}C_{2|\Delta x} \right) = \phi_r h_r W.\Delta x(C_{2|\Delta t} - C_{2|t}) \]

In a more compact form, Eqs. 1 and 2 become

\[
-\Delta (q_1 C_{1|x})/\Delta x + (C_1 \Delta q_1)/\Delta x + a.W.(C_2 - C_1) = \phi_r h_r W.\Delta C_{1|x}/\Delta t \]

\[
-\Delta (q_2 C_{2|x})/\Delta x - (C_1 \Delta q_1)/\Delta x - a.W.(C_2 - C_1) = \phi_r h_r W.\Delta C_{2|x}/\Delta t \]

where \(\phi_r\) is the fracture-matrix mass transfer coefficient calculated using the following equation (5)

\[Sh = 0.023 Re^{0.8} Sc^{0.4} \]

where

\[
\text{Sh} = \frac{a(LD)}{Re} \]

\[
\text{Re} = \frac{(U.L.\rho)}{\mu} \]

\[
\text{Sc} = \frac{(D.\rho)}{\mu} \]

Sh is the dimensionless mass transfer coefficient, Re
and Sc are Reynolds and Schmidt numbers, respectively.

In these equations, D, U, ρ and μ are diffusivity, velocity, density and viscosity. Since in gas-oil contact, D usually is in order of 10⁻⁵ (cm²/s), Sc is very small. Thus, we can neglect the interface mass transfer.

Total mass balance on the control volume yields

\[ \text{Total Fluid In} - \text{Total Fluid Out} = \text{Total Fluid accumulation} \]

\[ D \frac{\partial}{\partial x}(q_1|x - q_1|x + D x) = j_1 h_1 W Q X F_1 \]

\[ D x \]

(9)

in media 1 (fracture), and for matrix

\[ D \frac{\partial}{\partial x}(q_2|x - q_2|x + D x) = j_2 h_2 W Q X F_2 \]

\[ D x \]

(10)

The total fluid rate across the entire cross-section is constant; hence, \((q_1 + q_2)|_x = (q_1 + q_2)|_{x=L} = Q_t\). It follows from this and adding Eqs. 9 and 10 that \(Q_X F_1 = Q_X F_2\).

Note that crossflow causes the layer flow rates \(q_1\) and \(q_2\) to vary with position. Since crossflow originates from medium 1, appropriately the solute flow rate crossflowing between media should be a function of the solute concentration in medium 1.

Taking the limits as both \(\Delta x\) and \(\Delta t\) approach zero provides the two partial differential equations which describe the flow of solute and solvent in two media. However, these equations cannot be solved independently since they are coupled by the crossflow term. Hence Eqs. 11 and 12 must be solved simultaneously.

\[ -q_1 \frac{\partial C_1}{\partial x} = \varphi_1 h_1 w_1 \frac{\partial C_1}{\partial t} \]

(11)

\[ -q_2 \frac{\partial C_2}{\partial x} = \varphi_2 h_2 w_2 \frac{\partial C_2}{\partial t} \]

(12)

In addition to mass balance equations, we need to Darcy’s law, density-concentration and viscosity-concentration relations

\[ \frac{\partial P_1}{\partial x} = -\frac{\mu_1 q_1}{(k w_1 h_1)} + \rho_1 g \]

(13)

\[ \frac{\partial P_2}{\partial x} = -\frac{\mu_2 q_2}{(k w_2 h_2)} + \rho_2 g \]

(14)

\[ \rho_1 = \rho_{\text{solvent}} + C_1 \Delta \rho \]

(15)

\[ \mu_1 = \mu_{\text{solvent}} + C_1 \Delta \mu \]

(16)

\[ \rho_2 = \rho_{\text{solvent}} + C_2 \Delta \rho \]

(17)

\[ \mu_2 = \mu_{\text{solvent}} + C_2 \Delta \mu \]

(18)

\[ \Delta \rho = \rho_{\text{solute}} - \rho_{\text{solvent}} \]

(19)

\[ \Delta \mu = \mu_{\text{solute}} - \mu_{\text{solvent}} \]

(20)

In above equations, \(C_i\) is the solute concentration in medium \(i\) \((C_1 = 1\) for solute and \(C_1 = 0\) for solvent). Subscripts 1, 2 refer to medium 1 (fracture) and medium 2 (matrix). Other symbols are defined in the Nomenclature. The above equations have to be solved with proper initial and boundary conditions. The initial and boundary conditions are

\[ C_1 = C_2 = 1 \text{ at } t = 0 \text{ and } x = x_h \]

(21)

\[ Q_t = \text{constant at } t = t \text{ and } x = x_h \]

(22)

\[ C_1 = C_2 = 0 \text{ at } t = t \text{ and } x = 0 \]

(23)

Using dimensionless variables Eqs. 11 and 12 are transformed to

\[ -q_{1D} \frac{\partial C_1}{\partial x_D} = R_1 \frac{\partial C_1}{\partial t_D} \]

(24)

\[ -q_{2D} \frac{\partial C_2}{\partial x_D} + (C_2 - C_1) \frac{\partial q_{1D}}{\partial x_D} = R_2 \frac{\partial C_2}{\partial t_D} \]

For crossflow from matrix (medium 2) to fracture (medium 1) mass balance equations are

\[ -q_{1D} \frac{\partial C_1}{\partial x_D} + (C_2 - C_1) \frac{\partial q_{1D}}{\partial x_D} = R_1 \frac{\partial C_1}{\partial t_D} \]

\[ -q_{2D} \frac{\partial C_2}{\partial x_D} = R_2 \frac{\partial C_2}{\partial t_D} \]

(25)

where

\[ t_D = \frac{Q_t \phi}{\Delta H W L} \]

(26)

\[ x_D = x/L \]

(27)

\[ q_{1D} = q_1/Q_t \]

(28)

\[ R_1 = \varphi_1 h_1/\phi H \]

(29)
\[ R_2 = \varphi_2 h_2 / \phi H \]  
\[ \phi H = \varphi_1 h_1 + \varphi_2 h_2 \]  
(30)

The viscosity-concentration relations in the form of dimensionless variables become

\[ \mu_{1D} = (1 + (N - 1). C_1) / N \]  
(32)

\[ \mu_{2D} = (1 + (N - 1). C_2) / N \]  
(33)

where

\[ N = \frac{\mu_{\text{solute}}}{\mu_{\text{solvent}}} \]  
(34)

initial and boundary conditions with these dimensionless variables are

\[ C_1 = C_2 = 1 \quad @ \quad t_D = 0 \quad \text{and} \quad x_D = x_0 \]  
(35)

\[ Q_{1D} = 1 \quad @ \quad t_D = t_D \quad \text{and} \quad x_D = x_D \]  
(36)

\[ C_1 = C_2 = 0 \quad @ \quad t_D = t_D \quad \text{and} \quad x_D = 0 \]  
(37)

Also, the total material balance is

\[ q_{1D} + q_{1D} = 1 \]  
(38)

Eqs. 24 and 25 must be solved simultaneously. Here, we consider two limiting cases: (a) no crossflow, and (b) vertical crossflow equilibrium (VE). Solution can be obtained from the method of characteristics (MOC).

Case 1) No Crossflow

For the case of no crossflow, there is no mass transfer between matrix and fracture. Therefore,

\[ \frac{\partial q_{1D}}{\partial x_D} = 0 \]  
(39)

and

\[ \frac{\partial q_{2D}}{\partial x_D} = 0 \]  
(40)

In this case, we divide solution as two parts, before breakthrough time and after breakthrough time.

2.1 Before Breakthrough Time

Refer to Fig. 2. Assume \( x_{1D} \) and \( x_{2D} \) be the positions of solute-solvent interface for fracture and matrix. Integrating Darcey’s law from \( x_{2D} = 0 \) to \( x_{2D} = 1 \) yields

\[ P_1 = -(q_{1D}. \mu_{\text{solute}} Q_T / (k_1 h_1)) [x_{1D} + N(1-x_{1D})] \]  
+ \( \log [\rho_{\text{solute}} x_{1D} + \rho_{\text{solute}}(1-x_{1D})] \)  
(41)

\[ P_2 = -(q_{2D}. \mu_{\text{solute}} Q_T / (k_2 h_2)) [x_{2D} + N(1-x_{2D})] \]  
+ \( \log [\rho_{\text{solute}} x_{2D} + \rho_{\text{solute}}(1-x_{2D})] \)  
(42)

Note, at \( x = L \) (or \( x_D = 1 \)), pressure in matrix is equal to pressure in fracture, \( P_1 = P_2 \). Using this and Eq. 38, we have

\[ q_{1D} = \frac{B}{x_{1D} + N(1-x_{1D}) + \frac{B}{x_{2D} + N(1-x_{2D}) - \frac{h_2}{H}}} \]  
(43)

\[ B = K h_1 / h_2 \]  
(44)

\[ K = k_1 / k_2 \]  
(45)

\[ G = \Delta \rho g k_2 W H / \mu_{\text{solute}} Q_T \]  
(46)

Eqs. 24 and 25 yield

\[ q_{1D} = R_1 \frac{dx_{1D}}{dt_D} \]  
(47)

\[ q_{2D} = R_2 \frac{dx_{2D}}{dt_D} \]  
(48)

and, from Eqs. 43 and 47 we have

\[ \frac{dx_{1D}}{dt_D} = \frac{B}{R_1 \left\{ x_{1D} + N(1-x_{1D}) + \frac{B}{x_{2D} + N(1-x_{2D})} \right\} - \frac{h_2}{H}} \]  
(49)

Therefore, from Eq. 38,

\[ \frac{dx_{2D}}{dt_D} = 1 / R_2 - \frac{R_1}{R_2} \frac{dx_{1D}}{dt_D} \]  
(50)

Eqs. 49 and 50 are integrated with respect to \( t_D \) (by using Eq. 38). Initial conditions for this integration are

\[ x_{1D} = 0 \quad \text{at} \quad t_D = 0 \]

\[ x_{2D} = 0 \quad \text{at} \quad t_D = 0 \]

Fig. 2 Schematic of Displacement without Any Crossflow.

Fig. 3 The Plot of Dimensionless Length in Matrix and Fracture versus Dimensionless Time before Breakthrough.
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By these conditions, \( x_{1D} \) and \( x_{2D} \) are plotted versus dimensionless time, Fig. 3.

\[ x_{1D} = x_{2D} = 0 \quad @ \quad t_D = 0 \]  \hspace{2cm} (51)

2.2 After Breakthrough

At this time, in the matrix there are solute and solvent, but fracture is filled with solvent so that

\[
P_1 = -\left( q_{1D} \mu_{\text{solvent}} \cdot Q \cdot L \cdot (k_1 \cdot h_1) \right) + L_g \rho_{\text{solvent}} \quad \cdots \cdots (52)
\]

\[
P_2 = -\left( q_{2D} \mu_{\text{solvent}} \cdot Q \cdot L \cdot (k_1 \cdot h_1) \right) [x_{2D} + N(1 - x_{2D})] \\
+ L_g [\rho_{\text{solvent}} \cdot x_{2D} + \rho_{\text{solute}} \cdot (1 - x_{2D})] \quad \cdots \cdots (53)
\]

Thus, pressures equality and Eq. 38 yield

\[
q_{2D} = \frac{1 + G.N.B. (1 - x_{2D}) \cdot h_2}{H(1 + B.M)} \quad \cdots \cdots (54)
\]

Where

\[
M = x_{2D} + N(1 - x_{2D}) \quad \cdots \cdots (55)
\]

Again, by using Eqs. 47 and 48, \( x_{1D} \) and \( x_{2D} \) are plotted versus dimensionless time, Fig. 4.

Case 2) Vertical Crossflow Equilibrium (VCE)

The vertical crossflow equilibrium concept has been used extensively in the petroleum literatures (6-12). Generally, vertical crossflow equilibrium means that the sum of the driving forces in the vertical direction is zero for all fluid components. In other words, VCE means that the vertical pressure drop is zero at all time and positions in the reservoir. This means that the horizontal pressure gradients are equal at all vertical positions. It is not generally recognized that assuming VCE in a displacement implies perfect vertical communication, and this is the basis for claim that VCE implies the maximum degree of crossflow possible. Based on the results of Zapata and Lake (13), VCE will be a good assumption for reservoirs with effective length to thickness ratios of 10 or more.

Thus, from Eq. 28 and Darcey’s law we have

\[
q_{1D} = \frac{\mu_{1D} \cdot B + G.B. (C_1 - C_2) \cdot h_2}{H(\mu_{1D} + B\mu_{2D})} \quad \cdots \cdots (56)
\]

and

\[
q_{2D} = \frac{\mu_{1D} + G.B. (C_2 - C_1) \cdot h_2}{H(\mu_{1D} + B\mu_{2D})} \quad \cdots \cdots (57)
\]

Utilizing method of characteristics, Eqs. 24 and 25 are formulated as an eigenvalue problem. The solution can be obtained from this method as

\[
dx_{1D}/dt_D = q_{1D}/R_1 \quad \cdots \cdots (58)
\]

and

\[
dx_{2D}/dt_D = q_{2D}/R_2 \quad \cdots \cdots (59)
\]

as the characteristic directions for Eqs. 24 and 25, respectively. Also, the solutions along the characteristic for Eqs. 24 and 25 are

\[
\frac{dC_2}{dC_1} = \frac{R_2 \cdot (C_2 - C_1) \cdot \frac{dq_{1D}}{dC_1} - R_1 \cdot (C_2 - C_1) \cdot \frac{dq_{2D}}{dC_2}}{R_1 \cdot q_{1D} - R_2 \cdot (C_2 - C_1) \cdot \frac{dq_{1D}}{dC_1} - R_1 \cdot q_{2D}} \quad \cdots \cdots (60)
\]

\[
\frac{dC_1}{dC_2} = \frac{R_2 \cdot (C_2 - C_1) \cdot \frac{dq_{1D}}{dC_1} - R_1 \cdot (C_2 - C_1) \cdot \frac{dq_{2D}}{dC_2}}{R_1 \cdot q_{1D} - R_2 \cdot (C_2 - C_1) \cdot \frac{dq_{1D}}{dC_1} - R_1 \cdot q_{2D}} \quad \cdots \cdots (61)
\]

Note that, there is only one non-trivial characteristic direction for each equation.

A convenient way to represent these solutions in graphical form is the time-distance diagram (characteristic directions) and the concentration path diagram (solution along the characteristic).

As observed from Eqs. 60 and 61, the solutions along the characteristic require numerical integration in a step-wise fashion. In order to determine composition

![Fig. 4](image1) The Plot of Dimensionless Length in Matrix versus Dimensionless Time after Breakthrough.

![Fig. 5](image2) Compositional Paths.
paths, expressions for $q_{1D}$ and $q_{2D}$ from Eqs. 56 and 57 are used. Fig. 5 is an example of two media displacement with the necessary parameters given on the graph. The entire solution is a combination of the Region I solution integrated from the initial condition ($C_1 = C_2 = 1$) and the Region II solution integrated from the final condition ($C_1 = C_2 = 0$). The two solutions are intersected at the no crossflow point which represents a region of constant composition.

Every point in the concentration path diagram ($C_1$ versus $C_2$) is represented in the coordinates ($x_D$ versus $t_D$) from the characteristic direction in Fig. 6. The characteristic directions in their integrated form appear as an infinite number of straight lines fanning out from the common origin. A region of constant concentration conditions separates the spreading wave regions. The primary importance of a time-distance diagram is that it determines in ($x_D$ versus $t_D$) coordinates any pair of solute concentrations for which a constant velocity is associated.

The concentration path diagram and time-distance diagram provide all the information necessary to predict the solvent cut and oil recovery for two media miscible displacement with VCE. The results for the miscible displacement presented in Figs. 7 and 8.

### 3 Conclusions

In this work, we provide a theoretical analysis of the miscible displacement in fractured porous media. The development of equations accounts only for crossflow due to pressure gradients formed by the displacing (injected) and displaced (initial) fluids. Analysis of one dimensional miscible displacement theory shows that in a miscible injection process, injected fluids do not flow through the fracture. There is viscous crossflow between fracture and matrix. The results from laboratory study will be compared with the predicted results.
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from this model in future work (14).

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