This study investigated the feasibility of bridge health monitoring (BHM) using a linear system parameter of a time series model identified from traffic-induced vibration data of bridges, which data were obtained through a moving-vehicle experiment on scaled model bridges. In order to detect possible anomalies in bridges, this study adopted a parameter from autoregressive (AR) coefficients. Consideration was given to diagnosis of the bridge condition from pattern changes of identified system parameters due to damage. The Mahalanobis-Taguchi system (MTS) was successfully applied to make decisions on the bridge health condition. Observations demonstrate the feasibility of structural diagnoses of bridges from the identified system parameter.

**Key Words:** bridge health monitoring (BHM), linear system model, Mahalanobis-Taguchi system (MTS), traffic-induced vibrations

1. INTRODUCTION

Maintaining and improving civil infrastructure, including bridge structures, are important technical issues. Moreover, an effective maintenance strategy strongly depends on timely decisions on the health condition of the structure. It has been thought that structural health monitoring (SHM) using vibration data is one of the effective technologies that aid in decision making on bridge maintenance. Most precedent studies on SHM specifically examine the change in modal properties of structures. The fundamental concept of this technology is that modal parameters are functions of a structure’s physical properties. Therefore, a change in physical properties, such as reduced stiffness resulting from damage, will change these modal properties.

A challenge for bridge health monitoring (BHM) using vibration measurements is how to excite the bridge economically, reliably and rapidly. Ambient vibrations induced by traffic and wind, thus, are adopted as dynamic data for BHM. However, for short-span bridges, which cover majority of bridge structures, the wind-induced vibrations are usually too weak to use in the BHM of such bridges. On the other hand, traffic-induced vibrations are dominant for short-span bridges. It is noteworthy that the traffic-induced vibration of bridges is a kind of nonstationary vibration. Despite the nonstationary property of traffic-induced vibrations of bridges, the idea behind using traffic-induced vibration data of short-span bridges in BHM is that parameters identified repeatedly under moving vehicles could provide a pattern and give useful information to make a decision on the bridge condition.
Many studies focus on changes in system frequencies and structural damping constants for the structural diagnosis of bridges by utilizing a linear time series model \(^8\)-\(^15\). However, there exist drawbacks in modal parameter–based bridge diagnosis using time series models; e.g., the optimal time series model for vibration responses of bridge structures usually comprises a higher-order term, and as a result the optimal model identifies even spurious modal parameters, which causes false system frequencies and damping constants \(^16\). Those false modal parameters make it difficult to choose the proper modal parameters affected by structural damage. An interesting approach utilizing a micro shaker for damage identification of a steel truss bridge has also been reported \(^17\). However, the accuracy of the damage identification may vary according to traffic on the bridge.

The drawback of the classical method is the driving force behind this study. This study considered an alternative parameter from autoregressive (AR) coefficients as a damage-sensitive feature for the vibration-based BHM because both system frequency and damping constant are related to AR coefficients \(^16\), \(^18\).

The Mahalanobis-Taguchi system (MTS) \(^19\), which is one of the supervised learning schemes, was also adopted for making structural diagnosis of bridges.

2. AR MODEL AND DAMAGE INDICATOR

The linear dynamic system can be identified using the AR model \(^13\), \(^14\) as

\[ y(k) + \sum_{i=1}^{p} a_i y(k-i) = e(k) \]  \hspace{1cm} (1)

where \(y(k)\) denotes the output of the system, \(a_i\) is the \(i\)-th order AR coefficient and \(e(k)\) indicates the noise term.

To estimate AR parameters, the autocorrelation function of \(y(k)\), which is obtainable by multiplying each term of Eq. (1) with \(y(k-i)\) and taking the mathematical expectation, is used. This process yields the following Yule-Walker equation \(^13\), \(^14\), \(^20\):

\[ R_a = \sum_{i=1}^{p} a_i R_{i} \]  \hspace{1cm} (2)

where \(R\) is a Toeplitz matrix about \(R_{k,s} = E[y(s)y(k-i)]\), which is the autocorrelation function of the signal; \(a = [a_1; \ldots; a_p]\) and \(R = [R_1; \ldots; R_p]\); and \(p\) indicates the AR order.

The Levinson-Durbin algorithm \(^20\) is adopted to solve Eq. (2). It is noteworthy that the coefficient \(a_p\) is a pole of the system because the \(z\)-transformation of Eq. (1) can be written as

\[ Y(z) = H(z)E(z) = \frac{1}{1 + \sum_{i=1}^{p} a_i z^{-i}} E(z) \]  \hspace{1cm} (3)

where \(Y(z)\) and \(E(z)\) are \(z\)-transformations of \(y(k)\) and \(e(k)\), \(H(z)\) is the transfer function of the system in the discrete-time complex domain, and \(z^{-i}\) denotes the forward shift operator.

Values of \(z\) in which the elements of the transfer function matrix show infinite values are the poles. This means that the denominator of the transfer function is the characteristic equation of the dynamic system, given as

\[ z^n + a_1 z^{n-1} + a_2 z^{n-2} + \cdots + a_{p-1} z + a_p = 0 \]  \hspace{1cm} (4)

The poles on the complex plane are associated with the frequency and damping constant of the dynamic system of structures, as follows:

\[ z_k = \exp\left(-h_k\omega_k \pm \frac{j\omega_k}{2}\right) \]  \hspace{1cm} (5)

where \(h_k\) and \(\omega_k\) are the damping constant and circular frequency, respectively, of the \(k\)-th mode of the system, and \(j\) represents the imaginary unit.

The AR process with the model order \(p\) in Eq. (1) can be expressed in the \(z\)-plane as already given in Eq. (3). The \(H(z)\) in Eq. (3) is defined as the AR polynomial of the model transfer function relating the input to the output. The poles \(z_k\) of Eq. (3) are obtained by finding the roots of the AR coefficient polynomial in the denominator of \(H(z)\). Since the values of the coefficient of \(H(z)\) are real, the roots must be real or complex conjugate pairs. The number of poles in the \(z\)-plane equals the AR model order. Therefore \(z_k\) shown in Eq. (5) and AR coefficients have the following relationships according to Vieta’s formula \(^21\):

\[ \sum_{i} z_i = -a_1; \sum_{i,j} z_i z_j = a_2; \cdots \]  \hspace{1cm} (6)

Eq. (5) and Eq. (6) show that AR coefficients are directly associated with system frequencies and the damping constant. Therefore the parameter from AR coefficients is adopted as a damage-sensitive feature and defined as \(^16\)

\[ DI_j = \frac{|a_i|}{\sqrt{\sum_{i=1}^{p} a_i^2}} j > 1; \quad DI_1 = |a_i| \]  \hspace{1cm} (7)

where \(a_i\) denotes the \(i\)-th AR coefficient and \(DI_j\) is the damage indicator that considers up to the \(j\)-th AR coefficient.

Generally, damage in structures changes the modal parameters such as \(\omega_k\) and \(h_k\) in Eq. (5), and as a result changes \(z_k\). Moreover, according to Eqs. (5) and (6), AR coefficients are affected by damage, and structural damage also changes the \(DI_j\) values defined in Eq. (7).
Another way to explain the relationship between AR coefficient, circular frequency and damping constant is depicted in Appendix A.

3. LABORATORY MOVING-VEHICLE EXPERIMENT OF MODEL BRIDGE

(1) Model bridge

A laboratory moving-vehicle experiment was performed to verify the validity of the proposed approach. The experimental setup is shown in Fig. 1. The experimental setup comprised three simple beams representing the accelerating beam, observation beam and decelerating beam. The natural frequency of the bridge was considered as a factor in the scaling of the bridge model. The bridge modeled a single-span bridge with a span of 40.4m and a first natural frequency of 2.35Hz\(^2\).

Scale roadway profiles were paved on both of the left and right wheel paths of the vehicle with an electrical tape at the interval of 100mm, as shown in Fig. 1. The thickness of the tape was 0.2mm. The roadway profile was in fact taken from the single-span bridge\(^2\), with bumps added to realize a rougher level of road profile.

This study considered three kinds of damage: the lower parts of both flanges between 3L/8 and L/2 of the model bridge, a depth of 5mm, were removed for damage scenario D1; for damage scenario D2, an additional 5mm cut was applied to the damaged plate of the D1 scenario and removed; and for damage scenario D3, another 5mm cut was applied to the plate damaged by damage scenario D2 and was also removed. The concept and photo of the damage are shown in Fig. 2 and Fig. 3. The bending rigidity of the model bridge decreased to around
94% of the intact state due to damage scenario D1. For damage scenario D2, the bending rigidity decreased to around 80% of the intact state. Damage scenario D3 led to a decrease in the bending rigidity to around 65% of the intact state.

The first natural frequencies estimated from free vibrations were 2.66Hz for the intact bridge, 2.61Hz for damage scenario D1, 2.57Hz for damage scenario D2 and 2.51Hz for damage scenario D3. The third natural frequencies were 23.8Hz for the intact bridge, 23.3Hz for D1, 23.0Hz for D2 and 22.6Hz for D3.

It should be noted that, in this feasibility study, the focus is on verifying the feasibility of the present approach. Therefore, the artificial damage types were not intended to perfectly simulate real damage, but to make the bridges serve as damaged samples in comparison to intact ones, in terms of bending rigidity reduction.

(2) Model vehicle

Natural frequency and speed parameter are considered as factors in the scaling of the vehicle model. The vehicle model can be adjusted to obtain different dynamic properties; the spring stiffness of the axles can be varied by changing the springs while the body mass can be varied using steel plates. Three vehicles V1, V2 and V3 were considered in the experiment. Natural frequencies for the bounce motion of the vehicle models were 2.93Hz, 3.76Hz and 3.03Hz respectively. These frequencies are close to 3Hz, which is the frequency of the bounce motion of actual dump trucks.

Three different speeds of 0.93m/s (hereafter, S1), 1.16m/s (hereafter, S2) and 1.63m/s (hereafter, S3) were adopted to investigate the effect of the vehicle speed on the parameter identification. Nine scenarios of the laboratory experiment were considered as shown in Table 1. Assuming a real bridge whose first natural frequency was 2.35Hz and span length was 40.4m, the speeds of the model vehicles, S1, S2

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Vehicle type</th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCN 1</td>
<td>V1 (M=21.6kg, f=2.93Hz)</td>
<td>S1=0.93m/s</td>
</tr>
<tr>
<td>SCN 2</td>
<td>V1 (M=21.6kg, f=2.93Hz)</td>
<td>S2=1.16m/s</td>
</tr>
<tr>
<td>SCN 3</td>
<td>V1 (M=21.6kg, f=2.93Hz)</td>
<td>S3=1.63m/s</td>
</tr>
<tr>
<td>SCN 4</td>
<td>V2 (M=21.6kg, f=3.76Hz)</td>
<td>S1=0.93m/s</td>
</tr>
<tr>
<td>SCN 5</td>
<td>V2 (M=21.6kg, f=3.76Hz)</td>
<td>S2=1.16m/s</td>
</tr>
<tr>
<td>SCN 6</td>
<td>V2 (M=21.6kg, f=3.76Hz)</td>
<td>S3=1.63m/s</td>
</tr>
<tr>
<td>SCN 7</td>
<td>V3 (M=25.8kg, f=3.03Hz)</td>
<td>S1=0.93m/s</td>
</tr>
<tr>
<td>SCN 8</td>
<td>V3 (M=25.8kg, f=3.03Hz)</td>
<td>S2=1.16m/s</td>
</tr>
<tr>
<td>SCN 9</td>
<td>V3 (M=25.8kg, f=3.03Hz)</td>
<td>S3=1.63m/s</td>
</tr>
</tbody>
</table>

Fig. 4 Acceleration and Fourier spectra observed at Point I: a) Intact; b) D1; c) D2; d) D3 (SCN1, vehicle V1 at 0.93m/s).

Fig. 5 Road profile and wavelet transform of acceleration response observed at point I of intact bridge under SCN1.

Table 1 Scenarios of laboratory moving-vehicle test.
and S3, corresponded to 22.1km/h, 27.6km/h and 38.8km/h respectively according to the nondimensional speed parameter\(^{23}\) of Eq. (8):

\[
a = \frac{v}{2f_1 L}
\]

where \(L\) indicates the span length (m); \(f_1\), the first natural frequency of the bridge (Hz); and \(v\), vehicle speed (m/s).

The dimensionless parameter is important for the scaling of the experimental model as it is used to maintain a relationship between vehicle speed and frequency and span length of the 5.4m beam, a relationship similar to that for a 40.4m bridge subject to real traffic.

Three points—at 1/4, 1/2 and 3/4 of the span length—were observation points for acceleration responses as shown in Fig. 1. The sampling rate was 100Hz. Example acceleration responses of the model bridge before and after applying damage are shown in Fig. 4 with Fourier amplitude spectra. The dominant frequency near 23Hz, which was associated with the third bending mode of the model bridge, was excited when a vehicle axle hit one of the bumps on the road profile. The road profile and wavelet transform of acceleration response observed at point I of the intact bridge under scenario 1 (SCN1) are shown in Fig. 5, which demonstrates the dominant frequency near 23Hz when a vehicle passes over the bumps.

4. STRUCTURAL DIAGNOSIS USING DAMAGE INDICATOR

This section focuses on diagnosis of bridges using DI\(_j\) defined in Eq. (7), which was taken from AR coefficients. The optimal order of the AR model of each experiment was selected by means of the Akaike Information Criterion (AIC)\(^{24}\), shown in Eq. (9):

\[
\text{AIC} = N \log(2\pi\hat{\sigma}_M^2) + 2(M + 1) + N
\]

where \(N\) indicates the number of data; \(M\), AR order; and \(\hat{\sigma}_M^2\), mean square of \(M\) th prediction error.

The AIC consists of two terms. The first term is a log-likelihood function and the second term is a penalty function for the number of the AR order. Fig. 6 shows the AIC versus the AR order. An interesting observation is that, as shown in Fig. 6, the AIC value at the optimal AR order was changed apparently due to damage. Although it needs further comprehensive investigation, there is a possibility that the AIC could be utilized as a damage-sensitive feature.

(1) Sensitivity analysis

Considering random vibrations such as traffic-induced vibrations of bridge, it is not easy to decide which order of AR coefficient is the most effective to get a sensitive DI\(_j\) due to damage. Therefore a sensitivity analysis was performed to decide the most sensitive DI\(_j\) due to structural damage\(^{16}\).

After testing several different combinations with the AR coefficients, it was found that the first AR coefficient normalized by the square root of the sum of the squares of the first three AR coefficients gives the most sensitive DI\(_j\) due to damage, as shown in Fig. 7. Fig. 7 shows the average residuals of DI\(_j\) between intact and damaged bridges with respect to \(j\). The average residuals were estimated using 135 observations from combinations of 9 scenarios, 5 experiments and 3 observation points (9×5×3=135).

![Fig. 6 AIC w.r.t. the model order (\(p=\): optimal AR order).](image)

![Fig. 7 Average residuals of DI values from 135 observations w.r.t. to the order of AR coefficients in denominator of Eq. (7):](image)

a) Intact vs. D1; b) Intact vs. D2; c) Intact vs. D3.
The difference in DI values between intact and damaged bridges takes a peak when \( j \) takes 3, as shown in Fig. 7. Therefore this study adopted DI\(_3\) as the damage-sensitive feature.

### (2) Damage indicator according to severity of damage

This study examined the change in DI\(_3\) according to damage, observation points and vehicle speeds as shown in Fig. 8. The observed DI\(_3\) values at point I and point III under all vehicle speeds were changed due to damage. On the other hand, for the DI\(_3\) observed at point II, a relatively clear pattern change in DI\(_3\) due to damage was observed under vehicle speeds S2 and S3 in comparison with the DI\(_3\) under the vehicle speed S1.

### 5. FAULT DETECTION BY MTS METHOD

MTS is a pattern information technology, which is used in diagnostic applications to make quantitative decisions by constructing a multivariate measurement scale called Mahalanobis distance\(^{19}\) (hereafter MD). The concept of MD is shown in Fig. 9. The main object of the MTS method is to make accurate predictions of MD from unknown data by comparing with MD obtained from known data. Therefore, in this study, the MTS method was adopted for structural fault detection of bridges. The MTS method is summarized as follows.

The unit space, also called reference space, is obtainable from known data, as shown in Table 2. In Table 2, the columns indicate variables and the rows, observations. The mean value \( \bar{x}_i \) and standard deviation of each variable \( \sigma_i \) are applied in order to normalize the known data as

\[
X_{pi} = \frac{x_{pi} - \bar{x}_i}{\sigma_i}
\]

where \( i \) indicates the number of evaluated items, and \( p \) indicates the number of observations. \( \bar{x}_i \) and \( \sigma_i \) are definable by Eq.(11):

\[
\bar{x}_i = \frac{1}{n_p} \sum_{p=1}^{n_p} x_{pi}, \quad \sigma_i = \sqrt{\frac{1}{n_p} \sum_{p=1}^{n_p} (x_{pi} - \bar{x}_i)^2}
\]

The correlation coefficient matrix \( R_{MD} \) is obtainable from normalized data as shown in Eq. (12).

\[
R_{MD} = \begin{bmatrix}
1 & r_{12} & \cdots & r_{1k} \\
r_{21} & 1 & \cdots & r_{2k} \\
\vdots & \vdots & \ddots & \vdots \\
r_{k1} & r_{k2} & \cdots & 1
\end{bmatrix}
\]
The inverse matrix of $R_{MD}$ is defined as shown in Eq. (14):

$$A = R_{MD}^{-1}$$  \hspace{1cm} (14)

The MD of the normalized known data in the unit space can be estimated from Eq. (14) using matrix $A$ and normalized known data:

$$MD = \frac{1}{k} [X_{p1} \cdots X_{pk}] A \begin{bmatrix} X_{p1} \\ \vdots \\ X_{pk} \end{bmatrix}$$  \hspace{1cm} (15)

Next, the signal space is obtained from unknown data as shown in Table 3. Unknown data are normalized by utilizing $\bar{x}_i$ and $\sigma_i$, which are the mean value and standard deviation of known data described in Eq. (11) respectively:

$$Y_{pi} = \frac{y_{pi} - \bar{x}_i}{\sigma_i}$$  \hspace{1cm} (16)

The MD of the normalized unknown data in the signal space $MD_p$ is definable by Eq. (17) from the normalized unknown data and $A$, which is obtained from known data:

$$MD_p = \frac{1}{k} [Y_{p1} \cdots Y_{pk}] A \begin{bmatrix} Y_{p1} \\ \vdots \\ Y_{pk} \end{bmatrix}$$  \hspace{1cm} (17)

The required conditions to utilize MTS are as follows\(^{19}\): a) the number of evaluation items ‘$k$’ of known data is equivalent to that of unknown data; b) the number of observation data ‘$n$’ is larger than that of evaluation item ‘$k$’; and c) the standard deviation of known data ‘$\sigma$’ is not zero.

(1) Cross-validation

Cross-validation\(^{21}\) was adopted to decide on a threshold of the observed parameters and to identify faults in bridges. A scheme of the cross-validation is shown in Fig. 10. At first, $n-1$ data in the unit space are selected from known data and one known data item which is not selected as known data is assumed to be unknown. Next, by using the $n-1$ known data, the MD for the assumed unknown is estimated by means of the MTS method. Then, another data item is selected and assumed to be unknown, and cross-validation is performed. These steps for cross-validation are repeated $n$ times, and finally $n$ MDs in the signal space are obtained.

In this study, the largest and smallest values of MD taken from the cross-validation were removed, and the trimmed mean value was adopted as the threshold using ($n-2$) MD distances to reduce the effect of outliers on the MDs.

(2) Fault detection

The MTS method was applied for structural fault detection using DI\(_3\). Three evaluation items were used: 1) DI\(_3\) obtained from observation point I; 2)
DI$_3$ obtained from observation point II; and 3) DI$_3$ obtained from observation point III. The total number of observations was 15. Results of MTS according to vehicle speeds are shown in Fig. 11, where the horizontal red line indicates the threshold, and percentages denote the probability of MDs crossing the threshold. In this study, both percentage and the mean value of MDs crossing the threshold were considered. The percentage and mean value of MDs crossing the threshold are shown in Table 4 and Table 5 respectively.

In considering the percentage of the MD crossing the threshold summarized in Table 4 and Fig. 11(b) for D1 under vehicle speed S2, it is hard to read a clear pattern change in the MD due to the damage because the percentage of D1 is smaller than that of the cross-validation. On the other hand, the MD of D1 under vehicle speeds S1 and S3 (see Fig. 11a, Fig. 11c and Table 4) shows apparent changes in DI$_3$ due to damage as the percentage is greater than that of the cross-validation. For D2 and D3 under all vehicle speeds, it becomes easy to detect a fault since all MDs exceed the threshold.

In considering the mean value of the MDs crossing the threshold summarized in Table 5, the mean value of D1 under vehicle speed S2 is greater than that of the cross-validation even though the percentage of crossing the threshold is lower than that of the cross-validation as previously mentioned. For D1 under vehicle speed S1, on the other hand, the mean value is smaller than that of the cross-validation even though the percentage of crossing the threshold is higher than that of the cross-validation, as explained in Table 4.

The observations demonstrate that damage can be detected by the proposed method: both percentage and mean value of MDs crossing the threshold are utilized to make a decision on the health condition of bridges. For severe damage (D2 and D3), considering the percentage of MDs crossing the threshold would be useful for decision making. On the other hand, for light damage (D1), considering both percentage and mean value of the MDs crossing the threshold would be useful.

The Mahalanobis distances of system frequencies and damping constants identified by means of the AR model$^{14,16}$ are summarized in Appendix B for information. Since, in applying MTS, the number of evaluation items has to be unified across all observations, frequencies and damping constants have to be identified at all three observation points without fail. However, they were not always identified at every observation time by means of the AR model. The number of estimated MDs was, thus, different in each bridge condition (Intact, D1, D2 and D3).

This explains why the threshold could not be determined in Figs. B1c and B2c): following section 5(1), the number of observations in cross-validation becomes 3, while that number for the intact case is 4, which violates the required condition to utilize the MTS.

![Fig. 11 Mahalanobis distance of DI$_3$ with threshold: a) S1; b) S2; c) S3.](image)

| Table 4 Percentage of MD crossing the threshold. |
|----------------|----------------|----------------|
|              | S1  | S2  | S3  |
| Intact       | 27% | 27% | 27% |
| Cross-validation | 33% | 47% | 47% |
| D1           | 53% | 40% | 80% |
| D2           | 100%| 100%| 100%|
| D3           | 100%| 100%| 100%|

| Table 5 Mean value of MD crossing the threshold. |
|----------------|----------------|----------------|
|              | S1  | S2  | S3  |
| Intact       | 2.03| 1.79| 1.89|
| Cross-validation | 3.54| 2.60| 2.88|
| D1           | 2.36| 4.17| 3.23|
| D2           | 6.37| 6.97| 9.54|
| D3           | 6.61| 12.70| 13.65|
It is noteworthy that there is no information about the probability of exceeding the threshold in Fig. B1c) and Fig. B2c) because of the failure to decide the threshold.

The MDs of the identified system frequencies and damping constants demonstrate that the frequencies and damping constants for the first mode are insensitive to damage (see Figs. B1 and B2) compared to those for the third mode (see Figs. B3 and B4). A noteworthy point is that the probability of successfully detecting damage depends on the vehicle speed even in the case of the third mode: the total number of successfully identified frequencies and damping constants for the third bending mode of damage scenario D3 under vehicle speed S3 (1.63m/s) was less than the number for other scenarios, which leads to a smaller number of estimated MDs for damage scenario D3 appearing in Fig. B3c) and Fig. B4c).

Observations from frequencies and damping constants demonstrate that the classical modal parameters–based fault detection requires deciding damage-sensitive features which should be considered in health monitoring, such as mode, frequency and damping constant. In comparison, utilizing DI leads to successful fault detection. Therefore the approach combining DI and MTS methods provides an effective way of monitoring the health condition of bridges.

6. CONCLUSIONS

This paper investigated the feasibility of structural fault detection using traffic-induced vibration measurements on a model bridge through a laboratory experiment. This study considered the damage indicator (DI) obtained from AR coefficients as a damage-sensitive feature. The MTS method was applied to support decision making on bridge health conditions. The summarized results are as follows:

1) AR coefficients are directly associated with dynamic characteristics of the structural system.
2) According to the sensitivity analysis, considering AR coefficients up to the third order led to the most sensitive DI for structural damage in the model bridge considered in this study.
3) The DI would provide an effective way of monitoring health conditions of short-span bridges.
4) The MTS method successfully detected anomalies in the model bridge.
5) The proposed method successfully detected severe damage. For light damage, however, considering both probability and mean values of the MD crossing the threshold would be useful in making a decision on the health condition of bridges.

6) In making a decision for bridges’ health condition using the proposed approach, a higher vehicle speed leads to a better result. This may be because the faster the speed, the bigger the dynamic response, in general, of a bridge, and bigger responses of bridges could provide more information about bridge conditions than smaller responses. However, this also needs further investigation.

7) As an interesting observation, the AIC could be utilized as a damage-sensitive feature.

The next step for this study is to investigate the feasibility of the proposed approach for real-world applications. The authors are now considering, as a real-world application, fault detection of a real steel truss bridge by applying artificial damage. Another problem remaining to be solved is how to decide the optimal observation points and evaluation items, which needs further investigation.

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APPENDIX A

Dynamic equations of motion for a system can be written as Eq. (A1) and Eq. (A2):

\[ m\ddot{y}(t) + c\dot{y}(t) + k_y y(t) = 0 \quad (A1) \]
\[ \ddot{y}(t) + 2h\omega_0\dot{y}(t) + \omega_0^2 y(t) = 0 \quad (A2) \]

where \( h \) denotes the damping constant of the system, and \( \omega_0 \) is circular frequency.

A general solution is

\[ y(t) = Ge^{-\lambda t}\sin(\omega t + \varphi) \quad (A3) \]

\[ \omega = \omega_0\sqrt{1 - h^2} \quad (A4) \]
\[ \lambda = \omega_0 h \quad (A5) \]

where \( G \) is an unknown constant and \( \varphi \) indicates an unknown phase angle.

The time history of a dynamic system shown in Eq. (A2) can be modeled by the AR process as

\[ y_n = a_1y_{n-1} + a_2y_{n-2} + \cdots + a_ny_{n-w} + \cdots \quad (A6) \]

Using Eq. (A3), the dynamic response \( y_n \) can be rewritten as

\[ y_n = Ge^{-\lambda t} \sin(\omega t + \varphi) \quad (A7) \]

\[ y_{n+k} = Ge^{j\omega \Delta t}e^{-\lambda \Delta t} \sin\{\omega(t_n - k\Delta t) + \varphi\} \]
\[ = Ge^{j\omega \Delta t}e^{-\lambda \Delta t}\{ \sin(\omega t_n + \varphi)\cos(k\omega \Delta t) \}
- \cos(\omega t_n + \varphi)\sin(k\omega \Delta t) \} \quad (A8) \]
To simplify the problem, we consider the AR process up to the second order:

\[
y_{n+1} = Ge^{\Delta t}e^{-j\omega t} \left\{ \sin(\omega t_n + \varphi) \cos(\omega \Delta t) - \cos(\omega t_n + \varphi) \sin(\omega \Delta t) \right\} \tag{A9}
\]

\[
y_{n-2} = Ge^{\Delta t}e^{-j\omega t} \left\{ \sin(\omega t_n + \varphi) \cos(2\omega \Delta t) - \cos(\omega t_n + \varphi) \sin(2\omega \Delta t) \right\} \tag{A10}
\]

\[
G e^{\Delta t}e^{-j\omega t} \left\{ \sin(\omega t_n + \varphi)(1 - 2\sin^2(\omega \Delta t)) - \cos(\omega t_n + \varphi)(2\sin(\omega \Delta t)\cos(\omega \Delta t)) \right\} = y_n e^{2\Delta t} \left\{ [1 - 2\sin^2(\omega \Delta t)] \right\}
\]

From Eq. (A7), we obtain the following:

\[
G = \frac{y_n}{e^{-\Delta t}\sin(\omega t_n + \varphi)} \tag{A11}
\]

By substituting Eq. (A11) into Eq. (A9), we can obtain the following equation:

\[
y_{n+1} = y_n e^{2\Delta t} \left\{ \cos(\omega \Delta t) - \frac{\cos(\omega t_n + \varphi) \sin(\omega \Delta t)}{\sin(\omega t_n + \varphi)} \right\} \tag{A12}
\]

Substituting Eq. (A11) into Eq. (A10) gives

\[
y_{n-2} = y_n e^{2\Delta t} \left\{ [1 - 2\sin^2(\omega \Delta t)] \right\}
\]

\[
\left\{ \frac{y_{n-1}}{y_n} e^{-j\omega t} - \cos(\omega \Delta t) \right\} \left\{ 2\cos(\omega \Delta t) \right\} = y_n e^{2\Delta t} \left\{ [1 - 2\sin^2(\omega \Delta t)] \right\}
\]

Eq. (A14) can be rewritten in general formation as

\[
y_n = 2e^{-j\omega t} \cos(\omega \Delta t)y_{n-1} - e^{2\Delta t}y_{n-2} \tag{A15}
\]

Comparing Eq. (A15) with Eq. (A6) of up to the 2nd order,

\[
a_1 = 2e^{-j\omega t} \cos(\omega \Delta t), \quad a_2 = -e^{2\Delta t} \tag{A16}
\]

From Eq. (A16), we can obtain the following relationship:

\[
\lambda = \frac{\ln(-a_1)}{2\Delta t} \tag{A17}
\]

\[
\omega = \frac{1}{\Delta t} \cos^{-1} \left\{ \frac{a_1}{2\sqrt{-a_2}} \right\} \tag{A18}
\]

This also shows that AR coefficients are directly associated with dynamic characteristics of systems.

**APPENDIX B**

**Fig. B1** Mahalanobis distance of frequency of first bending mode with threshold: a) S1; b) S2; c) S3.

**Fig. B2** Mahalanobis distance of damping constant of first bending mode with threshold: a) S1; b) S2; c) S3.
REFERENCES


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