SHEAR FAILURE MECHANISM OF REINFORCED CONCRETE HAUNCHED BEAMS

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This study aims to clarify the shear resistance mechanism of reinforced concrete haunched beams (RCHBs). Four series of ten RCHBs with different parameters (positions of haunched portions, thickness of the concrete cover, presence of stirrups and the arrangement of the tensile rebar) were tested. The results demonstrated that the bending position of the tensile rebar as well as the different arrangement of the tensile rebar highly influenced the crack propagations which caused the variation in the shear capacities due to different contributions of the arch action. Different diagonal crack angles also resulted in different contributions by stirrups in RCHBs with stirrups. The thicker concrete cover at the mid span affected the crack propagation but had almost no effect on the shear capacity. Nonlinear FEM analyses were also conducted to complement the experiments and verify the results, which showed a good agreement including load-deflection curves, load capacities and crack patterns. The shape of the compression zone, which dominates the arch action in RC beams, was also evaluated by the analyses.

Key Words : haunched beam, arch action, angle of diagonal cracks, inclined compression zone, FEM analysis

1. INTRODUCTION

Reinforced concrete haunched beams (RCHBs) are widely used in simply supported and continuous bridges, structural portal frames, mid-rise framed buildings and cantilevers (Fig. 1). Such beams can reduce the weight of structures and contribute to the appearance from the aesthetic viewpoint, while using the concrete and steel bars more efficiently. Sometimes, they are also used to ease the placement of the facilities and equipment (electrical, air conditioning, piping, etc.) by providing more space under the ceiling.

Despite the fact that RCHBs are commonly used in current structures, the number of experimental data to predict the shear behavior of RCHBs is insufficient. Moreover, rational and economical design method in current JSCE (Japan Society of Civil Engineers) standard specifications for concrete structures1) or United States (ACI-318-11) building code requirements2) has not been established yet. Although the German code (DIN 1045-1, 2001)3) and some textbooks4) mention the necessity of accounting for the shear component of inclined tensile rebar and the compression force of concrete in the members with variable effective depth, there is no indication of the critical section in which the shear capacity should be calculated. Since the effective depth of RCHBs varies along the member axis from the support to the middle portion, it almost has no practical meaning if the critical section was not decided. As a result, structural engineers are mostly using such beams based on the empirical background. In order to ensure the reasonable design and to understand the shear behavior of haunched beams, it is necessary to explore the shear resistance mechanism of RCHBs.

Among the previous researches, Debaiky and Elniema5) investigated the position where the critical shear crack initiated and the influence of the
haunch’s inclination on the shear contribution of concrete. MacLeod and Houmsi proposed a method to predict the shear capacity of RCHBs without shear reinforcement based on the German code with a new assumption of critical section at which the shear strength should be calculated. However, the mechanism of the method is not clear, and the accuracy was not so high when applying to the RCHBs from the other researchers. Tena-Colunga et al. concluded that the shear capacity of RCHBs is affected mainly by the inclination of haunched portion and the effective depth at the mid span. After the static loading test, Tena-Colunga et al. also conducted the cyclic loading test to investigate the deformation and energy dissipation capacities. However, the experimental information is still not sufficient.

From the previous experimental information, normally the researchers fixed the effective depth at the support and changed the haunch’s inclination (the effective depth at the mid span was changed consequently). Since the splitting cracks or debonding cracks along the tensile longitudinal rebars and the main diagonal cracks initiating near the bending position of tensile rebars were observed in the previous researches, four new parameters that may affect such crack patterns and the shear behavior are investigated in this study. The positions of haunched portions, the thickness of the concrete cover, and the arrangement of the tensile rebar (using disconnected rebar instead of bent tensile rebar) may affect the propagation of the debonding cracks and diagonal cracks directly, while the presence of stirrups may affect them indirectly. Hence, with the objective of investigating the shear carried by concrete and stirrups in RCHBs to clarify the shear resistance mechanism, four series of ten RCHBs based on these four new parameters were tested. In addition, since the shape of the compression zone, which dominates the arch action in RC beams, could not be perfectly measured in the experiments, nonlinear FEM analyses were conducted to evaluate the compression zone and verify the results. It can be a good complement for the experiment and help to understand the shear resistance mechanism better.

2. EXPERIMENTAL PROGRAM

(1) Test specimens

A total of ten specimens belonging to the four series were subjected to a four-point bending test in the experimental program. Table 1 and Fig. 2 summarize and illustrate the details of the tested beams, including the dimension and reinforcing bars arrangement.

![Fig. 1 RC structures with haunched beams.](image)

<table>
<thead>
<tr>
<th>Table 1 Specimens’ details and material properties.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Series</strong></td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>I</td>
</tr>
<tr>
<td></td>
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<td></td>
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<tr>
<td></td>
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<tr>
<td>II</td>
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<tr>
<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>III</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

$f'_c$: compressive strength of concrete; $a$: shear span; $b$: distance between loading point and beginning of haunched portion; $c$: length of haunched portion; $e$: distance between support and end of haunched portion; $D_s$: beam depth at support; $d_s$: effective depth at support; $D_m$: beam depth at mid span; $d_m$: effective depth at mid span; $\rho_{sv}$: stirrup ratio.
angement of the RCHBs. The shear span \((a)\) was 650 mm and effective depth varied from 250 mm \((d_s)\) to 200 mm \((d_m)\) along the member axis. Consequently, the shear span-to-effective depth ratio also varied from 2.6 to 3.25. It satisfied the condition of \(a/d > 2.5\) for all beams with the purpose of not magnifying the characteristic arching mechanism. The haunch’s inclination \(\alpha\) was fixed to 11.3 degrees based on the dimension of real structures with large haunches as well as considering the feasibility of the formwork. All the specimens were designed to fail in shear by providing less or no stirrups in the left shear span while ensuring sufficient flexural capacity. The experimental parameters of these four series were the positions of the haunched portions from the loading point, thickness of the concrete cover, presence of stirrups and the arrangement of the tensile rebar, which were also used to name the specimens. For example, in the beam HS-100, “H” means a haunched beam, “S” means with stirrups, and 100 ...
represents the distance (b) between the haunched portion and loading point. “N” in series II means normal prismatic shape, while “D” in series IV means disconnected tensile rebar.

(2) Material Properties

In all ten specimens, D25 bars ($A_s=506.7 \text{ mm}^2$) having nominal diameter of 25.4 mm and yield strength of 411 N/mm$^2$ were used as tensile longitudinal bars. Two round bars having a diameter of 6 mm and yield strength of 328 N/mm$^2$ were used as compression bars. In series I, II, and IV, the D6 stirrups ($A_s=31.67 \text{ mm}^2$) having nominal diameter of 6.35 mm and yield strength of 322 N/mm$^2$ were arranged at the spacing of 200 mm in the non-test shear span to ensure the failure of the test shear span. In series III, the D6 stirrups were arranged at the spacing of 120 mm in the test shear span while D10 stirrups ($A_s=71.33 \text{ mm}^2$) having nominal diameter of 9.53 mm and yield strength of 363 N/mm$^2$ were arranged at the spacing of 120 mm in the non-test shear span to ensure the failure of the test shear span. The yield strength of steel bars slightly changed in different series of specimens as the ordering time for material was different. The above value is the average value for the four series.

To obtain the concrete strength of 30 N/mm$^2$, high-early strength Portland cement, fine aggregates, coarse aggregates, and air-entraining water-reducing agent were mixed in the proportion as shown in Table 2.

<table>
<thead>
<tr>
<th>$G_{\text{max}}$ (mm)</th>
<th>W/C</th>
<th>Unit weight (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>W</td>
</tr>
<tr>
<td>20</td>
<td>0.60</td>
<td>178</td>
</tr>
</tbody>
</table>

$G_{\text{max}}$: maximum size of coarse aggregate; W: water; C: cement (density = 3.14 g/cm$^3$); S: fine aggregate; G: coarse aggregate; AE: air-entraining water-reducing agent.

(3) Test setup and instrumentation

The specimens were subjected to a four-point bending with simply-supported condition. Steel plates with 50 mm width and 150 mm length were placed on the pin-hinge supports. Teflon sheets and grease were inserted between the specimen and supports in order to remove the horizontal friction. At the loading points, the steel plates with 65 mm width and 150 mm length were also placed. Figure 3 shows the actual loading test setup along with the locations of loading points.

During the four-point bending tests, the mid-span deflection was measured using four displacement transducers at the mid span and supporting points. The strain in tensile steel bars at various locations and the strain of concrete in several sections were measured by attaching strain gauges on the surface of tensile bars and concrete. About the stirrups in series III, two strain gauges were attached for each stirrup. One was near the bottom, close to the tensile rebar, and the other one was along the line from the support to the loading point (Fig. 2). Since the location of the strain gauges was slightly different in each specimen according to the dimensions, detailed information on the location will be explained for the certain specimen in Section 3(3). In addition, the crack propagation on the surface of test shear span during the loading test was captured in pictures.

3. EXPERIMENTAL RESULTS AND DISCUSSIONS

(1) Crack pattern and failure process

The crack patterns observed in the specimens just after the peak load are shown in Fig. 4. The white dashed lines at the bottom represent the positions of tensile longitudinal bars, while the white solid lines represent the positions of stirrups in the specimens. The red solid lines represent the yielded stirrups before the peak load during the loading test. For the four RCHBs without stirrups in series I, after the initiation of flexural cracks, the main diagonal cracks started from the bending position of the tensile rebar near the loading point and it proceeded along the inclined rebar and towards the loading point as well. The reason for such crack patterns is supposed to be that, the bending shape of tensile rebars caused the stress concentration near the bending position, while the tensile force tended to straighten the bent rebars and push over the concrete cover. After the initiation of the diagonal cracks, the load was observed to
increase until the concrete crushing near the loading point happened. Therefore, the failure mode was supposed to be the shear compression failure in series I.

For the beam HN-200 belonging to series II, the inclined shear crack started from the middle part of the haunched portion and propagated towards the loading point because the thick concrete cover caused some constraints in the beginning. During the formation of the whole diagonal cracks, the debonding cracks along the tensile steel bars suddenly occurred, resulting in the sudden drop of the load. However, the beam failed in the shear due to the crushing of concrete near the loading point eventually.

In series III, the main diagonal crack patterns showed similarity with that of RCHBs without stirrups, especially for the beams with same dimensions (for example H-0 and HS-0). Due to the existence of stirrups, more shear cracks and flexural cracks were observed, and the shear capacity was increased. Finally, the concrete crushed near the loading point.

For the two beams with disconnected tensile rebars in series IV, since the stress concentration near the bending positions was mitigated, the crack patterns of these two beams were totally different from the others. In the beam HD-100, the shear behavior and the diagonal cracks were similar to a normal slender beam with constant depth. Although some debonding cracks occurred along the inclined tensile rebar, almost no debonding cracks occurred along the horizontal tensile rebar and the region near the supporting point. The anchorage length with nut plate was also long enough to avoid the stress concentration in the end of the nut plate. It made the beam action govern the shear behavior and no arch action occurred before the failure. The peak came when diagonal cracks occurred. Therefore, the failure mode was diagonal tension failure. In the beam HD-300, because the debonding cracks in the inclined tensile rebar developed until the support, and the anchorage length with nut plate was not long from the joint point, a strong concrete strut was formed between the loading point and the anchorage.

**Fig. 4** Cracks just after peak load.
of the nut plate. As the load increased, a new critical shear crack was developed from the support. Finally, the collapse of the beam occurred right after the new critical shear crack penetrated into the concrete strut, making the failure very brittle. Therefore, both the use of nut plate and the arrangement of tensile rebar affected the shear behavior of HD-300 significantly.

(2) Load-displacement relationship

Figure 5 shows load-displacement curves of the RCHBs without stirrups in series I, II and IV. Figure 6 shows load-displacement curves of the RCHBs with stirrups in series III comparing with series I. The summary of experimental results of these ten beams is shown in Table 3. The shear capacity of the beam H-0 was the largest (68.1 kN) among all the four beams in series I. The shear capacities of H-100, H-200 and H-300 were smaller by 12%, 43% and 47% respectively than that of H-0. In the same manner as the similarity of crack patterns, the shear capacity of H-100 (60.3 kN) was close to the value of H-0, whereas the shear capacities of H-200 and H-300 did not show significant difference.

Although the cracks pattern and crack propagation of the beam HN-200 were different from other RCHBs of series I, the shear capacities of H-200, H-300 and HN-200 did not show significant difference.

Table 3 Summary of experimental results.

<table>
<thead>
<tr>
<th>Series</th>
<th>Specimen</th>
<th>P (kN)</th>
<th>V (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>H-0</td>
<td>136.2</td>
<td>68.1</td>
</tr>
<tr>
<td></td>
<td>H-100</td>
<td>120.5</td>
<td>60.3</td>
</tr>
<tr>
<td></td>
<td>H-200</td>
<td>77.1</td>
<td>38.5</td>
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<tr>
<td></td>
<td>H-300</td>
<td>71.9</td>
<td>36.0</td>
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<tr>
<td>II</td>
<td>HN-200</td>
<td>73.3</td>
<td>36.5</td>
</tr>
<tr>
<td>III</td>
<td>HS-0</td>
<td>154.7</td>
<td>77.4</td>
</tr>
<tr>
<td></td>
<td>HS-100</td>
<td>146.6</td>
<td>73.3</td>
</tr>
<tr>
<td></td>
<td>HS-300</td>
<td>137.0</td>
<td>68.5</td>
</tr>
<tr>
<td>IV</td>
<td>HD-100</td>
<td>129.6</td>
<td>64.8</td>
</tr>
<tr>
<td></td>
<td>HD-300</td>
<td>157.6</td>
<td>78.8</td>
</tr>
</tbody>
</table>

\( P \): Peak load; \( V \): Shear capacity.
The two beams in series IV showed a different tendency from series I in that the shear capacity of the beam HD-300 was 21.6% larger than that of HD-100. In series I, arch actions all occurred due to bent tensile rebars and debonding cracks. The different shear capacities only resulted from the different contributions of arch action. However, in series IV, as the occurrence of arch action in the two beams was different, the tendency changed. Another significant difference was the much more brittle shear failure in beams HD-100 and HD-300. As seen from the load-displacement curves, the load of the beam HD-100 suddenly dropped when the diagonal cracks occurred. After that, the arch action started to form in the beam HD-100, causing the load to increase slightly, though still smaller than the peak load caused by the diagonal crack occurrence. Thus, the failure mode of the beam HD-100 was diagonal tension failure. For the beam HD-300, the stiffness of the load-displacement curve changed when the debonding cracks occurred. Since the arch action developed in the shear span, the shear capacity was larger than that of the beam HD-100. Finally, the penetration of the critical shear crack into the whole concrete strut in the beam HD-300 made the shear failure much more brittle.

For the RCHBs with stirrups in series III, the shear capacity of the beam HS-0 was the largest (77.4 kN). For the beams HS-100 and HS-300, the shear capacities were smaller by 5.3% and 11.5%, respectively than that of the beam HS-0. At the same time, comparing the peak loads of the three RCHBs with stirrups and the peak loads of the three RCHBs without stirrups showed that the increased load due to the stirrups in each group decreased from 65.1 kN in the beam HS-300 to 18.5 kN in the beam HS-0. It also indicates that the load gaps between the three RCHBs with stirrups became smaller than the load gaps between the three RCHBs without stirrups. The reason behind such performances will be discussed in the following sections.

(3) Effect of the bent tensile rebars and the arch action

As shown in the load-displacement curves in series I and II, after the initiation of the diagonal cracks, the load still increased until the failure. However, the beams in series I and II were slender beams (a/d > 2.5) without stirrups. The diagonal tension failure is supposed to govern the failure mode in which the failure comes when the diagonal cracks occur\(^9\). Such inconsistency in the performance was caused by the forming of the arch action in the shear span due to the debonding cracks. The existence of arch action was also proved by the strain distribution of concrete and steel rebars. Figure 7 shows an example of arch action in the beam H-0. With the strain gauges attached on the surface of a concrete beam for five sections in the shear span, the distribution of the strain for these five sections just before the peak load can be derived, as shown in Fig. 7(a). The positive value of the strain means tension, while the negative value of the strain means compression in concrete. Through such strain distributions, the upper boundary of the inclined compression zone shown as the shadow part in Fig. 7 can be obtained as the red solid line. As the lower boundary cannot be measured, it is drawn subjectively as the red dashed line covering allpressive areas, as well as along the main diagonal cracks. Determining
the lower boundaries by using numerical analyses is one of the research works that will be introduced in Chapter 4. Figure 7(b) shows the strain distribution along the tensile rebar at several load levels. The horizontal axis shows the distance from the support to each strain gauge, which can also be found in Fig. 7(a). From the tendency of strain distributions, the bond loss as the load level increased can be observed clearly. When the load was small, the strain distribution started from zero at the support and developed proportionally to the bending moment. It means that a good bond existed in the beam. When the load was near the peak, the strain distribution started from around 1200µ, while the strains in the shear span were becoming flat. It means a partial loss of bond. Both of the inclined compression zone and the bond loss along the tensile rebar indicate the occurrence of arch action in RCHBs without stirrups⁹).

(4) Contribution of concrete to shear resistance

By using the same method introduced in the beam H-0, the existence of arch action in the other beams was confirmed (except HD-100). Figure 8 shows the inclined compression zones in all ten beams as the red shadow parts before the failure¹⁰). The method to determine the compression zones is the same as the one stated in Fig. 7 in which the upper boundary came from the strain distributions, while the lower boundary was drawn subjectively to cover all compressive areas as well as along the main diagonal cracks. For the specimens in series I, as the bending positions of the tensile rebars in the beams H-0 and H-100 were close to the loading point, the generation

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of main diagonal cracks shifted close to the loading point, making the concrete area above the main diagonal cracks very large. According to the strut-and-tie model\(^1\) in Fig. 9, the large concrete area, especially the area near the loading point where concrete crushing occurred, makes the cross-sectional area of the compression strut and horizontal compression zone larger, resulting in a stronger arch action to resist larger shear force. Since there was a high possibility of concrete crushing in both compression strut and horizontal compression zone in the strut-and-tie model, which depended on the cross-sectional area respectively, the failure due to concrete crushing near the loading point was considered as shear compression failure in this study.

The effect of the thick concrete cover in HN-200 was investigated by comparing it with the beam H-200 having the same arrangements of steel rebars. Because the thick concrete cover provided the constraint at the beginning, the load at diagonal cracks became 21% larger than that of the beam H-200. However, after the initiation of the debonding cracks, the arch action also developed due to the bond loss. Since the diagonal cracks before the crushing of concrete near the loading point were similar to those of the beam H-200, the compression zone and the developed arch action in the beam HN-200 were similar to those of the beam H-200. As the shear capacities of these two beams were almost the same with each other, it indicates that the situations of these two beams became almost the same after the occurrence of debonding cracks and the concrete cover made no contribution to shear capacity.

In the other three RCHBs with stirrups, the arch action was also developed to resist the shear force together with the contribution of stirrups. As the modified truss model only contained concrete member and stirrups in the shear resistance, the lattice model\(^2\) of RC beams with stirrups in Fig. 10 was utilized to explain the possibility of combining arch member and stirrups in the total shear resistance. In this study, because it is impossible to distinguish the shear contribution of arch member and concrete member, they were classified as the shear carried by concrete together. Since the main diagonal cracks, inclined compression zones, and failure modes in the beams with same dimensions (for example H-0 and HS-0) were almost the same, the contributions of concrete to shear in the beams with same dimensions are supposed to be the same.

For the two beams in series IV, it is difficult to compare the shear capacity with other RCHBs because of the different reinforcement ratio. In the same manner as the measurement of the beam H-0 introduced in series I, by measuring the strain distributions at five sections of concrete beams and strain distributions along the tensile steel bars, the existence of arch action was checked in both beams HD-100 and HD-300. For the beam HD-100, the strain distributions in concrete was more like the beam action in which the top part of concrete was in compression and the value of strain decreased from top to the neutral axis. The strain distribution along the tensile rebar also showed the same result; that is, no arch action existed in the beam HD-100 before the failure. The strain value increased almost linearly from zero as the distance from the support increased, which means that the bond was good and the tensile force was related to the moment. For the beam HD-300, the strain distributions of five concrete sections in the concrete indicated an inclined compression zone, which means that the arch action developed in the shear span. Therefore, the arrangement of the tensile rebars, as well as the anchorage length together with the usage of nut plates affect the shear behavior and shear capacity significantly. Thus, more attention to the tensile rebar’s arrangement should be paid in the future based on these interesting results.

(5) Contribution of stirrups to shear resistance

Considering the force acting at the diagonal crack in the RC beam with stirrups subjected to point loads, it can be seen that the shear force is resisted by the shear carried by concrete \(V_c\) (including the contribution of arch action) and the shear carried by stirrups \(V_s\). Consequently, the shear capacity \(V\) of RC beams is simplified in Eq. (1):

\[
V = V_c + V_s
\]

Therefore, as \(V_c\) in the beams with same dimensions was assumed to be same, the shear carried by stirrups \(V_s\) can be calculated by subtracting the shear capacity of the RCHB without stirrups from the
RCHB with stirrups in same dimensions (see $V_{s\text{-exp}}$ in Table 4). At the same time, Eq. (2) introduced from the truss theory with variable angle of diagonal crack was chosen to calculate the shear carried by stirrups $V_s$:

$$V_s = A_w f_{wy} \left( \zeta \cot \theta / s \right)$$  \hspace{1cm} (2)

where $A_w$ is the cross-sectional area of stirrups in the range of $s$, $f_{wy}$ is the yield strength of stirrup, $\zeta$ is the internal lever arm ($=jd$) with $j = 7/8$, $\theta$ is the angle of the diagonal crack to the axis of a beam, and $s$ is spacing of stirrups.

The main diagonal cracks in the beams HS-0 and HS-100 did not pass any stirrups, while the debonding cracks caused two stirrups near the loading point to yield before the peak load (Fig. 4). For the beam HS-300, the main diagonal cracks passed two stirrups and the debonding cracks passed one. All these three stirrups in HS-300 yielded before the peak (Fig. 4). However, the shear capacities of HS-0 and HS-100 did not increase so much compared with HS-300 (only about one-third of the increased load in HS-300). If the yielded stirrups in HS-0 and HS-100 contributed to shear completely, two stirrups will make the shear capacity increase around 36.4 kN. However, in the experiments, they only increased for about 10 kN. It indicates that the stirrups passed by debonding cracks did not contribute to shear significantly. In addition, by measuring the average angle of the main diagonal cracks within the height of stirrups in the shear span, the values of angles were obtained (Fig. 4). When the diagonal cracks passed the tensile rebars and the compressive rebars, the cross points were connected to measure the angle. When the diagonal cracks extended out of the shear span before reaching the compressive rebars (HS-0 and HS-100), only the average angle in the shear span was measured. In the case of HS-0, since the crack in the shear span was too short and the extended crack outside the shear span was straight, the whole straight crack was utilized to make the measurement easier and more accurate. The result shows that the smaller the angle was, the more the shear capacity increased. The angle of diagonal cracks dominated the increased shear capacity rather than the yielded stirrups passed by debonding cracks. Such experimental results match with the assumption of Eq. (2); that is, the smaller the angles of diagonal cracks are, the more contribution on shear the stirrups make. Therefore, the stirrups passed by debonding cracks were considered to perform as the stirrups passed by diagonal cracks partially in RCHBs. And the possibility of this assumption was checked in this study.

Using the angles and the Eq. (2) above, the value of $V_s$ was calculated (see $V_{s\text{-cal}}$ in Table 4). The mean value of the experimental value to the calculated value of the shear carried by stirrups was 0.98. It is an indication that the proposed method can evaluate the shear carried by stirrups in RCHBs. However, due to the limited number of specimens in the present study, the applicability of the proposed method should be evaluated by conducting more experiments with various ratios of stirrups in the future.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$V$ (kN)</th>
<th>$V_{s\text{-exp}}$ (kN)</th>
<th>Diagonal cracks’ angle (degree)</th>
<th>$V_{s\text{-cal}}$ (kN)</th>
<th>$V_{s\text{-exp}} / V_{s\text{-cal}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HS-0</td>
<td>77.4</td>
<td>77.4-68.1=9.3</td>
<td>68.2</td>
<td>10.2</td>
<td>0.91</td>
</tr>
<tr>
<td>H-0</td>
<td>68.1</td>
<td>-</td>
<td>68.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>HS-100</td>
<td>73.3</td>
<td>73.3-60.3=13.0</td>
<td>63.4</td>
<td>12.8</td>
<td>1.02</td>
</tr>
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<td>60.3</td>
<td>-</td>
<td>48.0</td>
<td>-</td>
<td>-</td>
</tr>
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<td>HS-300</td>
<td>68.5</td>
<td>68.5-36.0=32.5</td>
<td>39.8</td>
<td>32.3</td>
<td>1.01</td>
</tr>
<tr>
<td>H-300</td>
<td>36.0</td>
<td>-</td>
<td>30.3</td>
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$V$: shear capacity; $V_{s\text{-exp}}$: shear carried by stirrups observed in the experiments; $V_{s\text{-cal}}$: shear carried by stirrups calculated by Eq. (2).

4. NONLINEAR FEM ANALYSIS

(1) Nonlinear FEM analysis model

To get a better understanding of the shear resistance mechanism of RCHBs, as well as to obtain the shape of the inclined compression zone in the shear span, which was not measured in the experiments, a two-dimensional nonlinear FEM analysis using DIANA system (version 9.4.4) was carried out in this study. As the thick concrete cover at the mid span had almost no effect on the shear capacity of RCHBs without stirrups, the nonlinear FEM analysis was mainly focused on the series I, III, and IV. Two typical specimens were selected in each series, which are H-100, H-300, HS-100, HS-300, HD-100, and HD-300. In series I and III, the debonding cracks occurred mainly because of the stress concentration, which made the bent tensile rebar straighten and push over the concrete. Such mechanism made the bond reduce suddenly. In the nonlinear FEM analysis, the interface elements were introduced between
concrete and longitudinal tensile rebars to simulate such bond behavior\textsuperscript{13,14}. In series IV, since no significant stress concentration existed due to the disconnected tensile rebar, the bond in series IV was supposed to be good and it was chosen as the original bond model in the interface element. In addition, the sudden loss of the bond in series I and III was simulated by multiplying a factor less than 1.0 to the original bond model. In the 2D models, eight-node isoparametric plane stress elements were applied to the concrete. Three-node beam elements were applied to the longitudinal tensile rebars and embedded reinforcement elements were used for stirrups and longitudinal compressive rebars. The mesh size was 50 mm for the square mesh. The two-dimensional model of HS-300 is shown in Fig. 11 as one example.

The total strain fixed smeared crack model was used as the crack model in the analysis. It was developed along the lines of the Modified Compression Field Theory, originally proposed by Vecchio and Collins\textsuperscript{15,16} in which large tensile strains perpendicular to the principal compressive direction reduced the concrete compressive strength. As shown in Fig. 12, the parabolic curve model considering compressive fracture energy $G_{FC}$ and the Hordijk model considering tensile fracture energy $G_{F}$ were used for the compressive and tensile behavior of concrete, respectively. The compressive fracture energy $G_{FC}$ was obtained from the equation proposed by Nakamura and Higai\textsuperscript{17}:

$$G_{FC} = 8.8 \sqrt{f_c}$$  \hspace{1cm} (3)

\[\text{Concrete and longitudinal tensile rebars to simulate such bond behavior}^{13,14}. \text{In series IV, since no significant stress concentration existed due to the disconnected tensile rebar, the bond in series IV was supposed to be good and it was chosen as the original bond model in the interface element. In addition, the sudden loss of the bond in series I and III was simulated by multiplying a factor less than 1.0 to the original bond model. In the 2D models, eight-node isoparametric plane stress elements were applied to the concrete. Three-node beam elements were applied to the longitudinal tensile rebars and embedded reinforcement elements were used for stirrups and longitudinal compressive rebars. The mesh size was 50 mm for the square mesh. The two-dimensional model of HS-300 is shown in Fig. 11 as one example.}

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where $f'_c$: compressive strength of concrete (N/mm$^2$).

The tensile fracture energy $G_F$ was obtained from the relation recommended in JSCE specifications$^1$:

$$ G_F = 10(d_{\text{max}})^{\frac{1}{3}}f'_c^{\frac{\alpha_{\text{bl}}}{3}} \text{(N/m)} \quad (4) $$

where $d_{\text{max}}$: the maximum coarse aggregate size (mm), and $f'_c$: compressive strength of concrete (N/mm$^2$).

For the shear model, the constant shear retention model was used in the analysis as shown in Fig. 12(c). The two steel plates at the loading point and support of the test shear span were assumed to be elastic bodies, while the perfect elastic-plastic model was applied for steel rebars as shown in Fig. 13. Figure 14 shows the basic model of the interface element. The normal stiffness and shear stiffness were assumed to be independent. The normal stiffness was an elastic body with the large Young’s modulus. For the shear stiffness, several bond models were tried in the analysis work. Since some debonding cracks were observed even in series IV and the thickness of the side concrete in experiments was only 21 mm, the original bond model expressed as Eq. (5) was used in consideration of the consistency with the loading test results. By using Eq. (5), for a certain slip displacement, the shear stress between steel bar and concrete can be calculated to simulate the bonding performance. It was modified based on the research data and the equation proposed by Suga et al.$^{18}$:

$$ \tau = \alpha_{\text{bl}} \times 0.3 \times 0.90(f'_c)^{\frac{2}{3}} \left[ 1 - \exp \left( -40\left(\frac{s}{D}\right)^{0.5} \right) \right] \quad (5) $$

where $f'_c$: compressive strength of concrete (N/mm$^2$), $s$: slip displacement (mm), $D$: diameter of tensile rebar (mm), and $\alpha_{\text{bl}}$: bond loss factor.

Displacement control with the Quasi-Newton method (also called “Secant method”) was adopted to solve equilibrium equations. In each step with 0.02 mm displacement increment, when the variation in
internal energy became less than 0.01% of the internal energy of the first iteration in the step, the iteration process was terminated to move to the next step.

(2) Analytical results and discussions

As the constant shear retention model was used and the value of the shear retention factor was arbitrary, the specimens of HD-100 and HD-300 were used first to obtain a reasonable value. By using the trial-fitting method with these two beams in series IV, the shear retention factor was determined as 0.05. This value was fixed throughout the FEM analyses. With this shear retention factor, the bond loss factors of the two beams in series I were determined using the trial-fitting method. It was found that when the bond of the haunched portion was lost to around 80% in the beam H-100 (multiplying a factor of 0.8) and around 30% (multiplying a factor of 0.3) in the beam H-300, the analysis results were good. It indicates that the start position of the debonding crack in the B-Region and the large crack width resulted in the earlier occurrence of the diagonal crack and more bond loss in the beam H-300 than in the beam H-100. In the end, with the shear retention factor and the bond loss factor obtained from the previous analyses, the nonlinear FEM analysis was conducted in series III. Figure 15 shows the load-displacement curves of these six specimens obtained in the experiment and the FEM analysis. Similar to the loading test, after the occurrence of the main diagonal cracks and the drop of load, if no significant increase was observed (less than 5 kN increase per 1 mm), the FEM analysis would be set to stop by assuming that no more maximum load appeared. The results showed that by modifying the shear retention factor in series IV and the bond loss factor in series I and III, the tendency of the load-displacement and the shear capacity almost matched with the experimental results.

Figure 16 shows the contour-level graphs of the principal tensile strain and the principal compressive stress just before and after the peak load in series IV. Although the crack patterns at the failure were not completely same with the experiments, the crack propagations show similarity with the experimental results. The beam action in the beam HD-100 before the main diagonal shear crack and the forming of the inclined compression zone after the peak could be observed clearly. In the beam HD-300, the large compression zone resisting the shear force before the peak and the sudden collapse of the compression strut due to the penetration of the new critical shear
crack could also be seen, which matched the experimental measurements.

Figure 17 shows the contour-level graphs of the principal tensile strain and the principal compressive stress of the other four specimens just before the peak load. In the analysis of the two specimens in series I, by adjusting the bond loss factor in each beam, the crack patterns and compression zones matched with the experimental results. In the beam H-100, the bond difference in the steel bar between the haunched portion and the other portion resulted in a drop of the load after diagonal cracks, which was not observed in the experiment. In the beam H-300, the deformation capacity at the peak load was smaller than in the experiment. It is supposed to be due to the fact that the constant shear retention model overestimated the shear stiffness along the cracks, especially when the crack width became very large in the beam H-300. However, the main shear resistance mechanism of arch action was clarified. An inclined compression zone was developed after the diagonal cracks and debonding cracks, connecting the loading point and the support point. As the bond of the horizontal steel bar near the support was good, the compression zone normally started from there and above the diagonal cracks. It generally matched with the assumed compression zone in Fig. 8. In the analyses of two specimens in series III, as the concrete contribution to shear was assumed to be same in series I and III, the bond loss factors of HS-100 and HS-300 were set to be same as the factors of H-100 and H-300. From the comparison, a good agreement with the experimental results was obtained, which could be a good complement for Section 3.5. The reason for the different failure position in HS-300 was also clarified during the analyses in
this study. As shown in Figs. 17 and 18, only the diagonal cracks in HS-300 passed two stirrups directly. The presence of stirrups limited the development of the diagonal cracks in HS-300; that is, the diagonal cracks were limited to the shear span, not to the pure bending moment zone between two loading points. However, for the other specimens H-100, H-300, and HS-100, the main diagonal cracks entered into the pure bending moment zone, making the area of the horizontal compression zone, as explained in Fig. 9, smaller than that of HS-300. Moreover, the flat diagonal crack near the loading point in HS-300 made the area of the inclined compression zone relatively smaller than the area of the horizontal compression zone. It resulted in the concrete crushing first at the end of the inclined compression strut near the loading point. Although the quantitative evaluation was not obtained in the current study, it is an important issue in the future.

5. CONCLUSIONS

Efforts were made to clarify the shear resistance mechanism of RCHBs. The present study pointed out some parameters, which were not considered in the previous researches but still important for the shear behavior. Based on the experimental and analytical results, the following conclusions can be drawn:

1) In slender RCHBs (the value of $a/d$ is from 2.5 to 4.0), when the tensile rebar is bent and the stirrups ratio is comparatively small, main diagonal cracks start from the changing portion of the cross-section (same as the bending position of tensile rebar) near the loading point, proceed along the inclined tensile rebar and towards the loading point.

2) The bent tensile rebar has a negative contribution to the shear capacity due to stress concentration, while the debonding cracks result in the arch action even in slender beams. However, the contributions of arch action are different in different beams due to the variation in crack patterns and the areas of the compression zones.

3) The thick concrete cover of RCHBs can provide a limited constraint at the beginning, increasing the load at diagonal cracks and changing the crack propagations. However, it has almost no effects on the shear capacity, since the arch action does not change.

4) When stirrups are provided in RCHBs, more shear cracks and flexural cracks occur. As the angles of the main diagonal cracks are related with the changing portion of the cross-section (same as the bending position of tensile rebar) near the loading point, the number of stirrups contributing in shear is different. Thus, the shear carried by stirrups varies according to the diagonal crack’s angle as well as the bending position of the tensile rebar near the loading point.

5) Compared with the crack patterns of RCHBs with bent tensile rebars, the crack patterns become totally different, when the tensile rebar is changed into a disconnected one in RCHBs without stirrups.

6) By modifying the shear retention factor and the bond loss factor in the nonlinear FEM analyses, the load-displacement behavior and crack patterns of RCHBs can be simulated. The shape of the inclined compression zone is also determined roughly in the analyses. The mechanism between the crack pattern and the arch action in resisting shear of RCHBs is clarified eventually. When the main diagonal crack is closer to the loading point, the more contribution to shear the arch action makes.

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