APPLICATION OF THE KMR MODEL WITH SPATIAL GEOMETRIC SCALE ON COMPUTATIONS OF RIVER MORPHODYNAMICS

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A multilevel grid-type turbulent shallow flow model combined with an equilibrium sediment transport model (KMR-MB: Kinematic Mesh Reconstruction for Movable Bed) is proposed. The KMR method is a kind of multilevel grid model with dynamic refinement that combines grid cells to efficiently capture the unsteady flow phenomena. Saito et al. (2012)¹² have applied the KMR approach to open channel flows around a bridge pier and have shown the advantage of the model. They used the maximum value of the strain and rotation parameters for the threshold for quadtree and 3×3 grid divisions. We have extended the model for computations of morphological phenomena in rivers by incorporating an equilibrium sediment transport model as well as a bed continuity equation, and used new criteria for grid cell refinement. We used an average curvature of the bed surface as the criteria for refinement and combined quadtree-type grid cells. The present model has been applied to simulate the alternate bar formations. The experimental result by Akahori et al. (2011)³ was used to validate the model. The comparison between numerical and experimental results showed that the present KMR-MB can excellently simulate the phenomena with less CPU time than those computed with the fixed grid.

Key Words: KMR, shallow flows, alternate bar, multilevel grid model, two-dimensional (2D) depth-averaged model

1. INTRODUCTION

When we analyze the various flow and morphological phenomena in rivers, the uneven distributions of the dominant scale in time and space often make the prediction and clarification of the phenomena more difficult and complicated. Let us consider the flow around a bridge pier as an example. It is obvious that the order of the spatial scale in the mean flow and bar formations is one or more order larger than those in local phenomena, such as, the von Karman vortex shedding, the horseshoe vortex formation, and the local scour. However, the interactions of the phenomena with different spatial scales cannot be always negligible. For instance, it is known that the backwater elevation behind the bridge pier reaches far upstream of the bridge.

When we apply the numerical approach on the phenomena with different scales, the size of the computational grid cell should be set small enough to capture the smallest scale phenomenon. The application of the regular Cartesian grid thus becomes considerably uneconomic. The application of the stretching grid or a generalized curvilinear grid, which can locally increase the grid cell density, can overcome the problem to some extent. However, the sudden change in the grid size in those methods is likely to yield low accuracy and unstable solution.
The application of unstructured grid models can consider the spatial variation of the dominant scale more flexibly. However, numerical methods with temporally changing unstructured grid has not prevailed yet.

Application of the Cartesian grid with spatially variable grid has also been developed to avoid errors caused by the transformation of the governing equations. Nabi (2012) proposed a novel 3D (three-dimensional) model with coupling of a multilevel grid model and the IBM (Immersed Boundary Method). He also used LES (Large Eddy Simulation) as a turbulence model and successfully simulated the dune formation process in detail. This model concisely showed the great advantage of the multilevel grid model with the Cartesian coordinate. However, a drawback of the model is the very large computational load because it requires physically precise modeling in each component. Therefore, a multilevel grid-based model with moderate computational load and reasonable accuracy is strongly required for prediction and analysis of various practical river problems from the engineering point of view. Yasuda and Hoshino (2011) proposed the quadtree-type grid model combined with 2D (two-dimensional) shallow flow equations. They analyzed the flood flow in a river and showed its efficiency and accuracy. However, the grid division in their model was static and is therefore not suitable for unsteady phenomena with variations in the scale. Krámer and Józsa (2008) used a dynamic multilevel grid approach and simulated some unsteady flow phenomena, which showed the computational efficiency of the approach. Saito et al. (2012) used a similar dynamic multilevel grid model (KMR: Kinematic Mesh Refinement) with shallow water flow equations combined with a nonlinear turbulence model. They applied the model to a flow around the bridge pier with von Karman vortex shedding and clearly showed the advantage of the model.

The present study aims to extend the KMR by Saito et al. (2012) to the unsteady movable bed phenomena, and to show the advantage of the new model. Considering the computational efficiency, an equilibrium bedload model was used. The bed surface curvature was used for the criteria of grid division and combined to fit the grid size to the dominant scale of the bed topography. The result of the laboratory experiment on a single bar formation in a straight open channel performed by Akahori et al. (2011) was adopted as a test case to compare the accuracy and computational efficiency with existing numerical models with a fixed computational grid.

2. PHENOMENA FOR THE MODEL VALIDATION

We used an alternate bar formation in an open channel flow to validate the present computational model. The alternate bar formation is a classical but very important phenomenon from the engineering point of view because it often controls flow resistance as well as environment in rivers. In the present study, it is also important that the phenomenon is reproducible by a depth-averaged 2D shallow flow equations.

We therefore used the laboratory experiment on alternate bar formation performed by Akahori et al. (2011). The laboratory test was performed with a 50 m long 90 cm wide straight experimental flume located at the Civil Engineering Research Institute for Cold Region (CERI) in Sapporo, Japan. The slope of the bed was set at 1/200 and the average grain diameter of the bed material at 0.76 mm. The discharge was set constant (=6.4 l/s) and the total experimental duration time was 2400 min. The bed profile was measured several times during the experiment using a laser sand surface profiler at every 10 cm and 5 mm in the streamwise direction and in the lateral direc-
tion, respectively, after the discharge was carefully stopped and drained gradually. After the measurement, the flow was restarted again gradually to avoid disturbing the sand surface profile.

The hydraulic conditions in the present case was plotted as a circle on the chart by Kishi and Kuroki (1984), which classified the types of bed wave patterns as shown in Fig. 1. This chart indicates that the present hydraulic condition falls into the case of single alternate bar formation.

Figure 2 shows the photograph of the bed surface observed in the experiment at $t = 2800$ min. Periodic alternate bars were clearly generated. Figure 3 shows the contour profile of the bed surface height at $t = 2800$ min. This figure indicates that the wavelength of the alternate bar is about 6 m.

3. COMPUTATIONAL MODELING

(1) Outline of the KMR method
a) Dynamic dividing and combining of grid cells

We extended the KMR method proposed by Saito et al. (2012) to the phenomena with bed deformations. The KMR method by Saito et al. (2012) is a computational method to efficiently consider the spatial distribution of the turbulence scales with Cartesian coordinate with grid cell dividing/combining for shallow depth-averaged Reynolds-averaged equations combined with a nonlinear turbulence model. They showed that the KMR method could efficiently simulate the phenomena with large-scale vortex formations, such as von Karman vortex shedding downstream region of a bridge pier, with the URANS approach.

Although the KMR method can choose an arbitrary integer for the number of grids divided into $x$ and $y$ directions separately, we explain the basic concept of the method taking an example of a 2x2-type division case (quadtree type) for simplicity.

Figure 4 shows a snapshot image of an example of grid division at a certain computational stage. In this example, the largest grid cell ($L = 1$ level grid) is
partially divided twice and the division level of the smallest grid cell is \( L_{\text{max}} = 4 \). The grid cells at every division level are classified into three types, namely, “computational cell,” “interpolation cell,” and “extrapolation cell.” At the computational cells, the variables (velocity, water depth, bed height, etc.) are obtained by solving the governing equations. In the case of the division level \( L = 2 \), the computational cells are shown in Fig. 5. Since we adopted a staggered arrangement of hydraulic variables on the grid cells, the water depth and bed height are defined at the cell centers and the velocity and bedload flux are defined at the computational edges shown in Fig. 5.

To calculate the variables on the “computational cells” by solving the partial differential equations, the surrounding cells also should store the variables for boundary conditions. If the variables stored from the information form cells of a lower division level (larger cells), the cells are called “interpolation cells.” On the other hand, if the variables stored from cells of a higher division level (smaller cells), the cells are called “extrapolation cells.” The interpolation and extrapolation cells at the surrounding parts of the computational cells are chosen considering the situation of the maximum grid division level at the surrounding cells. Figures 6 and 7 show the arrangement of the extrapolation and interpolation cells, respectively, at the division level \( L = 2 \). For example, in the case of the interpolation cells, the depth and velocities are interpolated from the lower-level grid cells at the interpolated points and the interpolated edges, respectively, before the computation of the present time step.

As for the Reynolds stress tensor, the defining locations on the cell are different for each tensor component; that is, the normal components are defined at cell centers and the shear components are defined at grid points. This approach with separate definition points is chosen to accurately and efficiently calculate each component of the Reynolds stress. In Fig. 8, the definition points of the Reynolds stress components are shown. The normal component of the Reynolds stress also should be stored at the interpolation cells. For this purpose, we did not interpolate the Reynolds stress component itself, but calculated the Reynolds stress component from the stored velocities to avoid a decrease in computational accuracy.

Since the computations for the interpolation and extrapolation cells should be done at each time step, it is necessary to employ the interpolation/extrapolation algorithm with less CPU time. We used the simple linear methods following Saito et al. (2012) for simplicity. Compared to the extrapolation cells, the interpolation cells are more vulnerable to deterioration of accuracy because the information is transferred from coarser cells to finer cells. Therefore, the employment of an interpolation method with higher-order accuracy and smaller CPU time should be considered for future model improvement.

In the present KMR approach, the grid cells combining and dividing are made automatically at every time step. In this method, the computation for the interpolation cells should be done carefully because the variables are already stored if those cells were computational or extrapolation cells in the previous time step. In such case, the accuracy becomes worse if the values are overwritten by the interpolation. Since the computational algorithm for this process is complicated, the present model employs some flag variables to simplify this process.

b) Criteria for grid division/combination

In the KMR approach, the scale of the target phenomenon, which varies temporally and spatially, should be considered efficiently. Therefore, the criteria for dividing/combining grid cells should represent the scale of the phenomenon directly. The original KMR approach by Saito et al. (2012), which only calculates flows, uses the maximum strain and rotation parameters \( M \) as follows:

\[
M = \max[S, \Omega]
\]

(1)

where \( S \) is the strain parameter and \( \Omega \) is the rotation parameter, which expresses the rate of the shear and rotation in the turbulence phenomenon, respectively. Those parameters will be defined later. If the value of \( M \) is larger, basically the spatial scale of turbulence becomes smaller. Therefore, it is reasonable to divide grid cells into smaller sizes where the value of \( M \) becomes larger. If the value of \( M \) becomes smaller, the grid cells are combined to reduce the computational load. Although the computation of \( M \) is relatively complicated, the value of \( M \) is recycled for the computation of nonlinear turbulence mode. This is a great advantage of employing \( M \) value for the grid division criteria because then it does not increase the computational load.

On the other hand, it is reasonable to use the parameters expressing the scale of bed surface complexity as the grid division criteria for the computation of bed deformation with the KMR approach. Therefore, the slope of the bed, the distance from the bank or river structure, bedload flux and the Shields number, etc. may be possible candidates for the grid division criteria. We used the following average curvature of the bed surface, \( C_{\text{ave}} \), for the grid division criteria, which directly expresses the scales of the complexity of the bed surface:

\[
C_{\text{ave}} = \frac{1}{2} \left( \frac{\partial^2 z_b}{\partial x^2} + \frac{\partial^2 z_b}{\partial y^2} \right) \max[\Delta x_{\text{max}}, \Delta y_{\text{max}}]
\]

(2)
where \( z_b \) is bed height; \( x, y \) are the plane Cartesian coordinates; \( A_{\text{cell max}}, A_{\text{cell max}} \) are the maximum cell sizes in \( x \) and \( y \) directions, respectively, at the division layer \( L = 1 \) (maximum cell sizes). In equation (2), the curvatures in \( x \) and \( y \) directions are averaged after taking absolute values in order to avoid the cancellation of curvatures if the surface profile is like a saddle point, where the signs of the curvature in \( x \) and \( y \) direction are different. The value is nondimensionalized by multiplying the maximum size of grid cells.

The thresholds values for cell dividing/combining was determined via trial and error by Saito et al. (2012). However, in the present model, the values of the threshold were adjusted dynamically at every 1000 time steps during the computation to satisfy the following relations:

\[
\sum_{L=k}^{\text{max}} A_k / \sum_{L=k}^{\text{max}} A_k = R_k \tag{3}
\]

where \( A_k \) is the area of total cells with division layer \( L = k \), and \( R_k \) is the model constant expressing the rate of cell division. If \( R_k \) is set larger, the rate of smaller cells becomes larger and the accuracy increases though the computational time increases. In the present computation, \( R_k \) is commonly set as 0.7 at each division layer \((k = 1 \text{ to } L_{\text{max}} - 1)\). The effects of those values on computational efficiency and computational accuracy should be considered further to improve the model.

**2) Flow model**

**a) Governing equations**

The computational model for flows used in the present study was based on the depth-averaged 2D shallow water equations in the Cartesian coordinate (Kimura and Hosoda, 1997)\(^8\). The governing equations are described as

**[Continuity equation]**

\[
\frac{\partial h}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0 \tag{4}
\]

**[Momentum equation in \( x \)-direction]**

\[
\frac{\partial M}{\partial t} + \frac{\partial \beta u M}{\partial x} + \frac{\partial \beta v M}{\partial y} + gh \frac{\partial h}{\partial x} = gh \sin 0 - \frac{\tau_{u'v'}}{\rho} + \frac{\partial - u''^2 h}{\partial x} + \frac{\partial - u'v' h}{\partial y} + \nu \left[ \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right) \right] \tag{5}
\]

**[Momentum equation in \( y \)-direction]**

\[
\frac{\partial N}{\partial t} + \frac{\partial \beta u N}{\partial x} + \frac{\partial \beta v N}{\partial y} + gh \frac{\partial (h + z_b)}{\partial y} = - \frac{\tau_{u'v'}}{\rho} + \frac{\partial - v''^2 h}{\partial x} + \frac{\partial - u'v' h}{\partial y} + \nu \left[ \frac{\partial}{\partial x} \left( \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial v}{\partial y} \right) \right] \tag{6}
\]

where \((x, y)\) are spatial coordinates; \( t \) is time; \( h \) is depth; \((u, v)\) are depth-averaged velocity components in \((x, y)\) directions; \((M, N)\) are fluxes in \((x, y)\) direction defined as \((hu, hv); (u', v')\) are turbulence velocities in \((x, y)\) directions; \(-u''_j, v''_j\) is depth-averaged Reynolds stress tensor; \( \rho \) is dynamic viscosity coefficient; \( \sin \theta \) is bed slope; \( f \) is friction coefficient (function of Reynolds number); \((\tau_{ux}, \tau_{uy})\) are bed friction stress vectors; \( \beta \) is momentum coefficient; \( \theta \) is angle between stream line and \( x \)-axis; and \((\tau_{bx}, \tau_{by})\) are bottom shear-stresses in \((x, y)\) directions.

The components of the bottom shear-stress vector are evaluated by

\[
\tau_{ux} = \frac{f \rho u}{2} \sqrt{u'^2 + v'^2}; \quad \tau_{uy} = \frac{f \rho v}{2} \sqrt{u'^2 + v'^2} \tag{7}
\]

where \( f = \) friction factor related to local Reynolds number \( Re' \equiv uhl / \nu \), evaluated as follows:

\[
f = \frac{6}{R_e} \left( R_e' \leq 430 \right), \quad \frac{2}{\sqrt{f}} = A_k - \frac{1}{k} \left[ 1 - \ln \left( R_e' - \frac{f}{\sqrt{2}} \right) \right] \quad (R_e' > 430) \tag{8}
\]

where \( k = 0.41, A_k = 5.5. \)

**b) Turbulence model**

For the turbulence model, we used the modified second-order nonlinear 0-equation model developed by Kimura and Hosoda (2004)\(^9\), and Kimura et al. (2009)\(^10\). The constitutive equation of the present turbulence model is shown as

\[
-u''_j u'_j = \nu \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} k \delta_{ij} + \frac{h}{u_s} v \sum_{i=1}^3 C_j \left( S_{ij} - \frac{1}{3} S_{j\alpha\beta} \delta_{ij} \right) + C_j u'^2, \quad (i, j) = 1, 2 \tag{9}
\]

\[
S_{ij} = \frac{\partial U_i}{\partial x_j} \frac{\partial U_j}{\partial x_i}, S_{2ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} \frac{\partial U_j}{\partial x_i} + \frac{\partial U_j}{\partial x_i} \frac{\partial U_i}{\partial x_j} \right),
\]

\[
S_{3ij} = \frac{\partial U_i}{\partial x_j} \frac{U_j}{U_i} \frac{\partial U_i}{\partial x_j}
\]
\[ \alpha(M) = \min \left[ 0.2, \frac{0.3 \gamma_k \lambda_p}{1 + 0.09 M^2} \right] \]

\[ C_{\alpha} = \frac{\gamma_k}{3} (C_3 - 2C_1), C_{\beta} = \frac{\gamma_k}{3} (C_1 + C_3), C_{\gamma} = 0 \]

(10a,b,c)

\[ f_M(M) = \frac{1}{1 + 0.02 M^2} \cdot M = \max[S, \Omega] \]

\[ k = \gamma_k u^2, \gamma_k = 2.07, \lambda_p = \alpha/(\gamma_k C_p) = 1.07 \]

\[ S = \lambda_p \frac{h}{u^*} \left( \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \right)^2 \]

\[ \Omega = \lambda_p \frac{h}{u^*} \left( \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i} \right) \right)^2 \]

The eddy viscosity coefficient is evaluated by the following 0 equation model:

\[ v_i = \alpha(M) h u^* \]

(12)

where \( u^* \) = local friction velocity \( = \sqrt{f(v^2 + v^2)/2} \); and \( k \) = depth-averaged turbulent kinetic energy evaluated by the empirical formula proposed by Nezu and Nakagawa (1993)\(^{11} \), who proposed the universal expression in equation (13) for turbulent kinetic-energy distribution.

\[ k = 4.78 \exp \left( -2 \frac{z}{h} \right) \]

(13)

### (3) Bed morphological model

In the present model, only bedload is considered for the bed deformation. For the bedload flux model, we used the following formula by Yamaguchi and Izumi (2003)\(^{12} \), which simplified the Kovacs and Parker’s model (1994)\(^{13} \) considering the bed slope in the streamwise direction:

\[ q_{bx} = \frac{K}{1 + \frac{\partial z_b}{\partial s}/\mu_r} \left[ \tau^* - \tau_c^* \left( 1 + \frac{\partial z_b}{\partial s}/\mu_r \right) \right] \]

\[ \times \left[ \tau_c^* \frac{1}{1 + \frac{\partial z_b}{\partial s}/\mu_r} \right]^{1/2} \left[ \frac{(\sigma - 1)}{\rho} \right] gd^3 \]

(14)

where \( z_b \) = bed height; \( q_{bx} \) = bedload flux in the streamwise direction; \( K \) = model coefficient; \( s \) = coordinate in the streamwise direction; \( \mu_r \) = kinematic friction coefficient; \( \tau^* \) = Shields’ number; \( \tau_c^* \) = nondimensional critical traction force; \( \sigma \) = density of bed material \((= 2650 \text{kg/m}^3)\); and \( \rho \) = density of water \((= 1000 \text{kg/m}^3)\).

The bedload flux in the lateral direction perpendicular to the streamwise direction \((= q_{by})\) is evaluated by the following Hasegawa formula\(^{14} \):

\[ q_{by} = q_{by}^{AM} \left[ -\frac{h}{r} N_s \frac{\partial z_b}{\partial n} \sqrt{\frac{\tau_{c\perp}}{\mu_s \mu_r \mu_c}} \right] \]

(15)

\[ q_{by}^{AM} = K \left[ \tau^* - \tau_c^* \right]^{1/2} \left[ \frac{(\sigma - 1)}{\rho} \right] gd^3 \]

(16)

where \( n \) = coordinate in the lateral direction; \( \mu_r \) = static friction coefficient; \( r \) = curvature radius of the streamline; and \( N_s \) = model constant expressing the strength of the secondary current \((= 7)\).

The bed continuity equation is expressed as

\[ \frac{\partial q_{bx}}{\partial t} + \frac{1}{1 - \lambda} \left[ \frac{\partial q_{bx}^x}{\partial x} + \frac{\partial q_{bx}^y}{\partial y} \right] = 0 \]

(17)

where \( \lambda \) = void rate (porosity) of the bed material; \( q_{bx}^x \) = bedload flux in the \( x \)-direction; and \( q_{bx}^y \) = bedload flux in the \( y \)-direction.

### (4) Numerical procedure

The QUICK scheme with second-order accuracy in space is used for convective inertia terms in the momentum equation because it is a conservative scheme. The friction in the walls is evaluated by the log-law, and the periodic boundary conditions are specified at the inlet and the outlet of the flow domain. To stabilize computation, the computations of the depth and fluxes are done iteratively at every time step.

The arrangement of the variables related to the bed morphology in equation (17) closely affects the reproducibility of the dunes. Therefore, before applying the KMR approach, the influences of the variables definition points are initially examined with a fixed computational grid. The derivative term with \( x \)
in equation (17) is described as follows as the bed height $z_b$ is defined at the cell centers (see Fig.8):

$$\frac{\partial q_{bx}}{\partial x} \bigg|_{i+1/2} \Rightarrow \frac{q_b^i - q_b^{i-1}}{\Delta x}$$

(18)

It is important to adequately evaluate bedload flux $q_{bx}$ and $q_{ay}$ at the cell boundaries because the results closely depend on them. Bedload flux $q_b$ is easier to compute at cell center than at cell boundary on a staggered grid. We tried the following three different ways for preliminary computations and the results were compared.

Case 1: Bedload flux $q_{bx}$ is first calculated at cell centers (at $i-1/2$, $i+1/2$, etc.) and the fluxes at cell boundaries are then calculated by taking the average of values at two adjacent cell centers.

$$q_{bx}^i \Rightarrow q_{bx}^i \bigg|_{i+1/2} + q_{bx}^i \bigg|_{i-1/2} \over 2$$

(19)

Case 2: Bedload flux $q_{bx}$ is first calculated at cell centers (at $i-1/2$, $i+1/2$, etc.) and the value at cell boundary is then given as the value at the cell center just upstream (upstream of flow) of the cell boundary as follows:

$$q_{bx}^i \begin{cases} q_{bx}^i \bigg|_{i+1/2} : u_i \geq 0 \\ q_{bx}^i \bigg|_{i-1/2} : u_i < 0 \end{cases}$$

(20)

Case 3: Bedload flux $q_{bx}$ is initially evaluated at cell boundaries and is used directly for equation (17). This method takes more CPU time than the other two methods because $q_{bx}$ and $q_{by}$ should be calculated separately at the different cell boundaries.

The three different schemes are applied to the present case using a regular uniform grid ($1 \text{cm} \times 1 \text{cm}$). Figure 10 shows the results with three schemes at $t = 10000$ (sec). Only the result of Case 3, regular alternate bars like the experimental result (see Fig.2) are simulated. The result using Case 1 also shows an alternate bar like a bed form although the shape is quite unstable. The result of Case 2 is very irregular and cyclic features are not observed at all. The result indicates that the method defining the bedload flux at the cell centers is inadequate. One of the notable differences between Cases 1 and 2, and Case 3 is the stencil for calculating bed slope. When the bedload flux is calculated at cell centers (Cases 1 and 2), the stencil for evaluating the bed slope extends wider than in the case with the bedload defined at cell boundaries (Case 3). Therefore, the accuracy for evaluating the bed slope in Cases 1 and 2 are worse than that in Case 3. This likely explains why only the computation with Case 3 could generate adequate clear alternate bar, although further considerations for the mechanism are necessary. Hereafter, only the scheme of Case 3 is used for computations with the KMR-MB method.

Figure 11 shows the whole flowchart of the computation using the KMR-MB method. The interaction process is added between the depth and the discharge flux calculations to stabilize the computation.

4. RESULTS AND DISCUSSIONS

(1) Computational conditions

The computations were carried out under the same conditions with those in the laboratory test by Akahori et al. (2011)\(^3\). The length of the computational domain was set at $12.06 \text{ m}$ and cyclic boundary conditions were set at the inlet and outlet boundaries. The length of the computational domain was about double the wave length of the experimental result.

The base cell size (maximum cell size at level $L_1 = 1$) in both $x$ and $y$ directions was set at $18 \text{ cm}$. The
The number of base grid cells in \( x \) and \( y \) directions were 67 and 5, respectively. The grid division depth (maximum grid cell level) was set at \( L_{\text{max}} = 4 \). The number of grid cells at \( L = L_{\text{max}} = 4 \) in \( x \) and \( y \) directions were 536 and 40, respectively. The grid cell size at \( L = L_{\text{max}} = 4 \) became 2.25 cm in both \( x \) and \( y \) directions.

For the comparison, the computations with fixed grids, with similar sizes as those of \( L = 1 \) (\( \Delta x = \Delta y = 18 \text{ cm} = \text{const.} \)) and \( L = L_{\text{max}} = 4 \) (\( \Delta x = \Delta y = 2.25 \text{ cm} = \text{const.} \)) were also carried out.

The total computational time in all cases were set at 10000 sec, which was enough to achieve the equilibrium state of the bed form.

(2) Model validation

Figure 12 shows the computational results by the KMR-MB method at \( t = 3500, 4800, \) and 10000 sec. The color contour shows the bed elevation change from the initial condition. The grid division lines are shown together in those figures. Those figures demonstrate that the grid cells are gradually divided into finer sizes as the bed deformation increases. The results also show that the grid cells are divided into smaller sizes where the bed surface curvature becomes larger particularly at the edges of bars. The computational result at \( t = 1000 \) sec using the present KMR-MB method is in excellent agreement with the experimental results in Figures 2 and 3.

Figures 13 and 14 show the streamwise bed height profile at the centerline and 10 cm apart from the right bank in the experimental result and the three computational results; i.e., the result by KMR-MB method, the result by the fixed regular grid with the cell size of \( L = 1 \) (18 cm \( \times \) 18 cm), and the result by the fixed grid with the cell size of \( L = L_{\text{max}} \) (2.25 cm \( \times \) 2.25 cm). The bed profiles by the KMR-MB method are almost the same as the result with the fixed regular grid of \( L = L_{\text{max}} \), and are in good agreement with the experimental results. On the other hand, the computational result of the regular fixed grid of \( L = 1 \) can also reproduce the periodic fluctuations of the bed though the shape of the peaks are excessively smoothed. The amplitude at 10 cm away from the right bank by the regular fixed grid of \( L = 1 \) is smaller than the experimental result.

Figure 15 shows the lateral location (value of \( y \) coordinate, \( y = 0 \) means the location of the channel center) of the gravity center of the cross-sectional flow area plotted against the streamwise coordinate. The figure indicates the magnitude of meandering features of the flow center. Although the computation with fixed grid of \( L = 1 \) underpredicted the meandered feature of the flow, the results with KMR-MB method and the fixed grid of \( L = L_{\text{max}} \) were in reasonable agreement with the experimental results.

(3) Computational efficiency

The CPU time in the computation by KMR-MB method is 55% of CPU time of the computation with regular fixed grid of \( L = L_{\text{max}} = 4 \). The total number of grid cells in the computation with the KMR-MB method at the end of the computation is 39.8% of that of the computation of the fixed grid of \( L = L_{\text{max}} \). This result indicates that the rate of the decrease in CPU time is smaller than the rate of the decrease of grid cells. This difference seems to be caused by the overheads for the interpolation and extrapolation of grid cells. The reduction of the overhead is one of problems to be solved in the next step for model improvement.

5. CONCLUDING REMARKS

In this study, we extended the KMR method,
which has been proposed for computing shallow water flows efficiently by multilevel grid approach, and proposed the KMR-MB method for simulating bed morphology in shallow open channel flows. The present method was applied to simulate the alternate bar formations and the model performance and advantages were examined. For the grid cell division/combining criteria, the mean curvature at the bed surface was adopted. The threshold values of the mean curvature for the cell division/combining were adjusted dynamically through the computation to keep the rate of the cell divided area versus total computational domain constant.

The present model was applied to the alternate bar formations in shallow open channel flows for model validation. The computational result by the present approach was quite reasonable and was in excellent agreement with the existing experimental result. The CPU time by the present approach could reduce CPU time by about 45% compared with the case of the fixed regular grid, though the reduction rate was smaller than the rate of the decrease of total grid cell number.

The model should be applied to wider phenomena to check its applicability, and should be considered further to reduce the overhead for the interpolation and the extrapolation to improve the computational efficiency in the next step of model improvement.

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REFERENCES
2) Kimura, I., Saito, S. and Shimizu, Y.: Computations of unsteady river flows around a bridge pier by shallow multi-level grid model, International Conference for River


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