On the Possibility of the Existence of the Molten Portion in the Upper Mantle of the Earth

By

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§ 1. Introduction

In the previous paper (D. Shimozuru, 1962), the author obtained a possible temperature distribution in the upper mantle, in which the temperature reaches close to the melting point of forsterite between the depths of 150 and 250 km in the subcontinental mantle.

For the purpose of evaluating the velocity decrease due to the increasing temperature alone, percentage differences are calculated between the Gutenberg's velocity distributions and those computed from the finite strain theory at various depths. These values are illustrated in Fig. 1 for both P and S waves.

Fig. 1. Percentage differences between Gutenberg's velocity distribution in continental mantle and the velocities computed from finite strain theory provided that the upper mantle is composed of dunite.

We find from the figure that the velocity decrease is remarkable for S waves between the depths of 150 and 250 km. In this region, the temperature gradient is believed to be only several degrees per kilometer. Hence the large decrease of shear wave velocities must be the consequence of extraordinarily low rigidity of the material at these depths.

It is interesting to note that the depth of low rigidity portion in Gutenberg's subcontinental mantle almost coincides with that of Lehmann's S velocity distribution in the subcontinental mantle.

§ 2. Apparent shear modulus

From the preceding discussions, one might naturally imagine pockets of molten rocks at the said depths. There have been no definite means, however, to figure the shape and size of the molten pockets. It is very likely that if they actually exist, they are oblate spheroids with their axes of rotation directed vertically which are stable under the gravitational field.

In the following, numerical evaluation will be made on the effect of such oblate molten pockets.
pockets on the apparent shear modulus and velocity, assuming that the dotted molten pockets are far smaller linearly than the wavelengths. The spatial arrangement of such pockets is arbitrary, but their axes of rotation are assumed to be all vertical.

To find the apparent shear modulus in such a system, the ordinary elasticity theory encounters many difficulties. For an elastic medium containing small spherical holes filled with liquid, rigorous solutions were obtained by Y. Sato (1952). For the apparent shear wave velocity, he obtained

\[
\frac{V_s}{V_{so}} = \left\{ 1 - (1 - \rho) \right\} \frac{1}{2} \left[ 15 \frac{1 - \sigma_o}{7 - 5 \sigma_o} - (1 - D) \right],
\]

where \( V_s \) ..... apparent shear wave velocity,
\( V_{so} \) ..... shear wave velocity of solid portion,
\( (1 - \rho) \) ..... porosity,
\( \sigma_o \) ..... Poisson’s ratio of solid portion,
\( D \) ..... ratio of density of liquid to that of solid.

We have no rigorous elastic solutions for a medium containing spheroidal pockets of arbitrary arrangement. Here we treat the problem from the elasticity-electricity analogy.

Various formulae have been derived to calculate apparent bulk physical parameters such as thermal and electrical conductivities or dielectric constants of a mixture material as a function of those of constituent materials. Most of them, however, have a restriction on the arrangement and the state of mixing of constituent materials. We have various ways of expressing electrical equivalents of a mechanical system. If we take “electric current” to be an equivalent of “force”, then the free surface of the mechanical system on which no force is acting corresponds to the surface through which no electric current flows. Hence, in this equivalent system, the shape of a mechanical system may be retained without any change in the corresponding electrical system.

Since “displacement” may be represented by “voltage” in this equivalent system, rigidity in the elastic system corresponds to electrical conductivity and viscosity to dielectric constant.

F. Irie (1956), extending Wiener’s theory, obtained a simple formula that gives the existence domain of the complex dielectric constant of a binary mixture of given volume fractions of the components in alternating field without any information on the state of mixing. His formula is

\[
\begin{align*}
\varepsilon_a &= \varepsilon_1 \varepsilon_2 + \mu \varepsilon_p / \varepsilon_1 + \mu, \\
\varepsilon_p &= \delta_1 \varepsilon_1 + \delta_2 \varepsilon_2, \\
\delta_1 \varepsilon_1 + \delta_2 \varepsilon_2 &= 1.
\end{align*}
\]

In his equation, \( \mu \) is a parameter which depends on the shape of mixing material and is restricted to be

\[
\arg \varepsilon_1 \leq \arg \mu \leq \arg \varepsilon_2.
\]

This formula is also applicable without any change in form to the calculation of apparent electrical conductivity of such a mixed material. Hence, for the present purpose, \( \varepsilon_a, \varepsilon_1, \) and \( \varepsilon_2 \) in equation (2) should be replaced by \( \varepsilon_a, 0 \) and \( \varepsilon_2 \) resulting in the equivalent expression

\[
\mu_a = (\nu \cdot \delta_2 / \delta_1 + \nu) \cdot \mu_a, \quad \delta_1 + \delta_2 = 1,
\]

where

\[
\nu = u / \mu_a,
\]

\( \mu_a \) ..... apparent rigidity of the whole domain considered,
\( \mu_a \) ..... substantial rigidity of the solid portion of the said domain.

This formula yields the apparent rigidity modulus of the material containing pockets of zero rigidity. In the case of dense mixing, the interaction among the pockets cannot be neglected, and our formula is not valid.

Let the space within the ellipsoid

\[
x^2 / a^2 + y^2 / b^2 + z^2 / c^2 = 1
\]

be filled with liquid magma of zero rigidity,
and let the remainder of space be occupied by the solid rock of rigidity \( \mu_s \), then \( \nu \) in equation (3) is expressed as (SILLARS, 1937),

\[
\nu = (p_a h, n / 4\pi - 1),
\]

where

\[
p_a = \int_0^\infty \frac{dx}{(a^2 + x)} \cdot \beta,
\]

\[
p_b = \int_0^\infty \frac{dx}{(b^2 + x)} \cdot \beta,
\]

\[
p_c = \int_0^\infty \frac{dx}{(c^2 + x)} \cdot \beta,
\]

\[
\beta = \sqrt{(a^2 + x)(b^2 + x)(c^2 + x)} / 2\pi abc.
\]

\( p_a, p_b, p_c \) may be adopted according to the direction of applied field, i.e. the direction of particle displacement.

Since we consider the oblate spheroidal molten pockets with their axes of rotation vertical, semi-axes \( b \) and \( c \) of the ellipsoid are made equal and \( \nu \) takes the form

\[
\nu = 1 - \frac{1}{\sqrt{1 - e^2}} \arcsin e / e^2 - 1,
\]

when the direction of displacement is vertical, and

\[
\nu = 2e^2 (1 - e^2) \arcsin e - (1 - e^2) - 1,
\]

when the direction of displacement is horizontal,

\[
e = \sqrt{1 - a^2 / b^2} = \sqrt{1 - a^2 / b^2}.
\]

When \( a \) is much smaller than \( b \) or \( c \), \( \nu \to 0 \), \( \nu \to \infty \).

From (4) and (5), the relation between \( \nu \) and ratio of semi-axes \( \gamma (=a/b) \) can be calculated for both vertical and horizontal directions, as listed in Table 1.

Putting the above values of \( \gamma \) and \( \nu \) into equation (3), we obtain \( (\mu_a / \mu_s) \), the ratio of apparent rigidity to rigidity of solid portion for various volume fraction of molten pockets, as tabulated in Table 2.

\( (\mu_a / \mu_s) \) decreases as \( \delta_i \) increases, and this

Table 1. Shape factor for various axial ratio.

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>Vertical direction</th>
<th>Horizontal direction</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.16</td>
<td>13.24</td>
<td>Very flat</td>
</tr>
<tr>
<td>0.2</td>
<td>0.33</td>
<td>7.01</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.90</td>
<td>3.21</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>1.53</td>
<td>2.30</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>2.00</td>
<td>2.00</td>
<td>Sphere</td>
</tr>
</tbody>
</table>

Table 2. Ratio of apparent rigidity to rigidity of solid portion for various volume fraction of molten pockets with various axial ratio.

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( \delta_i )</th>
<th>0.02</th>
<th>0.04</th>
<th>0.08</th>
<th>0.10</th>
<th>0.12</th>
<th>0.13</th>
<th>0.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.8710</td>
<td>0.7680</td>
<td>0.6133</td>
<td>0.5538</td>
<td>0.5028</td>
<td>0.4800</td>
<td>0.4387</td>
<td>Vert.</td>
</tr>
<tr>
<td>0.2</td>
<td>0.9785</td>
<td>0.9571</td>
<td>0.9145</td>
<td>0.8933</td>
<td>0.8721</td>
<td>0.8616</td>
<td>0.8405</td>
<td>Hori.</td>
</tr>
<tr>
<td>0.5</td>
<td>0.9240</td>
<td>0.8563</td>
<td>0.7404</td>
<td>0.6906</td>
<td>0.6453</td>
<td>0.6241</td>
<td>0.5844</td>
<td>Vert.</td>
</tr>
<tr>
<td>0.8</td>
<td>0.9772</td>
<td>0.9546</td>
<td>0.9096</td>
<td>0.8873</td>
<td>0.8652</td>
<td>0.8542</td>
<td>0.8323</td>
<td>Hori.</td>
</tr>
<tr>
<td>1.0</td>
<td>0.9687</td>
<td>0.9191</td>
<td>0.8449</td>
<td>0.8100</td>
<td>0.7765</td>
<td>0.7602</td>
<td>0.7286</td>
<td>Vert.</td>
</tr>
<tr>
<td>0.1</td>
<td>0.9679</td>
<td>0.9482</td>
<td>0.8976</td>
<td>0.8728</td>
<td>0.8483</td>
<td>0.8361</td>
<td>0.8121</td>
<td>Hori.</td>
</tr>
<tr>
<td>0.2</td>
<td>0.9716</td>
<td>0.9366</td>
<td>0.8762</td>
<td>0.8471</td>
<td>0.8196</td>
<td>0.8046</td>
<td>0.7771</td>
<td>Vert.</td>
</tr>
<tr>
<td>0.5</td>
<td>0.9702</td>
<td>0.9412</td>
<td>0.8846</td>
<td>0.8571</td>
<td>0.8302</td>
<td>0.8169</td>
<td>0.7907</td>
<td>Sphere</td>
</tr>
</tbody>
</table>

Vert. = Vertical, Hori. = Horizontal
is more remarkable with the vertical direction. If the ratio of axes is 0.1, i.e., very flat spheroid, apparent rigidity measured in vertical direction becomes almost half of the substantial rigidity having molten pockets of only 11 per cent of the domain considered. 

Ratio of apparent rigidity in horizontal direction to that in vertical, \( \frac{(\mu_a)_H}{(\mu_a)_V} \), can be calculated from the above table as listed in Table 3. These values are illustrated in Fig. 2.

### Table 3. Ratio of apparent rigidity in horizontal direction to that in vertical for various axial ratio of spheroidal molten pockets.

<table>
<thead>
<tr>
<th>( \eta )</th>
<th>0.02</th>
<th>0.04</th>
<th>0.08</th>
<th>0.10</th>
<th>0.12</th>
<th>0.13</th>
<th>0.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.1234</td>
<td>1.2462</td>
<td>1.4911</td>
<td>1.6130</td>
<td>1.8345</td>
<td>1.7950</td>
<td>1.9159</td>
</tr>
<tr>
<td>0.2</td>
<td>1.0576</td>
<td>1.1148</td>
<td>1.2285</td>
<td>1.2848</td>
<td>1.3408</td>
<td>1.3687</td>
<td>1.4242</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0159</td>
<td>1.0317</td>
<td>1.0624</td>
<td>1.0775</td>
<td>1.0925</td>
<td>1.0998</td>
<td>1.1146</td>
</tr>
<tr>
<td>0.8</td>
<td>1.0038</td>
<td>1.0075</td>
<td>1.0147</td>
<td>1.0182</td>
<td>1.0205</td>
<td>1.0235</td>
<td>1.0269</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Fig. 2. The flatter the molten pockets are, the larger the ratio \( \frac{(\mu_a)_H}{(\mu_a)_V} \) is, which becomes almost 20 when the ratio of the axes is 0.1 and the volume fraction is 16 per cent.

It must be noted that in equation (3) the compressibility of the substance is not taken into account. Therefore comparison with the elastic solution, equation (1), is needed in the case of media involving spherical pockets filled with liquid magma.

If we put \( \sigma = 0.26 \) and \( D = 1/1.028 \) (Bowen, N. L., Scharer, J. F., 1935), equation (1) takes the form

\[
\frac{V_s}{V_{so}} = \left(1 + 0.9595 \times \frac{\delta_1}{1 + \delta_1}\right). \quad (6)
\]

Since \( \nu = 2 \) in the case of spherical pockets, equation (3) becomes

\[
\frac{V_s}{V_{so}} = \left(\frac{2 - 2\delta_1}{\delta_1 + 2}\right)^{1/2} \times (1 + 0.028\delta_1)^{1/2}. \quad (7)
\]

Equations (6) and (7) show the ratio of apparent shear velocity to the shear velocity of solid portion, and the results of calculation for various volume fraction of spherical pockets \( \delta_1 \) are listed as follows,

<table>
<thead>
<tr>
<th>( V_s/V_{so} )</th>
<th>0.02</th>
<th>0.04</th>
<th>0.08</th>
<th>0.10</th>
<th>0.12</th>
<th>0.13</th>
<th>0.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic solution</td>
<td>0.9812</td>
<td>0.9831</td>
<td>0.9289</td>
<td>0.9128</td>
<td>0.8972</td>
<td>0.8897</td>
<td>0.8749</td>
</tr>
<tr>
<td>Electric analogue</td>
<td>0.9853</td>
<td>0.9708</td>
<td>0.9415</td>
<td>0.9271</td>
<td>0.9127</td>
<td>0.9054</td>
<td>0.8911</td>
</tr>
</tbody>
</table>

Fig. 2. Ratio of apparent shear modulus in horizontal direction to that in vertical direction for various volume fraction of involved molten pockets.
Above results show that the formula derived from electric analogue agrees with the rigorous elastic solution within the error of 2 per cent for the media containing spherical liquid pockets by 10 per cent.

§ 3. Application to the mantle surface wave evidences

Recent accumulation of observational data of Rayleigh and Love wave phase and group velocities, as well as observed periods of free oscillations of the earth yields in general the consistency of the mantle having P and S velocity distribution similar to those proposed by Gutenberg and a density distribution similar to that of Bullen's model A.

Strictly speaking, the group and phase velocities of mantle Rayleigh waves agree well with those calculated from the Gutenberg-Bullen's model A of the earth, while a slight discrepancy is found in the shorter periods of Love wave phase and group velocities between the observed and predicted curves. The discrepancy is remarkable at the periods shorter than 200 seconds for the phase velocity curve and 250 seconds for the group velocity curve (Fig. 3, 4). The inconsistency of these surface wave data for a given mantle structure seems to be significant.

It seems that there are two ways of approaching this problem. One is to construct a mantle structure which fits better with both Love and Rayleigh wave data, consistently, and the other is to search for some mechanism which produces the above observed facts.

According to H. Takeuchi (personal communication), Love wave group velocity data agree better with those predicted by Lehmann's model than with those calculated by Gutenberg's model. He pointed out also that Lehmann's model and Gutenberg's model show no apparent difference for Rayleigh wave group velocity. Hence, Takeuchi considers that the mantle structure can be approximated rather by Lehmann model than by Gutenberg's model.

Even adopting Lehmann's model, a slight discrepancy seems to remain between Love wave group velocity data and the theoretical curve in shorter periods, which is systematic and seems to be significant.

Brune, Benioff and Ewing (1961) have stated that Gutenberg-Bullen A model has a
continental crust and hence the observed phase velocity values may be expected to deviate at shorter periods.

On the other hand, by treating a transversely isotropic model, Anderson (1961) attempted to point out how anisotropy will introduce apparent discrepancy between Love and Rayleigh wave data.

The author would like to attribute this discrepancy to the existence of molten pockets in the upper mantle.

Preceding computation requires, when oblate spheroidal molten pockets exist, lower rigidity in vertical direction than in horizontal. In other words, in such medium, velocities of waves of which particle displacement is horizontal are expected to be higher than those of waves of which particle displacement is vertical. S waves, used for the construction of the velocity distribution inside the earth, arrive in general in both SH and SV types. In this connection, predicted dispersion curve computed from both Gutenberg’s and Lehmann’s model should rather agree with mantle Rayleigh waves.

Since Love waves may be considered to be of purely SH type, observed higher velocities in shorter periods suggest the existence of oblate spheroidal molten pockets with their axes of rotation vertical. Since observed differences in velocities are found only in shorter periods, such molten pockets are likely to exist in the upper mantle. Further, in orogenic region, velocities of mantle Rayleigh waves are expected to be lower than the theoretical curves in shorter periods.

Basic assumption of our analogical calculation is that the dimension of spheroidal molten pockets are far smaller than the wavelengths. For Love waves having the periods of 100-300 seconds, the corresponding range of wavelength is approximately 450-1500 km. Thus, observed discrepancy between Love and Rayleigh wave data suggests that the molten pockets are flat and far smaller than the above wavelength.

We have, at present, no further geophysical evidences to estimate the size and shape of molten pockets so far discussed. In the previous paper, Shimozuru (1961) estimated the necessary volume fraction of molten pockets to interpret the Lehmann’s low velocity zone, and found it to be several percent of the whole region considered.

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**Fig. 4.** Love wave phase and group velocity data reproduced from the paper by Brune, Benioff and Ewing (1961). Lehmann-Bullen curve is plotted by the present author according to Takeuchi’s computation.
Beneath the Precambrian shield, no evidence of a low velocity zone has been reported (Bolt, Doyle and Sutton, 1958) and we cannot expect the existence of molten pockets beneath any Precambrian shields. On the other hand, at the orogenic region, for instance in Japan, low velocity zone seems to extend up to Moho (Aki, 1961) which suggests the existence of molten pockets up to the considerably shallower depth. This does not mean shallower origin of magma, but only reflects the present profile of the process of upward movements of magma.

§ 4. Concluding remarks

Estimated temperature distribution and the existence of low rigidity portion suggest the possibility of the existence of partially molten pockets in the upper mantle. Subcontinental mantle seems to be dotted with these molten pockets at a depth probably between 150 and 250 km. In suboceamic mantle, shallower low velocity zone suggests a shallower depth of such molten pockets between the depths of 60 and 150 km, which may be favourable for the interpretation of unexpected large heat flow through oceanic basins.

Observed higher velocities of mantle Love waves in shorter periods requires us to conclude that the molten pockets have a shape of an oblate spheroid with the axis of rotation directed vertically provided that their size is far smaller than the wavelength of mantle surface waves of shorter periods. The present calculations also suggest that the velocities of mantle Rayleigh waves of shorter periods having a path in orogenic region are comparatively low.

Acknowledgement

The present author expresses hearty thanks to Dr. H. Takeuchi of Tokyo University who kindly let him know the unpublished results of his recent calculation of the velocities of mantle Love and Rayleigh waves.

References


1962 “Low velocity zone and the temperature distribution in the upper mantle of the earth” J. Phys. of the Earth.