TRANSFORMATION OF WAVE GROUPS IN A DIRECTIONAL SEA

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The purpose of this study is to determine wave group evolution by investigating the transformation of multi-directional spectra at constant depth and on a sloping beach. The transformation of wave groups can be evaluated by computing multi-directional nonlinear waves based on the third-order Zakharov equation. Initial conditions for numerical simulation were characterized by the Gaussian spectrum for several values of significant wave heights and water depths. The numerical results show that the distribution of the energy processes affects the evolution of the wave groups, and the characteristics of directional wave groups can be reproduced through the Zakharov equation model. Finally, field observation data at Akabane beach were used to demonstrate the transformation of wave groups in a real directional sea.

Key Words : wave groups, directional spectra, wave-wave interaction, Zakharov equation

1. INTRODUCTION

Waves in an actual sea are composed of an infinite number of components which have different frequencies and directions. Therefore, ocean surface waves have a complex pattern and are random in amplitude, period and direction. As surface waves propagate from deep to shallow water, the directional wave spectrum is transformed due to both linear and nonlinear processes. Changes in the directional spectrum due to a linear effect can be accurately predicted by linear models, which are still widely used in coastal and offshore applications. However, Elgar et al.1) found that the frequency-directional spectra of shoaled waves observed in the field and in the laboratory are significantly different from those predicted by linear theory.

According to Goda2), although sea waves may look random, inspection of wave records indicates that high waves fall into groups rather than emerge individually. The interest in wave groups is stimulated by the fact that wave grouping and associated nonlinear effects play an important role for many coastal engineering problems. For instance, the stability of coastal structure, irregular wave run-up and the rate of overtopping depend on the number of waves in such a group. However, the number of works dealing with nonlinear aspects of wave group transformation is still limited.

Wave group evolution may be governed by a balance of nonlinearity, dispersion and dissipation. It has been observed in many field investigations that the distribution of waves in a group approaching the shore becomes more uniform, so that the maximum wave height in the group tends to decrease. This wave amplitude modulation may result from dissipation in the bottom boundary layer as well as from nonlinear and dispersive effects. Barnes and Peregrine3) numerically obtained the reduction in the maximum wave height with a decrease in the water depth. They used a full irrotational fluid motion solver and found that the maximum wave height in the group becomes less than its initial value as the nonlinearity and dispersion cannot be balanced in shallow water, after the critical depth $kh = 1.36$. But furthermore, Janssen and Onorato4) found that a transfer of energy occurs as an effect of nonlinear interactions in shallow-water, with $kh = 1.36/2$.

The nonlinear interaction of gravity waves has been a subject of interest for many years. According to Longuet-Higgins5), the interaction produces only a small modification to the motion in the second-order,
which remains bounded in time. In the third approximation, it is possible for a transfer of energy to take place from three primary waves to a fourth wave, in such a way that the amplitude of the fourth wave increases linearly with time. Nwogu\(^{16}\) conducted an investigation on the nonlinear evolution of directional wave spectra in shallow water using the second-order of the Boussinesq equation, while Kit et al.\(^{7}\) investigated wave group evolution in shallow water of constant depth by applying the Korteweg-de Vries equation. Good agreements between the experimental and numerical results were obtained.

Evaluation of nonlinear wave groups can also be demonstrated by the third-order Zakharov equation, which was derived by Zakharov\(^{8}\) in 1968. Numerous investigations were subsequently executed\(^{2,9,10,11,12}\). Stiassnie and Shemer\(^{13}\) extended the derivation to a finite depth and to the next order. Deterministic and stochastic of Zakharov equation are described in detail in Stiassnie and Shemer\(^{14,10,15}\), Janssen\(^{16}\), and also Eldeberky and Madsen\(^{17}\). The Zakharov equation can evaluate nonlinear wave fields that are free of any constraints on the spectral width. A numerical study based on the third-order Zakharov nonlinear equation, which was further modified to describe slow spatial evolution of unidirectional waves, was conducted by Shemer et al.\(^{17}\). Their model accurately describes the variation of the group envelopes along the tank at an intermediate water depth.

Furthermore, Kioka et al.\(^{18,19}\) investigated the transformation of a deterministic wave group over a 1:30 sloping beach, both experimentally in a wave tank and theoretically by a numerical solution of the Zakharov nonlinear equation. They found that wave groups of high wave steepness undergo weak defocusing in shallow water. The maximum wave heights no longer yield to the shoaling curves given from the finite amplitude theory. The spectrum widening and front tail asymmetry are more pronounced at these finite amplitude theory. The spectrum widening and no longer yield to the shoaling curves given from the

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d= \frac{\partial B(k,t)}{\partial t} = \iint \int T(k,k_1,k_2,k_3)B'(k_1,t)B(k_2,t)B(k_3,t) \delta(k + k_1 - k_2 - k_3) \exp \left\{ i(\omega_1 - \omega_2 - \omega_3) t \right\} , (1)

\text{where } B \text{ denotes the complex amplitude, } \ast \text{ is the complex conjugate, } \delta \text{ is the Dirac } \delta \text{-function and the kernel } T(k,k_1,k_2,k_3) \text{ is given in Stiassnie and Shemer}^{13} \text{ and corrected by Mase and Iwagaki}^{20}.

Energy transfer is induced due to the fact that four trains of waves interact if the following resonance conditions are met among free waves:

\begin{align}
  k + k_1 - k_2 - k_3 &= 0, \ |\omega_1 - \omega_2 - \omega_3| &\leq O(\epsilon^2) ,
\end{align}

where \(\epsilon\) is a small parameter representing the magnitude of nonlinearity, and the wave vectors \(k, k_1, k_2, k_3\) and the frequencies \(\omega_1, \omega_2, \omega_3\) each satisfy the following dispersion relation, with \(h\) being the water depth:

\begin{align}
  \omega^2 &= g |k| \tanh |k| h ,
\end{align}

In order to describe the spatial evolution, Shemer et al.\(^{17}\) have modified the third-order Zakharov model into the form

\begin{align}
  i c_g \nabla_h B &= \iint \int T(k,k_1,k_2,k_3)B'(k_1,k_2,k_3) \delta(\omega_1 - \omega_2 - \omega_3) \exp \left\{ -i(k + k_1 - k_2 - k_3) x \right\} , (4)
\end{align}

\text{where } c_g \text{ is the group velocity and } \nabla_h \text{ is the horizontal gradient. This spatial Zakharov equation de-}
scribes the evolution of the complex amplitude $B$ of each free wave in the spectrum due to four-wave interaction in a mild slope (|$\nabla h$| $\leq O(\varepsilon^2)$) space domain, which satisfies the near-resonant condition:

$$\omega_1 + \omega_2 - \omega_3 = 0, \quad |k + k_1 - k_2 - k_3| \leq O(\varepsilon^2), \quad (5)$$

The mode-coupled discrete Zakharov equation can be written as

$$ic \nabla B = T(k_j, k_{j'}, k_{j''}, k_{j'''})B_j \int B_j B_{j'} B_{j''} B_{j'''} \exp \left\{ -i(k_{j'} - k_{j''} - k_{j'''})x \right\}$$

$$(n, p, q = 1, 2, \ldots, N). \quad (6)$$

The set of mode-coupled nonlinear complex ordinary differential equations is solved using the fourth-order Runge-Kutta method. When calculating the kernel in Eq. (4), we have introduced Stokes’ corrections to remove near-resonance singularities. Nevertheless, Eq. (4) is invalid for water of very shallow depth; the equation requires that the dispersion remains sufficiently strong (see Agnon21)). The first-order free surface elevation $\eta(x, t)$ is related to the quantity $B$ and computed through

$$\eta(x, t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left( \frac{\omega(k)}{2g} \right)^{1/2} B(k, t) \exp \left\{ i(k \cdot x - \omega(k) t) \right\} + \ast , \quad (7)$$

(2) Wave group structure

The structure of wave groups can be quantitatively described using a wave envelope. The wave envelopes of various frequency bands can be calculated using a Hilbert transform. If the sea surface elevation $\eta(t)$ is a stationary random function of time, then the Hilbert transform $\zeta(t)$ is given by

$$\xi(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\eta(t)}{t-x} , \quad (8)$$

where $P$ indicates the Cauchy value. With the Hilbert transform $\zeta(t)$ of the function $\eta(t)$, the analytic function is given as

$$S(t) = \eta(t) + i \xi(t) = A(t) \exp \{ i \varphi(t) \} , \quad (9)$$

The wave envelope $A(t)$ can then be obtained by

$$A(t) = \left[ \eta^2(t) + \xi^2(t) \right]^{1/2} , \quad (10)$$

The envelope $A(t)$ is always symmetrical with respect to the $t$-axis, as $\eta(t)$ is composed of only first-order free waves. Only the fundamental frequency band $0.5f_p \sim 1.5f_p$, which produces free waves only and does not include the bound waves, is considered and calculated.

The amplitude $A_{ave}$ denotes the average value of the envelope amplitude (see Fig.1). The zero-up cross method relative to $A_{ave}$ is used to determine the wave group period $T_g$. The wave group amplitudes $A_{ave}$ and $A_{max}$ denote the average and maximum of the envelopes, respectively. The wave group period $T_{ave}$ is the average value of $T_g$ and $T_{max}$ corresponds to the period of the wave group containing $A_{max}$.

3. NUMERICAL SIMULATIONS

The wave conditions for the numerical simulation, characterized by the peak period $T_p$, relative water depth $k_p h$, wave amplitude $a(\omega, \theta)$ and principal direction $\theta$ were defined for input in the nonlinear wave interaction modeling. The principal wave direction $\theta = 0$ was used for all the simulations except the field data. The wave model requires initial condition information, describing the initial state of the sea. In this study, the initial sea state was described as a Gaussian spectrum in the form
\[ S(\omega, \theta) = \frac{m_0}{2\pi \sigma_\omega \sigma_\theta} \exp \left\{ -\frac{(\omega - \omega_0)^2}{2\sigma_\omega^2} - \frac{\theta^2}{2\sigma_\theta^2} \right\}, \quad (11) \]

where \( m_0 \) is the zero-th moment of the spectrum, \( \omega \) is the angular frequency, and \( \sigma_\omega \) and \( \sigma_\theta \) are standard deviations for frequency and direction, respectively.

By taking a finite range of frequency \( (\omega_{\text{min}}, \omega_{\text{max}}) \) and direction \( (\theta_{\text{min}}, \theta_{\text{max}}) \), the initial amplitude \( a(\omega, \theta) = \left(2S(\omega, \theta)d\omega d\theta\right)^{1/2} \) was determined for calculating the complex amplitude \( B \), which is obtained by
\[
B = \pi \left( \frac{2g}{\omega} \right)^{1/2} a(\omega, \theta) \exp(i\phi), \quad (12)
\]

where \( \phi \) is the random phase.

The wave steepness \( ak_p = 0.07 \sim 0.2 \) (\( a \) and \( k_p \) being the carrier wave amplitude and number) were used for simulation. The relative water depths, denoted by \( k_p h \), in deep water and intermediate water depth are equal to 5.0 and 1.0, respectively, and the relative water depth on the sloping beach is \( 1.0 \geq k_p h \geq 0.5 \), with slope calculation \( k_p h(i) = k_p \left(26 - 13(i/2)^2\right)\), where \( z \) is the number of segments and \( i = 1,2,3,...,z \).

At intermediate water depth \( k_p h = 1.0 \), we are not considering the adjustment of the spectrum as the effect of water depth, as in the Wallops spectrum. We just assume that the same shape of the Gaussian spectrum is used in deep water and at intermediate water depth.

Regarding the number of wave components of the directional spectrum, Japan Meteorological Agency (JMA) has improved their operational wave model, so that 400 components have become 900 components. The directional components of wave spectrum become fine and can express isotropic and smooth spreading, unlike the previous model\(^{[25]}\). As the purpose of this study is to investigate the transformation of wave groups, which strongly depends on the resonance of the wave components, a larger number of wave components were used in this simulation. At constant depth, directional spectra were simulated with 1550 components, which consisted of 50 components of frequency and 31 directional components. However, in sloping cases the directional spectra were simulated with 50 frequency components and 21 directional components. Additionally, refraction effects on sloping cases were calculated based on linear theory. The directional spectra were normalized by the peak of the initial directional spectrum \( S_0(f_p, \theta_p) \). Finally, evolution of wave groups as a result of the directional spectrum was compared with unidirectional simulation, which consisted of 100 frequency components.

The Runge-Kutta method, which solves a differential equation numerically, gives the integration of the spatial evolution of the nonlinear waves.

4. RESULTS AND DISCUSSION

Now we present the results of the simulations as well as an analysis of these results. Nonlinear wave interaction effects on the evolutions of directional spectra were analyzed to investigate the transformation of wave group structures. Attention is paid mainly to the transformation of directional spectra; then the evolution of the wave groups due to the nonlinear wave-wave interaction both for directional spectra and unidirectional spectra are compared.

1. Constant depth

The Gaussian spectra for \( ak_p \) equal to 0.07 and 0.13 were simulated at constant water depth with relative water depth \( k_p h = 1.0 \); this is presented in Fig.2. The first row in Fig.2 illustrates the directional spectrum at \( x = 0 \) or the initial condition, whereas the second and third rows illustrate the directional spectrum at \( x = 50L_p \). The variable \( L_p \) denotes the length of the wave corresponding to the peak period \( T_p \) at the spectrum. The directional spectrum for \( ak_p = 0.07 \) shows a very small evolution, while the evolution of the directional spectrum for \( ak_p = 0.13 \) indicates that the directional spectrum grows at the peak frequency, increases in energy, and then distributes its energy to the side near the main direction. This energy is absorbed more from lower frequencies than from higher frequencies because the low-frequency part is affected more significantly than the high-frequency part.

With the same significant wave height for the spectra, another case was simulated in deep water with \( k_p h = 5.0 \). The transformation of the directional spectrum until \( x = 100L_p \) indicates that the evolution of the directional spectrum is very slow, as shown in Fig.3(a). In fact, the directional spectrum undergoes only minor modification. This indicates that the nonlinear wave interactions are affected more significantly in relatively shallow water than in deep water.

With an increase in significant wave height, \( ak_p = 0.2 \), evolution of the nonlinear wave interaction in deep water can be seen more clearly, as shown in Fig.3(b). At \( x = 100L_p \), spatial evolution of the directional spectrum indicates that interchange energy transfers occur near the spectra peak and generate another peak of frequency. Directional spreading occurs near the spectra peak. In contrast to the case
for intermediate water depth, in deep water the high-frequency part of the spectrum is affected more significantly than the low-frequency part. Evolution of the directional spectrum is much more pronounced at high frequency.

**Fig.2** Evolution of directional spectra from $x = 0$ to $x = 50L_p$ for the case of $k_p h = 1.0$.

**Fig.3** Evolution of directional spectra at $x = 100L_p$ for $k_p h = 5.0$.

**Fig.4** Evolution of directional spectra as in **Fig.3(a)** with different initial random phases.

**Fig.5** Evolution of wave groups from $x = 0$ to $x = 100L_p$ for the case of $k_p h = 5.0$ with $a_{k_p} = 0.1$. 

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According to Shemer et al., the initial-phase in the complex wave spectra is essential in determining the eventual shape of the surface elevation variation. In this respect, the directional spectrum in Fig.3(a) was simulated again with different random phases, as presented in Fig.4. The result indicates that the energy distribution on the spectrum gives a totally different spectral shape, which will provide a different form of the wave groups.

Transformation of wave groups regarding the distribution of energy in the directional spectrum is given from Fig.5 to Fig.7. Wave group structures are allocated using a wave envelope. The envelopes are formed only by free waves, not including the bound waves. The initial variation of the surface elevation at \( x = 0 \) can be compared with the surface elevation at \( x = 50L_p \) for \( k_p h = 1.0 \) and at \( x = 100L_p \) for \( k_p h = 5.0 \).

Wave group transformation for the case of \( k_p h = 5.0 \) with \( \alpha k_p = 0.1 \) is displayed in Fig.5. Wave group envelopes express that the group shape is almost the same at \( x = 0 \) and \( x = 100L_p \), which indicates that nonlinear effects are weak. As presented before, the distribution of energy on this condition, as seen in Fig.3(a), is a minor modification. The result of the unidirectional spectrum also illustrates a slight evolution.

At high steepness, the nonlinear effects are clearly pronounced and exhibit themselves in the evolution of the shape of wave groups, as shown in Fig.6. The shape of the wave groups is totally different; the maximum value of the wave groups’ amplitude \( A_{g\text{max}} \) increased slightly, accompanied by a reduction in the \( T_g \) (\( T_g\text{mean} \) and \( T_g\text{max} \)).

Comparison of the evolution of the wave group structures based on directional simulation and unidirectional simulation is presented in Table 1. As the effect of the initial random phase, one case is simulated three times and the average values of wave groups’ structures are presented.

Transformation of wave groups at intermediate water depth \( k_p h = 1.0 \) is displayed in Fig.7. Nonlinear effects become stronger with increasing wave steepness, and the crest of the maximum wave group amplitude \( A_{g\text{max}} \) increases. The period of the max-
mum wave group amplitude becomes wider; however, the mean period is still the same.

(2) Sloping beach

Directional spectra for sloping beach cases are studied by simulated numerical calculation from intermediate through shallow water depths. For modeling the field condition, the initial condition is specified at Station A, and further, the waves propagate to Station B with distance $30L_p$.

The transformations of the Gaussian spectra with increasing wave steepness are presented in Fig.8. Evolution of directional spectra indicates that the distribution of the energy is dominant in the middle range of the frequency spectrum. Transformation of the frequency spectrum describes that the nonlinear interaction transfers energy from the peak frequency of the spectrum to the lower frequency and higher frequency. At the lower frequency, energy is received from the peak frequency and absorbed, thus down-shifting the peak frequency.

The effect of shoaling on the evolution of the directional spectrum is to enhance the wave height as the wave approaches the coast. Regarding the influence of shoaling, the low-frequency part of the spectrum is affected more significantly than the high-frequency part. In fact, the highest frequency may not be affected at all, because the water depth may be larger for these frequencies.

As the waves propagate to the coast, the directional spreading becomes narrow owing to the wave refraction effect. Because the waves are propagating perpendicularly to the coast, the energy is increasing in this direction. However, at the highest steepness, $ak_p = 0.2$, the energy at the peak frequency decreases, generating another peak at a lower frequency, which affects the wave profile.

The wave profiles evolve into a form with sharp and high crests and shallow and flat troughs, as shown in the last wave profiles of Fig.9. Because the heights of larger waves are more enhanced than those of smaller waves, the distribution of individual wave heights becomes stretched. Evolution of wave groups at lower steepness indicates that the shape of the wave groups is almost the same, and slightly changes at $ak_p = 0.13$. Transformations of directional wave groups on the sloping beach are also compared with unidirectional spectra. Evolution of wave group structures is also presented in Table 1.

With the same initial peak energy density on the frequency spectrum, the unidirectional simulation produces a higher initial amplitude than directional simulation. The maximum amplitude $A_{g_{\text{max}}}$ is not the real maximum wave height, because the simulation does not include the bound wave component; therefore, the value is slightly smaller than the real wave.

Comparison of evolutions of wave groups on directional spectra with those on unidirectional spectra indicates that evolution of wave groups in deep water and at intermediate water depth significantly affects nonlinear interaction in directional simulations.

When the directional effect is considered, transformation of wave groups in deep water is much more pronounced at $ak_p = 0.2$. The effects of wave interaction are enhanced in relatively shallow water;
however, the nonlinear interaction is reduced on a sloping beach, which decreases the maximum wave height.

In deep water, by increasing the wave steepness, the variability of the characteristic wave amplitude is increased. This is followed by a reduction in the wave group period ($T_g$), whereas at intermediate depth the maximum wave amplitude in the group is increased, but the maximum wave group period becomes slightly longer. However, when the nonlinear parameter increases on the sloping beach, the wave groups become stretched. This reduces the maximum wave group amplitude and increases the maximum wave period.

The wave profiles have almost the same period but gradually varying amplitudes. This is caused by the energy of the wave spectrum, which is concentrated within a narrow range of frequency. The variability of the characteristic wave height increases as the spectral peak becomes sharp. However, if the frequency spectrum gets narrower, the envelope becomes longer, and if the directional spread decreases, the wave crest widens.

(3) Field Observation

The wave data were collected 6km offshore (Station A; 26m deep) and 1km offshore (Station B; 13m deep) in an extension line perpendicular to the coastal line at Akabane in the Atsumi Peninsula, Japan on the Pacific coast. The bathymetry is nearly uniform, and the bottom slope changes gradually from 1/400 to 1/100. The measurements were performed by 2 wave gauges located at intermediate water depths, with $1.0 \geq k_p h \geq 0.5$. The sea surface elevation and bottom velocities were recorded every two hours for one hour long at a sampling data rate of 0.5 s for the observation period$^{23}$. 

Fig. 9 Evolution of directional wave groups from St. A to St. B.
Table 1  Comparison of the evolution of wave groups structures for directional spectra with that for unidirectional spectra.

<table>
<thead>
<tr>
<th>Wave group structures</th>
<th>$\Delta G_{\text{mean}}/\Delta G_{\text{ave}}$</th>
<th>$\Delta G_{\text{max}}/\Delta G_{\text{ave}}$</th>
<th>$T_{g_{\text{mean}}}/T_p$</th>
<th>$T_{g_{\text{max}}}/T_p$</th>
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<tbody>
<tr>
<td><strong>Directional</strong></td>
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<td>$k_h = 5.0$</td>
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<tr>
<td>$\alpha_k = 0.1$</td>
<td>$x = 0$</td>
<td>1.35</td>
<td>2.04</td>
<td>4.17</td>
</tr>
<tr>
<td></td>
<td>$x = 100 L_p$</td>
<td>1.33</td>
<td>2.02</td>
<td>4.17</td>
</tr>
<tr>
<td>$\alpha_k = 0.2$</td>
<td>$x = 0$</td>
<td>1.49</td>
<td>2.08</td>
<td>4.17</td>
</tr>
<tr>
<td></td>
<td>$x = 100 L_p$</td>
<td>1.72</td>
<td>2.77</td>
<td>3.33</td>
</tr>
<tr>
<td><strong>Unidirectional</strong></td>
<td></td>
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<tr>
<td>$\alpha_k = 0.1$</td>
<td>$x = 0$</td>
<td>1.45</td>
<td>2.64</td>
<td>2.94</td>
</tr>
<tr>
<td></td>
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<td>1.56</td>
<td>2.85</td>
<td>2.94</td>
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<tr>
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<td>1.40</td>
<td>2.58</td>
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<td></td>
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<td>2.50</td>
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<td>$k_h = 1.0$</td>
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<tr>
<td>$\alpha_k = 0.13$</td>
<td>$x = 0$</td>
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<td>2.02</td>
<td>4.17</td>
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<td></td>
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<td>1.43</td>
<td>1.90</td>
<td>4.17</td>
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<tr>
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<td>1.55</td>
<td>2.19</td>
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<tr>
<td></td>
<td>$x = 50 L_p$</td>
<td>1.32</td>
<td>2.38</td>
<td>4.17</td>
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<tr>
<td><strong>Unidirectional</strong></td>
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<tr>
<td>$\alpha_k = 0.13$</td>
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<tr>
<td></td>
<td>$x = 50 L_p$</td>
<td>1.53</td>
<td>2.78</td>
<td>2.94</td>
</tr>
<tr>
<td>$\alpha_k = 0.2$</td>
<td>$x = 0$</td>
<td>1.45</td>
<td>2.65</td>
<td>2.94</td>
</tr>
<tr>
<td></td>
<td>$x = 50 L_p$</td>
<td>1.48</td>
<td>2.95</td>
<td>2.94</td>
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<tr>
<td><strong>Sloping</strong></td>
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<tr>
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<td>2.33</td>
<td>5.20</td>
</tr>
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<td></td>
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<tr>
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<td>2.58</td>
<td>2.94</td>
</tr>
<tr>
<td>$\alpha_k = 0.2$</td>
<td>St. A</td>
<td>1.45</td>
<td>2.59</td>
<td>2.94</td>
</tr>
<tr>
<td></td>
<td>St. B</td>
<td>1.44</td>
<td>2.18</td>
<td>3.13</td>
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</table>

Using the measurements from field observations at Akabane Beach, data on August 8, 2006 were calculated. The initial condition of the directional spectrum was determined by the incident wave spectrum which was obtained at Station A, as shown in Fig.10. The initial directional spectrum at Station A indicates that the spectrum centered on peak frequency $f_p = 0.084$ and $\theta = -9^\circ$. Waves propagated from Station A to Station B with a relative water depth of $1.0 \geq k_h \geq 0.5$. An angle of 0 indicates a line perpendicular to the shoreline.

The directional spectrum of field data at Station B was used to verify the numerical result, and it indicates that the spectrum increases the peak frequency. The directional spectrum of field data was calculated using the EMLM estimation method\(^{24}\). All available theories are based on linear theory, therefore we admit the linear theory analysis, but only the fundamental frequency band is considered, and that composes the free wave only.

The numerical results of the spatial evolution directional spectrum at Station B indicate that the energy transfer occurs from the peak frequency to the lower frequency and higher frequency; therefore, the spectrum slightly widens in frequency and narrows in direction. The results of numerical simulation at Station B show a trend close to the directional spectrum of field data; however, the energy at peak frequency looks smaller than in the field data. This discrepancy is caused by the effects of the linear calculation on the field data.

Lin and Lin\(^{25}\) introduce a new wave-breaking function to calculate the wave breaking as a result of white-capping at intermediate depths. We have used this formulation to see effects of the white-capping
on the nonlinear directional spectra; however, the result only gives minor evolution. Therefore, the effect of white-capping in this simulation is weak.

Wave group envelopes from measurement data at Station A and Station B are shown in Fig. 11. In this figure, the wave group envelopes were governed only by the fundamental frequency, without the bound waves. Using the Hilbert transform wave group structure at Station A, the values $A_{ave} = 1.08m$, $A_{g\text{mean}} = 1.31m$, $A_{g\text{max}} = 2.16m$, $T_{g\text{mean}} = 80s$ and $T_{g\text{max}} = 72s$ were obtained; further at Station B, $A_{ave} = 1.03m$, $A_{g\text{mean}} = 1.25m$, $A_{g\text{max}} = 2.15m$, $T_{g\text{mean}} = 80s$ and $T_{g\text{max}} = 147s$.

Wave groups from the simulation results at Station B are presented in Fig. 12. As discussed previously, the initial random phase that affects the energy distribution is different, which will affect the wave profiles and, further, the wave groups. Therefore, we calculated the directional spectrum at Station A three times, as shown in Fig. 12.

The envelopes of the wave group were also formed by free waves only. By the Hilbert transform calculation, wave groups’ structures at Station A were obtained: $A_{ave} = 1.07m$, $A_{g\text{mean}} = 1.30m$, $A_{g\text{max}} = 2.12m$, $T_{g\text{mean}} = 80s$ and $T_{g\text{max}} = 72s$ and the wave groups’ structures as a result of directional simulation at Station B: $A_{ave} = 1.13m$, $A_{g\text{mean}} = 1.38m$, $A_{g\text{max}} = 2.10m$, $T_{g\text{mean}} = 80s$ and $T_{g\text{max}} = 150s$.

Wave group structures from the simulation results are in good agreement with the field data; therefore, this model could be used to predict the evolution of a wave group. Although the field data and simulation results have different shapes of directional spectra and wave group envelopes, they produce almost the same wave group structures.

5. CONCLUSIONS

The transformation of wave groups has been investigated by numerical simulation based on the third-order Zakharov equation. The main conclusions can be summarized as follows.

The third-order Zakharov equation model is able to predict the effect of the nonlinear interaction on the transformation of directional spectra for constant depth and for a sloping beach. The nonlinear transfer of energy was found to control the shape of the spectrum, including the development of the peak and the wave groups.

In relatively shallow water, nonlinear wave interactions appear to have a more significant effect than in deep water. The low-frequency part of the spectrum is affected more significantly than the high-frequency part; however, in deep water the high-frequency part is affected more significantly than the low-frequency part.

On the sloping beach, the transformation of directional spectra indicates that the lower frequencies are enhanced more than the higher frequencies. This results in a higher energy in the principal direction, and the peak of the spectrum slightly shifts to the lower frequency.
By increasing the wave steepness, the effects of the nonlinear wave interaction become stronger. The evolution of the directional spectrum is much more pronounced at a high steepness than at a lower steepness.

Wave groups can be characterized by the wave envelopes. The Zakharov equation, which contains initial-phase information, can be advantageous for prediction of the evolution of the wave groups’ envelope. Initial random phases significantly affect the distribution of energy on the spectrum and the eventual shape of wave groups; however, they produce almost the same wave group structures.

The transformation of the wave groups is in accordance with the evolution of the directional spectrum. If the energy of the wave spectrum is concentrated within a narrow range of frequency, the wave profiles have almost the same period, but gradually varying amplitudes.

The variability of the characteristic wave height increases as the spectral peak becomes sharp. However, if the frequency spectrum gets narrower, the envelope becomes longer, and if the directional spread decreases, the wave crest widens.

The comparison of the wave group evolutions on directional spectra to those on unidirectional spectra indicates that evolutions of wave groups in deep water and at intermediate water depths are significantly affected by nonlinear interactions between directional components. When a three-dimensional model is considered, transformation of wave groups in deep water is much more pronounced at \( ak_p = 0.2 \).

The effects of wave interaction are enhanced in relatively shallow water; however, the nonlinear interaction is suppressed on a sloping beach, which decreases the maximum wave height.

By increasing the wave steepness, nonlinear wave-wave interaction becomes more pronounced and induces wave breaking. This strong wave instability may be investigated on the basis of fully nonlinear theory in future studies.

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REFERENCES


22) Tauchi, T., Kohno, N. and Kimura, M.: The improvement
of JMA operational wave models, 10th International Workshop on Wave Hindcasting and Forecasting and Coastal Hazard Symposium, Oahu, Hawaii, 2007.


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