STEPPED-END DESIGN FOR CARBON FIBER-REINFORCED PLASTIC (CFRP) BONDED ONTO A STEEL PLATE UNDER UNI-AXIAL LOAD

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In the application of multiple Carbon Fiber-Reinforced Plastic (CFRP) plates onto a structural steel member, stepping of the CFRP plate ends is often adopted to prevent debonding. This approach requires the nonuniform extension of CFRP plate lengths. In this study, to shorten the length of CFRP plates, a nonuniform overlapping arrangement of several CFRP plate ends was proposed and the theoretical conditions of arrangement were proposed. Additionally, the design for the stepped-end arrangement of CFRP plates under a uni-axial load was shown. It was confirmed with stress analysis that the stress in the steel plate was reduced to the design value by bonding the CFRP plate with the proposed stepped ends.

Key Words : CFRP, steel plate, overlapping bonding, design

1. INTRODUCTION

When multilayered carbon fiber sheets, carbon fiber-reinforced plastic (CFRP) plates or carbon fiber strand sheets are made to adhere to steel plates under a uni-axial load, research has clarified that the shear stress in the adhesive end becomes small by applying stepped ends (1-7). Namely, the debonding load increases by applying stepped ends. If stepped ends are applied as shown in Fig.1, shear stresses in adhesive layer ends become smaller than in the case without stepped ends. However, the length of the CFRP plate required to reduce the stress to the design value at the center of the CFRP plate becomes longer than in the case without stepped ends.

Recent researches have reported on steel plates strengthened with multiple carbon fiber materials with a proposed step-length of 25 mm (2, 3). In this method, each step-length is fixed at 25 mm regardless of the difference in the stiffness of steel members and carbon fiber materials and in the thickness of adhesive layers; but this does not turn out to be optimum for all cases.

On the other hand, the tensional rigidities of CFRP plates and step-lengths have been theoretically clarified by considering the same shear stresses in the adhesive layer of all stepped ends (6). In this method, the required CFRP length for reducing steel stress becomes shorter than when using stepped ends for CFRP plates with the same extensional rigidity. However, CFRP plates with different extensional rigidities have to be provided depending on the design, and it might be difficult to use ready-made CFRP plates.

Because of these factors, in this study the conditions for applying multiple stepped ends of CFRP plates with the same extensional rigidity is proposed to establish the functional stepped-end design.

2. STRESS IN A STEEL PLATE BONDED WITH STEPPED-END CFRP PLATES, AND SHEAR STRESSES IN ADHESIVE LAYERS

In the case of a steel plate subjected to a uni-axial load and strengthened by bonding several CFRP plates of the same extensional rigidity with stepped ends, the shear stress in adhesive layers at the stepped ends can be approximated by the following equations...
as proposed in Ref.6):

\[ t_{si} = c_i l_i \frac{1 - \xi_i}{2F} \left( \prod_{j=1}^{i} \xi_j \right) \sigma_{sm} \quad (1) \]

where

\[ c_i = \sqrt{\frac{G_{si}}{h_i}} \sqrt{\frac{2}{1 - \xi_i}} \sqrt{\frac{F}{E_{si} t_{si}}} \quad (2) \]

\[ \xi_i = \frac{1}{1 + 2FE_i t_i / (E_{si} t_{si})} \quad (3) \]

\[ t_{si} = t_s \left[ 1 + \frac{2(i-1)FE_i t_i}{E_{si} t_{si}} \right] \left( \prod_{j=1}^{i} \xi_j \right) \quad (4) \]

\[ F = \frac{h}{b} \quad (5) \]

\[ E_i \] and \( t_i \) are Young’s modulus and thickness of the steel plate, respectively; \( E_s \) and \( t_s \) are Young’s modulus and thickness of the CFRP plate, respectively; \( b \) and \( h \) (\( b \geq h \)) are respectively widths of the steel and CFRP plates; \( G_{si} \) and \( h_i \) are shear modulus and thickness of adhesive layer \( i \), respectively; \( \sigma_{sm} \) is the applied stress; \( i \) (\( 1 \leq i \leq N \) and \( \prod_{j=1}^{i} \xi_j = 1 \) at \( i = 1 \)) is the number of CFRP plates or adhesive layers on the steel plate; \( N \) is the total number of CFRP plates; and \( j \) is the range of multiplication.

Eq.(1) can be used when each stepped-end length, \( l_{s(i+1)} \), satisfies the following equation:

\[ l_{s(i+1)} = \frac{1}{c_i} \cosh^{-1} \left( \frac{1}{\eta_i} \frac{2FE_i t_{si}}{E_{si} t_{si}} \right) \quad (6) \]

where

\[ c_i = \sqrt{\frac{G_{si}}{h_i}} \sqrt{\frac{2}{1 - \xi_i}} \sqrt{\frac{F}{E_{si} t_{si}}} \sum_{j=1}^{i} (D_j / D_i) \quad (7) \]

\[ h_i = h + \sum_{j=1}^{i} \left( \frac{h_j G_{sj}}{G_{si}} \right) = h_i \sum_{j=1}^{i} \left( \frac{D_j}{D_i} \right) \quad (8) \]

\[ D_j = \frac{h_j G_{sj}}{h_i G_{si}} \quad (9) \]

The variable \( \eta_i \) is the accuracy that the steel stress at the left end of the \( i \) th CFRP plate converges to \( (\prod_{j=1}^{i} \xi_j) \sigma_{sm} \). In the case of \( i = N \), \( l_{s(N+1)} \) is defined as the half-length of the \( N \) th CFRP layer, \( L_N \).

Since the overall thickness of adhesive layer 1 to \( i \) is considered in Eq.(6), \( l_{s(i+1)} \) becomes longer than the length required for converging to the design value of steel plate stress at the end of the \( i+1 \) th CFRP plate. Therefore, when all the stepped-end lengths are designed using Eq.(6), the stress in the steel plate at the center of the \( N \) th CFRP plate, \( \sigma_{s,center} \), becomes smaller than that the design value, as given by

\[ \sigma_{s,center} \leq \eta_0 \xi_0 \sigma_{sm} \quad (10) \]

where

\[ \xi_0 = \prod_{j=1}^{N} \xi_j = \frac{1}{1 + 2NFE_i / E_{si}} \quad (11) \]

\[ \eta_0 = \prod_{j=1}^{N} \eta_j \quad (12) \]

The value of \( \eta_i \) can be arbitrarily determined so as to satisfy Eq.(12) under \( \eta_0 > \eta_i > 1 \). In this study, all of the values of \( \eta_i \) denoted by \( \eta \) is determined by the following equation:

\[ \eta = \sqrt[N]{\eta_0} \quad (13) \]

On the other hand, the half-length of the \( i \) th CFRP plate is given by

\[ L_i = \sum_{j=1}^{i} l_{j(i+1)} \quad (14) \]

For the case of steel plate strengthening using CFRP plates with two stepped ends and material properties as listed in Table 1, shear stresses in the adhesive layer at the end of each CFRP plate is calculated using Eq.(1) and plotted as filled circles in Fig.1. In this figure, the shear stresses in all adhesive layers, calculated using the stress analysis method proposed in Ref.7), are also included. The x axis in Fig.1 shows the distance from the center of the CFRP plates. Each stepped-end length was calculated under \( \eta_0 = 1.01 \). The designed stepped-end lengths, \( L_{12} \), \( L_{23} \) and \( L_3 \), were 41.3 mm, 65.0 mm and 83.8 mm, re-

<table>
<thead>
<tr>
<th>Table 1 Material properties.</th>
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<tr>
<td>Young’s modulus of steel ( E_s ) [GPa]</td>
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<tr>
<td>Young’s modulus of CFRP plate ( E_c ) [GPa]</td>
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<tr>
<td>Shearing modulus of adhesive ( G_{\sigma} ) [GPa]</td>
</tr>
<tr>
<td>Thickness of steel plate ( t_s ) [mm]</td>
</tr>
<tr>
<td>Thickness of CFRP plate ( t_c ) [mm]</td>
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<tr>
<td>Thickness of adhesive ( h ) [mm]</td>
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<tr>
<td>Width of steel plate ( b_s ) [mm]</td>
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<tr>
<td>Width of CFRP plate ( b_c ) [mm]</td>
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respectively. In other words, the length of the first CFRP plate, $L_1$, became 190.1 mm.

As one can observe from Fig. 2, all the shear stresses calculated by Eq.(1), $\tau_i$, agree with those of the stress analysis results. Additionally, it was shown that the shear stress in the first adhesive layer was higher than in the other layers when three CFRP plates with the same extensional rigidity were bonded onto a steel plate with two stepped ends.

Fig. 3 shows the stress distribution in the steel plate calculated by stress analysis\(^\text{7)}\). From this figure, it is shown that the steel plate stress at the center of the CFRP plates, $\sigma_{\text{center}}$, becomes 5009.0 MPa, which is smaller than $\eta_0 \sigma_0 = 505.000 \sigma_{\text{m}}$.

3. DESIGN FOR ALIGNING THE ENDS OF MULTIPLE CFRP PLATES

As mentioned in Section 2, in the case of multiple stepped ends of CFRP plates with the same extensional rigidity, the shear stress in the $i$th adhesive layer at the end of the CFRP plate decreases as $i$ increases. Therefore, the CFRP plate would debond at a higher shear stress, $\tau_{ie}$, in the first adhesive layer under the same debonding resistance strength in all adhesive layers.

As mentioned in Ref.6), in the case that debonding of all CFRP plates occurs simultaneously, the tensile debonding load would reach its maximum. Therefore, it is considered that the shear stress in the $i$th adhesive layer can be increased until it reaches the shear stress $\tau_{ie}$ in the first adhesive layer, when multiple CFRP plates with the same extensional rigidity are adhered onto the steel plate with stepped ends. Generally, the shear stress at the end of the CFRP plate increases as the thickness of the CFRP plate increases\(^\text{8)}\). Accordingly, it will be possible to align the ends of the CFRP plates with the same extensional rigidity for the second or subsequent stepped ends.

In this section, the conditions for aligning the ends of several CFRP plates are shown.

(1) Conditions for aligning the ends of multiple CFRP plates

The shear stress introduced in the adhesive layer at the end of the CFRP plate, with aligned ends of
multiple plates, is assumed by modifying Eq.(1) as follows:

$$\tau_{i,j+X-1e} = c_{i,j,X-1} \frac{1 - \xi_i}{2F} \left[ \prod_{j=1}^{i-1} \xi_j \right] \sigma_{\nu j}$$  \hspace{1cm} (15)

where

$$c_{i,j,X-1} = \frac{G_{ai}}{h_i \sqrt{1 - \xi_i}} \frac{2}{1 - \xi_{i,j,X-1} - 1 \xi_i}$$  \hspace{1cm} (16)

$$\xi_{i,j,X-1} = \frac{1}{1 + 2XFE_t e} \frac{E_{i,j}}{E_{i,j}}$$  \hspace{1cm} (17)

The variable $X$ is the number of CFRP plates whose ends can be aligned ($X \geq 2$).

By substituting $\xi_i$ with $\xi_{i,j,X-1}$ in Eq.(1), Eq.(15) is derived. As shown by Eq.(17), $\xi_{i,j,X-1}$ includes the total extensional rigidity of $X$ layers of CFRP plates. Namely, Eq.(15) gives the estimated shear stress in the $i$th CFRP plate with a total thickness of adhesive layers between the steel plate and the $i$th layer. However, as shown in Section 4, $\tau_{i,j,X-1e}$, as calculated by Eq.(15), becomes higher than that given by the stress analysis result, because the shear lag effects of adhesive layers between the $i$th and $i+1$th CFRP plates are not considered. The shear lag effects of adhesive layers between CFRP plates are discussed in detail in Refs. 8) and 9).

Eq.(15) is also used when each stepped-end length, $l_{i,j,X-1(i+X)}$, satisfies the next equation:

$$l_{i,j,X-1(i+X)} = \frac{1}{c_{i,j,X-1}} \cosh^{-1} \left( \frac{1}{\eta - 1} \cdot \frac{2XFE_t e}{E_{i,j}} \right)$$  \hspace{1cm} (18)

where

$$c_{i,j,X-1} = \frac{G_{ai}}{h_i \sqrt{1 - \xi_i}} \frac{2}{1 - \xi_{i,j,X-1} - 1 \xi_i}$$  \hspace{1cm} (19)

$$h_i = h_i + \sum_{j=1}^{i-1} \frac{h_i G_{ai}}{G_{aj}} \sum_{j=1}^{i-1} \frac{h_i G_{ai}}{G_{aj}} = h_i \sum_{j=1}^{i-1} \frac{D_j}{D_i}$$  \hspace{1cm} (20)

In the case of $i = N - X + 1$, $l_{i,j,X-1(i+X)}$ becomes $L_{N-X+1} = L_{N-x+2} = ... = L_N$.

In the calculation of $l_{i,j,X-1(i+X)}$, $c_{i,j,X-1}$ in Eq.(18) considers the overall thickness of adhesive layers between the $i$th and $i+1$th CFRP plates. Therefore, as the same condition of Eq.(6), $l_{i,j,X-1(i+X)}$ becomes longer than the stepped-end length required for converging to the design stepped-end length of the steel plate stress at the end of the $i + X$th CFRP plate.

By aligning the ends of several CFRP plates, the number of stepped ends is reduced. Therefore, $\eta$ for Eq.(18) can be defined as follows:

$$\eta = \sqrt{\frac{\eta_0}{N}}$$  \hspace{1cm} (21)

where $N'$ is the number of stepped ends, taking into account the reduction of steps by aligning the ends of CFRP plates.

The upper limit of shear stress, $\tau_{i,j,X-1e}$, is given by the following equation, which includes the difference in debonding resistance between adhesive layers.

$$\tau_{i,j,X-1e} \leq \frac{T_e}{T_i}$$  \hspace{1cm} (22)

The variable $T_i$ is the debonding resistance of the $i$th adhesive layer.

Eq.(22) is derived by relating the debonding in the $i + X - 1$th adhesive layer and that in the first adhesive layer.

On the other hand, Eq.(15) can be represented as

$$\tau_{i,j,X-1e} = \tau_e \frac{1 - \xi_i}{1 - \xi_{(i+X)-1}}$$  \hspace{1cm} (23)

By substituting $\tau_{i,j,X-1e}$ given by Eq.(23) and $\tau_e$ given by Eq.(1), the relation between $i_x$ and $2FE_t e / (E_{i,j})$ is given as

$$i_x \geq \frac{X \left[ XZ^2 + 4(Z + 1) \left( \frac{T_e}{T_i} \right)^2 \right] - (X - 2)Z + 2}{2Z}$$  \hspace{1cm} (24)

where

$$Z = \frac{2FE_t e}{E_{i,j}}$$  \hspace{1cm} (25)

The variable $i_x$ is the layer number of the CFRP plate which is able to align the ends of $X$ layers of CFRP plates.

(2) Conditions for aligning the ends of two CFRP plates

By substituting $X = 2$ into Eq.(24), the conditions for aligning the ends of two CFRP plates can be expressed as

$$i_2 \geq \sqrt{\frac{Z^2 + 2(Z + 1) \left( \frac{T_e}{T_i} \right)^2 - 1}{Z}}$$  \hspace{1cm} (26)

The variable $i_2$ is the layer number of the CFRP plate which is able to align the ends of two CFRP plates.
The conditions for aligning the ends of two CFRP plates, given by this equation, are shown in Table 2 and Fig.5. In these conditions, it is assumed that the debonding resistance in all adhesive layers is the same.

The shaded area in Fig.5 indicates the $i_2-Z$ relation, which is applicable for aligning two CFRP ends. As can be seen from Fig.5, if the value of $Z$ becomes 0.333 or greater, the value of $i_2$ becomes greater than 2. Therefore, if the $i_2$ ($i_2 \geq 2$)th CFRP plate end is aligned with the $i_2+1$th CFRP plate, the shear stress in the $i_2$th adhesive layer would not exceed the shear stress in the first adhesive layer.

(3) Conditions for aligning the ends of three CFRP plates

By substituting $X = 3$ into Eq.(24), the conditions for aligning the ends of three CFRP plates are derived as follows:

$$i_3 \geq \sqrt{\left[3Z^2 + \frac{4(Z + 1)}{D_{i_3}} \left(\frac{T_{i_3}}{T_{i_1}}\right)^2\right] - (Z + 2)}$$ (27)

The variable $i_3$ is the layer number of the CFRP plate that is used to align the ends of three CFRP plates.

The conditions for aligning the ends of three CFRP plates, given by Eq.(27), with the same debonding resistance in all adhesive layers are shown in Table 3 and Fig.6.

As can be seen from Fig.6, if the value of $Z$ be-
comes 0.5 or greater, the value of \( i_3 \) becomes greater than 2. Therefore, if the \( i_4 \) \((i_4 \geq 2)\) th CFRP plate end is aligned with the \( i_4 + 1 \) th and \( i_4 + 2 \) th CFRP plates, the shear stress in the \( i_4 \) th adhesive layer would not exceed the shear stress in the first adhesive layer.

(4) Conditions for aligning the ends of four CFRP plates

By substituting \( X = 4 \) into Eq. (24), the conditions for aligning the ends of four CFRP plates is given as

\[
i_4 \geq \frac{Z^2 + Z + 1}{D_{i_4}} \left( \frac{1}{T_{i_4}} - \frac{1}{T_{i_3}} \right)^2 - (Z + 1)
\]

The variable \( i_4 \) is the layer number of the CFRP plate which is able to align the ends of four CFRP plates.

The conditions for aligning the ends of four CFRP plates, given by Eq. (28), with the same debonding resistance in all adhesive layers are shown in Table 4 and Fig. 7.

As can be seen from Fig. 7, if the value of \( Z \) becomes 0.6 or greater, the value of \( i_4 \) becomes greater than 2. Therefore, if the \( i_4 \) \((i_4 \geq 2)\) th CFRP plate end is aligned with the \( i_4 + 1 \) th to \( i_4 + 3 \) th CFRP plates, the shear stress in the \( i_4 \) th adhesive layer would not exceed the shear stress in the first adhesive layer.

4. EXAMPLES OF STEPPED-END DESIGN FOR CFRP BONDED ONTO STEEL PLATE UNDER UNI-AXIAL LOAD

In this section, examples of a stepped-end design for CFRP bonded onto a steel plate under a uni-axial load as proposed in Section 3 are shown. It is assumed that the debonding resistance strengths in all adhesives are the same in this section. A further assumption is that the first CFRP layer does not peel off under the design load. Additionally, \( \eta_b = 1.01 \) is employed for the design of each stepped-end length.

(1) Stepped-end design procedure

The stepped-end design procedure of the CFRP plate is shown in Fig. 8. As explained in this figure, since the value of the required reduction stress in the steel plate and the material properties of the CFRP plate and adhesive are given, the number of CFRP plates required and the value of \( NFEt_{i_4} \) can be calculated. Furthermore, by substituting the number of CFRP plates aligned, \( X \), into Eq. (24), the layer number \( i_x \) of the CFRP plate applicable for aligning \( X \) CFRP layers can be calculated. Next, by using \( N' \), which is the number of stepped ends taking into account the reduction of steps by aligning the ends, the stepped-end lengths are calculated by providing the value of \( \eta_b \) with the condition that the length of CFRP plate 1 becomes shortest.

(2) Bonding of three CFRP plates

For bonding three CFRP plates with similar extensional rigidity as given in Table 1, the design of each stepped-end length is discussed herein.

In the case of bonding three CFRP plates, it is necessary to examine whether it is possible to align the ends of two CFRP plates. From Table 1, the value of \( 2FEt_{i_4}(1/E_{i_4}) \) becomes 0.333. Since the material properties, thickness and debonding resistance in all adhesive layers are similar, \( i_2 \geq 2 \) is given as in Table 2. Namely, the ends of the second and third CFRP plates can be aligned. By aligning the ends of the second and third CFRP plates, \( N' \) becomes 2. The value of \( \eta \) is given by substituting \( N' \) into Eq. (21). Furthermore, the first step-length, \( l_{i_2} \), is calculated as 38.2 mm by Eq. (6). Similarly, the lengths of the second and third CFRP plates, \( L_2 \) and \( L_3 \), are both calculated as 95.4 mm. Therefore, to reduce the stress in the steel plate at the center of the CFRP plate to less than \( 0.505EW_{i_4} \), the length of the first CFRP plate becomes 133.6 mm, which is 0.7 times shorter than the case with two stepped ends as discussed in Section 2. The total
length of the CFRP plate for aligning the ends of two CFRP plates is also 0.77 times shorter than the case with two stepped ends.

The stress distribution in the steel plate and shear stress distributions in adhesive layers based on stress analysis\(^7\) are shown in Fig.9. Additionally, the analysis results of two CFRP plates with one stepped end, of which the extensional rigidity of the second CFRP plate is twice that of the first CFRP plate, are shown in Fig.10. In this figure, the first stepped-end length, \(L_{1e}\), and the length of the second CFRP plate, \(L_2\), are 38.2 mm and 95.4 mm, respectively. The shear stresses, \(\tau_{1e}\) and \(\tau_{2(2)2e}\), calculated by Eqs. (1) and (15) are also plotted in Figs. 9 and 10 as solid circles.

In Fig.9(a), by aligning the stepped ends as proposed in this research, the stress in the steel plate at the center of the CFRP plate calculated by stress analysis, \(\sigma_{\text{center}}\), is reduced to 0.5009\(\sigma_m\), which is smaller than \(\eta_{1e}\sigma_m = 0.505\sigma_m\). On the other hand, \(\sigma_{\text{center}}\) in Fig.10(a) becomes 0.5006\(\sigma_m\). The difference in \(\sigma_{\text{center}}\) in Figs. 9(a) and 10(a) is caused by the existence of adhesive layer 3. Namely, due to the shear lag effect in adhesive layer 3, the distributed force in CFRP 3 near the plate’s end is smaller than that in CFRP 2.

From Fig.10(b), in the case of CFRP 2 with double extensional rigidity of the first CFRP plate, the shear stresses at the ends of adhesive layers 1 and 2, \(\tau_1\) and \(\tau_2\), correspond to \(\tau_{1e}\) and \(\tau_{2(2)2e} (= \tau_{1e})\), respectively. On the other hand, from Fig.9(b), while the shear stress at the end of adhesive layer 1, \(\tau_1\), corresponds to \(\tau_{1e}\), the shear stress at the end of adhesive layer 2, \(\tau_2\), is smaller than \(\tau_{2(2)2e}\). The reason for this is that the shear stress introduced in adhesive layer 3, as shown in Fig.9(b), is not considered in Eq.(15). Thereby, by applying Eq.(24), which is proposed for aligning the CFRP ends, the shear stress \(\tau_{(i,i+X-1)e}\) is provided conservatively. However, as the number of CFRP plates that can be aligned, \(X\), becomes larger, the difference between \(\tau_{(i,i+X-1)e}\) in Eq.(15) and the actual shear stress becomes larger. This is because the number of adhesive layers is not considered in Eq.(15).

(3) Bonding of five CFRP plates

For the bonding of five CFRP plates with similar
extensional rigidity as listed in Table 1, the design of each stepped-end length is discussed herein. In the case of bonding five CFRP plates, one has to check whether it is possible to align the ends of two, three or four CFRP plates beforehand. For \( \eta \xi \sigma_{ss} \), the condition for aligning the ends of two CFRP plates is provided as \( i = 2 \) from Table 2. Namely, the ends of CFRP 2 and 3, as well as the ends of CFRP 4 and 5, can be respectively aligned. The conditions for aligning the ends of three CFRP plates are given as \( i = 3 \) from Table 3. Namely, the ends of plates CFRP 3 to 5 can be aligned. Since the conditions for aligning the ends of four CFRP plates are given as \( i = 4 \) from Table 4, the aligning of the ends of the four CFRP plates cannot be carried out.

Fig. 11 shows the stress in the steel plate and shear stresses in adhesive layers calculated by stress analysis, in which the ends of plates CFRP 2 and 3, as well as those of CFRP 4 and 5, are respectively aligned. Fig. 12 shows the stress in the steel plate and shear stresses in adhesive layers for the case of alignment of the ends of plates CFRP 3 to 5.

Stresses in the steel plate at the center of the CFRP plate in Figs. 11(a) and 12(a) become \( 0.3753 \sigma_{ss} \), which is smaller than \( \eta i \xi \sigma_{ss} = 0.379 \sigma_{ss} \). In the case of aligning the ends of plates CFRP 2 and 3 and the ends of CFRP 4 and 5, the length of CFRP plate 1, \( L_1 \), becomes 291.8 mm, which is 0.66 times shorter than the case with four stepped ends. The total length of the CFRP plate for aligning the ends of CFRP 2 and 3 and those of CFRP 4 and 5 is also 0.72 times shorter than the case with four stepped ends. Similarly, in the case of aligning the ends of CFRP 3 to 5, the length of CFRP plate 1, \( L_1 \), becomes 268.6 mm, which is 0.61 times shorter than the case with four stepped ends. The total length of the CFRP plate after aligning the ends of CFRP 3 to 5 is 0.65 times shorter than the case with four stepped ends. Therefore, the bond length of the CFRP plate becomes shortest by aligning the ends of CFRP 3 to 5.

From Fig. 11(b), in the case of aligning the ends of CFRP 2 to 3 and the ends of CFRP 4 to 5, the shear stress in adhesive layer 2, \( \tau_{(2,3)\sigma} \), is close to \( \tau_{4\sigma} \), which is introduced in adhesive layer 1. As can be seen from Fig. 12(b), in the case of aligning the ends of CFRP 3 to 5, the shear stress in adhesive layer 3, \( \tau_{(3,5)\sigma} \), is also close to \( \tau_{\sigma} \). Accordingly, the length of
the CFRP plate can be shortened by aligning the ends of CFRP plates in which the shear stress in the adhesive layer at stepped ends is controlled to be close to that in adhesive layer 1, with the same debonding resistance in all adhesive layers.

5. CONCLUSIONS

In this research, a design procedure for stepped ends of multiple CFRP plates under a uni-axial load was proposed. The main conclusions are as follows:

1) The shear stress in the adhesive layer at the stepped ends, obtained by aligning the $X$ layer of CFRP plates, was approximated using Eq.(15). The shear stress calculated by Eq.(15) becomes slightly higher than the actual shear stress because the adhesive layers between the aligned CFRP plates are ignored in Eq.(15). Furthermore, the length of each stepped end, which allows the approximation of shear stress by Eq.(15), were given by Eq.(18). Each stepped-end length calculated using Eq.(18) is estimated as slightly longer than the actual length because the adhesive layers between the aligned CFRP plates are included.

2) The conditions for aligning the ends of multiple CFRP plates were given by Eq.(24).

3) Examples of stepped-end design of bonding between three or five CFRP plates were shown. Furthermore, it was shown that the stress in the steel plate at the center of the CFRP plate becomes smaller than the design value by applying the proposed stepped ends.

APPENDIX

If in the first step, the aligned ends of several CFRP plates do not debond under the design load, the second step can be used to align the ends of the same number of CFRP layers as in the first step, with the same debonding resistance and thickness in all the adhesive layers. Furthermore, in this appendix, the condition that aligns the ends of CFRP plates not in the first layer but in the second and subsequent layers is shown.

With Eqs. (1) and (23) as reference, the shear stress introduced in the first adhesive layer at the end of aligned CFRP 1 to $X_i$, $\tau_{(1,X_i)k}$, is assumed as

$$
\tau_{(1,X_i)k} = \tau_{kc} \sqrt{\frac{1 - \varepsilon_{i,X_i}}{1 - \varepsilon_i}}
$$

(29)

where

$$
\varepsilon_{i,X_i} = \frac{1}{1 + 2 \frac{X_i FE t_k}{E t_s}}
$$

(30)

In these equations, the range of $i$ becomes $i > X_1$. $X_i$ is the number of CFRP plates with aligned ends in the first step.

In the case of aligning the stepped ends of $X_1$ layers of CFRP plates in the first step and the stepped ends of $X$ ($X > X_1$) layers of $i$ th to $i + X - 1$ th CFRP plates in the second step or more, the upper limit of shear stress, $\tau_{(i,i+X-1)k}$, is given by the following equation, which includes the difference in debonding resistance between adhesive layers.

$$
\tau_{(i,i+X-1)k} \leq \frac{T_i}{T_{cs}} \tau_{(1,X_i)k}
$$

(31)

By substituting $\tau_{(i,i+X-1)k}$ given by Eq.(23) and $\tau_{(1,X_i)k}$ given by Eq.(29), the relation between $i_X$ and $Z = 2FE t_k/(E t_s)$ is presented.

$$
i_X \geq -\sqrt{\frac{X \left\{ X^2Z^2 + 4(XZ+1) \left( \frac{T_i}{T_{cs}} \right)^2 \right\} - (X-2)Z + 2}{2Z}}
$$

(32)

For the same condition in Section 4, in the case of $X_1 = 2$, which has two CFRP ends aligned in the first step, the value of $2FE t_k/(E t_s)$ of $i_X \geq 3$ given by Eq.(32) becomes greater than 0.1. Therefore, in the case of the bonding of five CFRP plates as designed in Section 4(3), three ends of CFRP plates can be aligned in the second step, while the two ends are aligned in the first step. In this case, the half-length of the first CFRP plate and the total length of all plates are 0.52 times and 0.60 times shorter than in the case with four stepped ends. Therefore, the step-length can be made shorter than that designed in Section 4(3) by aligning the ends of multiple CFRP plates in the first step.

REFERENCES


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