Magnitude-Frequency Relation for Earthquakes in and near Japan.

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Abstract

By means of the formulas determined by the present author previously, the magnitudes of 735 shallow earthquakes which took place in and near Japan during 1931-1950 were determined. The mean annual numbers of earthquakes having the magnitude M were found to be expressed by

$$\begin{aligned}\log N &= -1.60 + 1.06(8-M), \text{ for the area A,} \\ \log N &= -1.57 + 0.72(8-M), \text{ for the area B,} \\ \log N &= -1.61 + 0.66(8-M), \text{ for the area C,} \\ \log N &= -1.46 + 1.04(8-M), \text{ for the area (A+B),} \\ \log N &= -1.33 + 1.01(8-M), \text{ for the area (A+B+C),}\end{aligned}$$

the class interval of M being taken as 0.1. It is interesting to note that while the first constant does not differ much from area to area, the second does so. The second constant for an area is nearly proportional to the total number of earthquakes which occurred in that area.

In his previous paper (TSUBOI, 1951), the present writer obtained formulas which can conveniently be used for determining the RICHTER-GUTENBERG's instrumental magnitude M of an earthquake, the maximum vibration amplitude A of the ground due to which is known at a known distance Δ from its epicentre. The formulas obtained are:

$$M = 0.20 \Delta + 0.67 \log A + 3.80 \quad (\Delta < 5),$$

$$M = 0.03_3 \Delta + 0.60 \log A + 5.00_5 \quad (\Delta > 5),$$

where Δ is measured in 100 km and A in micron. Log is the common logarithm. The former formula is applicable to the case when the epicentral distance is smaller than 500 km, while the latter to the case when it is greater than 500 km. In determining these formulas, only those earthquakes which had occurred in and near Japan and which had been observed in Japan were taken into consideration. It is not clear therefore whether or not their validity can be claimed for earthquakes in other areas.

During 20 years from 1931 to 1950 inclusive, 735 shallow earthquakes were reported by C.M.O. to have taken place in and near

Japan with such magnitudes as to be felt beyond 200 km from the respective epicentres. By means of the formulas given above, the magnitudes of all these earthquakes were determined, taking as materials the seismometrical observations obtained at the following six principal seismological stations in Japan: viz., Sapporo, Sendai, Tokyo, Nagoya, Kobe, and Fukuoka. The observational data at each of the six stations yield independently the magnitude for one earthquake. In most cases these six independent determinations result in a satisfactorily consistent magnitude of one and the same earthquake but in several other cases they do not so. In these latter cases, the mean of the six values is adopted.

Among the 735 earthquakes, given in Table I, 38 were found to have had the magnitudes greater than 7.0.

In the table, M_T is the magnitude as determined by means of our formulas using seismometrical data obtained in Japan, while M_G is that determined by GUTENBERG and RICHTER and published in their book "Seismicity of the Earth" (1949).

Table I. 38 Earthquakes with Magnitudes greater than 7.0.

	Origin Time (JST)	Epicentre		M_T	M_G
			λ	ϕ	
1931	Feb. 20 14 34	138.0	44.0	7.0	
	Mar. 9 12 49	141.9	40.6	7.2	7.7
1932	Nov. 13 13 48	137.3	43.5	7.0	
1933	Mar. 3 2 31	144.7	39.1	7.7	8.5
	June 19 6 37	142.8	38.5	7.2	7.3
1935	Sept. 11 23 4	145.1	42.7	7.0	7.6
	Oct. 13 1 45	143.3	40.4	7.0	7.1
	18 9 12	143.8	40.2	7.0	7.2
1936	Nov. 3 5 46	142.0	38.4	7.4	7.2
1937	Feb. 21 16 3	150.0	44.5	7.5	7.4
	July 27 4 56	142.0	38.2	7.1	
1938	Jan. 12 0 12	135.1	33.4	7.0	6.5
	May. 23 16 18	141.5	36.7	7.2	7.4
	Oct. 12 9 35	144.3	39.8	7.0	
	Nov. 5 17 43	141.7	37.1	7.5	7.7
	5 19 50	141.7	37.2	7.0	7.7
	6 17 54	141.8	37.6	7.4	7.6
	7 6 39	141.9	37.2	7.0	7.1
1939	May 1 14 53	139.5	39.6	7.1	7.0
	Oct. 11 3 32	143.0	38.4	7.0	7.4
1940	Aug. 2 0 8	139.1	44.3	7.3	7.7
1941	Nov. 19 1 46	132.4	32.3	7.4	7.8
1943	June 13 14 12	142.7	40.9	7.0	7.4
	Sept. 10 17 37	134.2	35.5	7.3	7.4
	10 18 5	133.8	35.4	7.2	
1944	Dec. 7 13 35	136.2	33.7	7.4	
	19 23 10	124.0	38.0	7.1	

1945	Jan.	13	3	38	137.2	34.7	7.0	7.3
	Feb.	10	13	58	142.1	40.9	7.3	
	Apr.	10	10	22	143.0	41.8	7.1	
		24	23	36	139.5	31.0	7.0	
	Oct.	9	23	37	147.0	43.8	7.0	
1946	Dec.	21	4	19	135.6	33.0	7.7	
1947	Apr.	14	16	16	149.2	41.2	7.0	
	Oct.	4	9	9	141.0	43.8	7.1	
1948	June	15	20	44	135.5	33.8	7.2	
		28	16	13	136.2	36.1	7.2	
1950	Feb.	28	19	21	143.8	46.0	7.4	

The frequencies of earthquakes are tabulated in Table II according to the magnitude and the year in which they occurred. The frequencies are given separately for earthquakes which took place in the areas A, and B, and C as indicated in Fig. 1. This grouping of epicentral areas is not arbitrarily made. On the contrary, it is suggested naturally by the characteristically uneven distribution of epicentres among the areas. In the table, the frequencies for (A+B) and (A+B+C) are also given.

Table II. Number of Earthquakes.

a. Area A.

M	5.1	.2	.3	.4	.5	.6	.7	.8	.9	6.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	7.0	.1	.2	.3	.4	.5	.6	.7	Total
Year																												
1931				1	3	2	4	4	2	4	2	2	1	2	3	3		2				1						36
2					1	3	2	1	3	1	1	2	2	3	1			1										21
3			1	1	3	2	4	4	3	3	2	1	3	2	2			3			1						1	38
4				1	1	1	1			3	1	2	1		2													14
5								3	1	3	1	2			4	1	3	1	1	3								24
6						3	1	1	2	3	1		2	2			1								1			17
7						1	2	2		2	3		1	2			2	1			1					1		18
8				2				3	3	2	2	10	6	3	3	10		2	2	3	1			1	1			54
9					1		1	2	5	1	3		3	1					1									16
40						1	1	1	3	3	4		3	5	1			1										17
1				1		1		1	3	1	1	3	1		1													18
2				2			2	2	1	6	2	1	3	1				2	1									20
3					1	1	1	2	3	2	4	2	3		7					1								26
4		2			2	1	1	1		2	3	1		2	1			1										19
5			1		2	1		2	3	4	1	1	1	1	2			3		1	1			1				25
6				3	1	1	1	1	1	1	2	3	3		2			2			1	1						20
7	1				2	1	2	1		1	1	2	3		1			2	1		1	1						19
8			1	1	1		2	1	2	3	2	4	2	2				2	1									24
9			1	3	3	3	3	2	1	3		1		1				1										24
50		1	3	2	3	2	2	2	3	4	1		1					1							1			26
Total	1	3	7	18	23	24	30	35	43	43	36	41	36	27	21	34	10	13	8	10	3	3	1	3	2		1	476

b. Area B.

<i>M</i> Year	4.8	.9	5.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	6.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	7.0	.1	.2	.3	.4	.5	.6	.7	Total
1931											2			2	1	2			1			1									9
2							1			2	1		2		2																11
3			1			1	1															1									7
4											1											1									3
5												2		1					1	1											3
6											1																				5
7								1		1	1								2												8
8								1			1		2						1												2
9															2		1	1				1									1
40									1																						5
1					1	2							1														1				9
2					1				1				1	1																	8
3	1												1			1			3												4
4													1			1															9
5							1	1	1	2			1			1		1	2	1	1		2			1					11
6									1		1	1	3	1	1			2	1												12
7											1	1	1	1	2	1	2	1	1	1									1		19
8											1	1	1	1	1	1	1	1	1						1						16
9											1	1	1	1	1	1	1	1	1												7
50											2		1	1	1							1									6
Total	1		1	1	2	3	4	6	10	9	16	12	15	6	14	11	8	8	10	4	5	2	3	1	2						155

c. Area C.

<i>M</i> Year	5.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	6.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	7.0	.1	.2	.3	.4	.5	.6	.7	Total
1931			1								2			1							1								5
2					1						1	1			1				1		1								5
3						1				1	1	2		2															5
4							1			2	1	1				1													3
5								1			1	1																	7
6									1								1												2
7								1				1	1		1				1										4
8					1				1		1	1	1	1	1														7
9	1									1				1				1		1		1							6
40				1				1												1	1			1					5
1				1							1	1																	5
2						1					2	1		1		1													4
3										2	1	2	1	2	2	2							1	1					14
4							1	1		1	1	1																	3
5						1	1			1	1																		5
6													1						1										2
7					1	2	1		1	1	1																		6
8							1	1	1			1																	6
9								1	1				1			1	2												6
50										1	1				1	1													4
Total	1		1	2	4	6	3	5	5	12	11	8	7	9	6	8	2		5	1	2	2	2	2					104

d. Area (A+B).

M	4.8	.9	5.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	6.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	7.0	.1	.2	.3	.4	.5	.6	.7	Total
Year																															
1931							1	3	2	4	6	2	4	4	3	3	2	3	4	2	1	1			1						45
2							1	1	4	4	2	4	3	1	4	2	3	1	1	1											32
3			1			2	2	3	2	5	4	3	3	2	3	3	2	2	2	1	4				1						45
4							1	1	1	1			3	3	1	2	1	1	1	1										1	17
5												3	3	3	1	2		1	5	1	3	1	1	3							27
6									3	2	2	2	3	1		3	2	2			1							1			22
7								1	2	3	3	2	2	3		2	2	1	2	1					1				1		26
8					2		1	1			3	3	2	2	10	6	3	3	10		2	2	4	1	1				1		56
9										1	2	5	3	3	4	2	3	1	1		1	1									17
40								1		2	1	1	3	3	4	2	3	1	1		1										22
1					1	2	1		1		1	4	1	1	6	6	1	1									1				27
2					1		2		1	2	2	1	1	7	2	2	1	3	2	1											28
3	1							1	1	1	2	3	3		4	3	4		7			1									30
4					2			2	1	1	1	2	3	3	1	1	3	1	1	2			1				1				28
5						1	1	3	2	2	2	3	5	1		2	1	1	4		3		3	1		1					36
6							3	1	2	1	2	2	4	3	4	3	2	1	2			1							1		32
7							1	3	2	2	6	2	2	2	4	4	2	2					1	1							38
8					1		2	2	1	3	2	4	6	3	4	2	3								1						40
9						1	3	4	5	4	3	1	3	1	1	1	1	1	2	1											31
50					1	3	2	3	2	2	4	3	5	1	1	1		2			1						1				32
Total	1		1	2	5	10	22	29	34	39	51	55	58	42	55	47	35	29	44	14	18	10	13	3	4	1	5	2		2	631

e. Area (A+B+C).

M	4.8	.9	5.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	6.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	7.0	.1	.2	.3	.4	.5	.6	.7	Total
Year																															
1931					1		1	3	2	4	6	2	6	4	3	4	2	3	4	2	1	1		1							50
2						2	1	4	4	2	4	3	2	2	4	2	4	1	1	1	2		1								37
3			1			2	2	3	3	5	4	4	4	2	3	5	2	2	2		4				1						50
4							1	1	1	1	1	3	4	2	1		3	1	1	1											20
5								1			3	5	4	2	2		1	6	1	3	1	1	3								34
6									3	2	3	2	3	1		3	2	2	1		1							1			24
7							1	2	4	3	2	2	4	1	2	2	1	2	1	1									1		30
8							3	1		4	3	3	3	11	7	4	3	10		2	2	4	1	1		1		1			63
9			1					1		1	2	6	3	1	3	1	1		1		2	1	1	1							23
40						1			2	2	1	3	3	4	2	3	1	1		1	1	1				1					27
1					1	3	1		1		1	5	1	2	6	7	1	2									1				32
2					1	1	2	1	1	2	2	3	1	7	2	2	1	1	3	2	1										32
3	1							1	1	1	2	5	4	2	5	6	2	7					1		1	1					44
4					2			2	1	1	1	2	3	3	2	2	3	3	1	1	2			1			1				31
5						1	1	4	3	2	2	4	6	1	1	2	1	1	4		3		3	1		1					41
6							3	1	2	1	2	2	4	3	5	3	2	1	2		1	1							1		34
7						1	2	5	2	2	7	3	3	2	4	4	2	2		2			1	1							44
8						1	2	3	2	4	3	4	6	4	4	2	3				2	3									46
9						1	3	4	5	5	4	1	3		2	1	2	3	2	1											37
50					1	3	2	3	2	2	4	4	6	1	1	1	1	1	2		1						1				36
Total	1		2	2	6	12	26	35	37	44	56	67	69	50	62	56	41	37	46	14	23	11	15	5	6	3	5	2		2	735

Fig. 1. The areas A, B and C.

In Fig. 2, the logarithm of the mean annual number N of earthquakes is plotted against M . The value is at its maximum at about $M=6.0$ and decreases toward its both sides. While it is only natural that the number decreases toward larger M , that it does so also toward smaller M is due to the circumstance that not all earthquakes whose magni-

Fig. 2a. Logarithm of the mean annual number of earthquakes N against magnitude M (Area A).

Fig. 2b. (Area B).

Fig. 2c. (Area C).

Fig. 2d. (Area A+B).

Fig. 2e. (Area A+B+C)

tudes are less than 6.0 are reported and there are therefore many of them which fall out of the statistics. The number of earthquakes having magnitudes greater than 6.0 only can be trusted upon.

As to the relation between N and M , GUTENBERG and RICHTER proposed a formula of the form

$$\log N = a + b(8 - M)$$

in which $\log N$ is linear with respect to M . Here it is important to note that N is the mean annual number of earthquakes having the magnitudes between $(M - \Delta M)$ and $(M + \Delta M)$, where ΔM is half the class interval of M for which N is counted.

The numerical values of the constants a and b in the above formula were determined by the method of least squares so that the mean annual number of earthquakes in the areas A, B, C, (A+B), and (A+B+C) can be expressed, taking those earthquake which have greater magnitudes than 6.0. The results obtained corresponding to $\Delta M = 0.05$ are:

Area	a	b
A	-1.60	1.06
B	-1.57	0.72
C	-1.61	0.66
(A+B)	-1.46	1.04
(A+B+C)	-1.33	1.01

It is interesting to note that while the constant b differs considerably from area to area, the constant a does not so very much. As is clear from the expression:

$$\log N = a + b(8 - M),$$

the constant a is nothing but the logarithm of the mean annual number of earthquakes having the magnitude $M = 8.0$. The approximate constancy of the value of a for different areas therefore amounts to mean that the mean annual number of such large earthquakes does not differ much from area to area, although there are considerable differences among the total numbers of earthquakes. The total number of earthquakes is decidedly the largest in the area A, and is followed by B and C as may be seen in Table

III. The constant b is also the largest in A. It was found that b is almost proportional to the total earthquakes number in the respective areas as is shown in Fig. 3. This would indicate that if the total number of earthquakes in a certain area is twice as much as that in another, for instance, it does not mean that earthquakes of various magnitudes are equally double. Earthquakes with smaller magnitudes increase more rapidly than those with greater magnitudes do.

Table III.

Number of earthquakes of various magnitudes in the areas A, B, and C, (1931-1951).

M	A	B	C	Total
6.0	43	15	11	69
6.1	36	6	8	50
6.2	41	14	7	62
6.3	36	11	9	56
6.4	27	8	6	41
6.5	21	8	8	37
6.6	34	10	2	46
6.7	10	4	0	14
6.8	13	5	5	23
6.9	8	2	1	11
7.0	10	3	2	15
7.1	3	0	2	5
7.2	3	1	2	6
7.3	1	0	3	4
7.4	3	2	0	5
7.5	2	0	0	2
7.6				
7.7	1	1	0	2
Total	292	90	64	446

On the other hand, basing on their own materials, GUTENBERG and RICHTER gave for Japan

$$\log N = -0.90 + 0.80(8 - M),$$

corresponding to $1/4$ units of ΔM . From the data we have been using, the following relation is obtained:

$$\log N = -0.85 + 1.22(8 - M),$$

corresponding also to $1/4$ units of ΔM . It is not surprising that while the values of a agree rather well, those of b differ sensibly from each other. The difference is presumably due to the circumstance that relatively smaller earthquakes with the magnitude a little over 6.0 are apt to be partly dropped out of the

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Fig. 3. Dependence of the constant b on the total number N of earthquakes.

statistics of GUTENBERG and RICHTER based on the distance earthquake observations.

As was stated before, GUTENBERG and RICHTER proposed a formula of the form

$$\log N = a + b(8 - M)$$

for expressing N in terms of M . Strictly speaking, precautions must be paid before this expression is used, because it does not admit a linear superposition with respect to N . Suppose the constants a, b for a certain interval of time T_1 are a_1 and b_1 , and those for the same area but in another interval of time T_2 are a_2 and b_2 . Then,

$$\log N_1 = a_1 + b_1(8 - M),$$

$$\log N_2 = a_2 + b_2(8 - M),$$

where a_1 and b_1 may differ from a_2 and b_2 respectively. If we take two statistics altogether, $\log \left(\frac{N_1 T_1 + N_2 T_2}{T_1 + T_2} \right)$ comes into question, but if the above two expressions are put into this one, the resultant expressions is no more linear with respect to M . In spite of this circumstance, it is always possible, by the very nature of the method of least squares, to determine the values of the constants a and b for expressing $\log \left(\frac{N_1 T_1 + N_2 T_2}{T_1 + T_2} \right)$ in the form

$$\log \left(\frac{N_1 T_1 + N_2 T_2}{N_1 + N_2} \right) = a + b(8 - M).$$

The constants a and b may differ again from a_1, a_2 , and b_1, b_2 , respectively. With such variance, the constants have little significant meanings. a_1, a_2, a , and b_1, b_2, b can be equal or at least approximately equal respectively with each other only when the time interval taken for the statistics is of such a sufficient length that the frequency distribution of earthquakes with respect to M in that interval may be regarded as truly representing the average tendency peculiar to the area under consideration. If we are to attribute to the constants a, b any physically

Table IV.

The constants a and b determined for various time intervals.

Interval	a	b	Interval	a	b
1931	-0.40	0.52	1950	-0.18	0.21
1931-1932	-0.97	0.79	1950-1949	-0.03	0.36
1931-1933	-0.67	0.60	1950-1948	-0.90	0.64
1931-1934	-0.80	0.65	1950-1947	-1.26	0.88
1931-1935	-0.86	0.69	1950-1946	-1.32	0.91
1931-1936	-1.05	0.79	1950-1945	-1.28	0.90
1931-1937	-1.25	0.90	1950-1944	-1.30	0.91
1931-1938	-1.10	0.85	1950-1943	-1.37	0.96
1931-1939	-1.15	0.86	1950-1942	-1.46	1.02
1931-1940	-1.23	0.90	1950-1941	-1.49	1.03
1931-1941	-1.25	0.91	1950-1940	-1.56	1.07
1931-1942	-1.30	0.95	1950-1939	-1.56	1.07
1931-1943	-1.35	0.98	1950-1938	-1.51	1.08
1931-1944	-1.36	0.99	1950-1937	-1.46	1.03
1931-1945	-1.44	1.04	1950-1936	-1.47	1.04
1931-1946	-1.39	1.00	1950-1935	-1.48	1.05
1931-1947	-1.41	1.01	1950-1934	-1.52	1.07
1931-1948	-1.43	1.03	1950-1933	-1.42	1.01
1931-1949	-1.46	1.04	1950-1932	-1.45	1.03
1931-1950	-1.46	1.04	1950-1931	-1.46	1.04

Fig. 4. Dependence of the constants a and b on the number of years taken in the statistics.

significant meaning related to the nature of mechanism of earthquake occurrence, much care must be paid therefore as to the length of the time interval to be taken for the statistics. In this connection, it will be interesting to investigate how the constants a and b tend to their respective final values as we lengthen the time interval of statistics successively. Taking the area $(A+B)$, the

constants a and b were determined for representing the mean annual earthquake frequencies in 1931, (1931-1932), (1931-1933),(1931-1950), adding one year successively each time. The same thing was done starting from 1950 and going back the other way, the interval being taken as 1950, (1950-1949), (1950-1948), (1950-1931). The results of calculations are given in Table IV and in Fig. 4.

As is clear from Fig. 4, both a and b change conspicuously at first, but gradually tend to their respective asymptotic values when more and more years are added into the statistics. Beyond about $n=15$, the values of a and b may be regarded as approximately stationary. How many years are necessary in order to get the stable value of a and b depends on the fluctuation in the mode of occurrence of earthquakes with particular reference to magnitude. This presents an interesting problem and the writer expects to publish the results of his study of this problem in a near future.

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