Variation of Elastic Wave Velocity and Attenuative Property near the Melting Temperature

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§ 1. Introduction
In view of the fact that the mechanical behavior of materials around the melting point has been qualitatively used to set a certain limit to the possible distribution of temperature and anelastic property within the earth's interior, its quantitative investigation is expected to give further comprehensive informations in the related problems. Experimental studies along this line have so far been made by T. Murase (1962), and K. Kumazawa, H. Furuhashi and K. Iida (1964). However, the experimental difficulties encountered therein prevented them from covering a wide temperature range around the melting point and also from determining the shear wave velocity and attenuation coefficient. The purpose of this work is to give a quantitative basis to the discussion of elastic and anelastic property of materials around the melting point by experimentally determining the elastic and anelastic constants of an alloy having a low melting temperature. The advantage of using such an alloy is its easy accessibility to the melting temperature. The working hypothesis on which the present study is based is that although the absolute values of the elastic constants of the alloy used here are not necessarily representative of those of earth materials, their relative variations around the melting temperature can be regarded as indicating the behavior of earth materials.

§ 2. Experimental Method
The alloy adopted in the present study consists of Pb (37.7%), Bi (42.5%), Sn (11.5%), and Cd (8.5%), and has the melting point at 72°C.

A schematic diagram showing the experimental arrangement and the sample configuration is given in Fig. 1. An electric pulse produced by a pulse generator is converted to a mechanical pulse by the lead zirconate ultrasonic transducer attached to one end of the sample. After being trans-
mitted lengthwise through the sample, the mechanical pulse is picked up by the other transducer at the opposite end. The signal from this transducer together with a triggering pulse from the pulse generator, gives rise to an ordinary pulse-echo trace on an oscilloscope. The signal from this transducer together with a triggering pulse from the pulse generator, gives rise to an ordinary pulse-echo trace on an oscilloscope. The travel time was read on the oscilloscope trace by using a 1 Mc/sec quartz crystal oscillator.

The temperature of the sample was measured by a thermister pasted on the outer surface of the sample vessel. The temperature was varied slowly enough to attain a thermal equilibrium between the sample and the thermister. It took about three hours to raise the temperature from 20°C to 120°C and approximately the same time to lower it again. As the influence of the temperature change in the proximity of the melting point is very critical to the change of the physical properties of materials, the temperature should be varied slowly. In this study, the temperature was varied at the rate of 5°C/1 hour in the range around the melting point.

The resonance frequency of the transducer is 1 Mc/sec for P waves and 400 kc/sec for S waves. In order to determine the influence of the temperature on the electro-mechanical coupling efficiency of the transducer, the calibration was made by replacing the sample by an aluminium sample whose attenuative property, we assumed, exhibits no appreciable temperature variation in the range covered by the present study.

![Diagram](image-url)  
*Fig. 2. Variation with temperature of compressional wave velocity \( V_p \), shear wave velocity \( V_s \). \( T/T_m \) denotes the ratio of the temperature to the melting temperature.*
The alloy was first heated to the molton state and molded in a vessel. The upper transducer (See Fig. 1) was buried in the molded sample so that a good mechanical contact between the transducer and the sample was attained. When the temperature is raised and the sample gets molten the upper transducer can be displaced either upward or downward so that the effective sample length can be arbitrarily adjusted.

§ 3. Variation of Elastic Wave Velocity with Temperature

The result with respect to the temperature variation of $P$ wave velocity $V_P$ and $S$ wave velocity $V_S$ is shown in Fig. 2. No $S$ wave velocity determination was made in the temperature range above the melting point. As is easily seen in Fig. 2, the elastic wave velocity exhibits but a slight change with temperature except in the limited range around the melting point. Detailed inspection, however, shows that the variation of $S$ wave velocity is large compared with that of $P$ wave velocity. The change of $P$ wave velocity is about 2.4% for the temperature change of 60°C before melting and that of $S$ wave velocity is about 9.3% which is four times as large as the $P$ wave variation. It is remarkable that both $P$ and $S$ wave velocities undergo a sharp change over a narrow temperature range of 2~3°C around the melting temperature. The $P$ wave velocity decreases about 20% on passing through the melting point. The $S$ wave signal became undetectable in the melting

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**Fig. 3.** Variation of elastic constants with temperature. $k$, $\lambda$, $\mu$, and $\sigma$ denote incompressibility, Lamé constant, rigidity and Poisson's ratio respectively.
region and the velocity could not be determined. This is due to the combined effect of attenuation and a large reduction of the wave velocity. Although there is no clue to the estimation of the shear wave velocity above the melting point, we may safely conclude that the S wave velocity is effectively zero.

From the wave velocity thus obtained we can calculate the bulk modulus $k$, Lamé constants $\lambda$ and $\mu$, and Poisson’s ratio $\sigma$. The results are shown in Fig. 3. In these calculations the material is assumed to be an isotropic, perfectly elastic body. It can be said from Fig. 3 that although $\mu$ varies sharply on passing from solid to liquid state, it is unaccompanied by any appreciable change in $\lambda$. It should be noted here that it is not $k$ but $\lambda$ that is effectively kept constant on melting.

§ 4. Measurement of $Q$

As is well known the amplitude of elastic wave decays almost exponentially with the distance travelled, due to the internal friction of materials. If we denote the amplitude of a plane wave at a point $P_0$ by $A_0$, the amplitude $A_x$ at another point $P_x$ distance $x$ apart, can by written as

$$A_x = A_0 \exp(-\alpha x) \quad (1)$$

where $\alpha$ is the attenuation coefficient, which is a positive constant characteristic of the material. It is customary, however, to use a nondimensional quantity $Q$ called quality factor instead of $\alpha$. The usual definition of $Q$ is

$$Q = (f\sigma)/(\alpha V) \quad (2)$$

where $f$ and $V$ are frequency and velocity of the elastic wave respectively.

In order to determine the attenuation coefficient we adopted the method similar to the one described by M. AUBERGER and J.S. RINEHART (1961). In this method the attenuation coefficient is determined by measuring the amplitude decay of the elastic wave travelling through samples of various lengths.

Let $A_1$ and $A_2$ be the amplitude after travelling through a sample of length $L_1$ and $L_2$ respectively, the attenuation coefficient $\alpha$ can then be given by

$$\alpha = (\ln(A_1/A_2))/(L_2-L_1) \quad (3)$$

We first consider the frequency dependence of $Q$ for the purpose of investigating the attenuative nature of the material used herein. In Fig. 4 we plotted the logarithmic amplitude against the sample length for various frequencies. The gradient of linear curves fitted to the data can be used to determine the attenuation coefficients (KRISHNAMURTHI, M. and BALAKRISHNA, S.: 1957.)

![Fig. 4. Logarithmic decrement of amplitude of compressional wave for various frequencies.](image-url)
The scattering of the points in Fig. 4 can be regarded as indicating the accuracy of the method.

Fig. 5 shows the result in terms of the variation of $Q$ with frequency. Only the quality factor for $P$ wave was studied here. Unlike the general result that the values of solid are substantially independent of frequency (Knopoff, L.: 1964), the values of $Q$ obtained here increase almost linearly with frequency. However, in the frequency range covered by this study there have been no data available for comparison.

The linear increase of $Q$ with frequency is characteristic of the Maxwellian material for the frequency range higher than the critical frequency. In view of the fact that the Maxwell model pertains mainly to liquids or materials that have flow under statically applied stresses (Mason, W.P.: 1958), this result is not unexpected for the alloy which has low melting temperature and is easy to flow even at normal temperatures.

In the preceding discussion we have assumed that the attenuation of the elastic wave is solely due to the internal friction of the material. However, the reflections at the outer surface of the sample might also result in the distortion of the transmitted wave form. As the transducer is excited by a pulse as narrow as $0.1 \mu$ sec duration, the wave form at the emitting transducer can be regarded as a sort of the impulsive response; a damped oscillation whose frequency is about the same as the resonance frequency of the transducer but slightly modified by the acoustic coupling with the sample. The wave form will gradually be distorted while travelling through the sample due to the multiple reflections at the outer surface. The distortion occurs mainly in the later phases rather than in the initial rise. Since in the present study the sample length is comparatively small and only the initial rise was considered in the determination of the attenuation coefficient, the effect of the multiple reflection was neglected. This might not always be valid but when the internal friction of the sample is large and dominating over the effect of the multiple reflection this will be a reasonable assumption.

§ 5. Variation of $Q$ with Temperature

The variation of $Q$ with temperature was studied at the frequency of 1 Mc/sec. Amplitude-versus-length curves from which the
The attenuation coefficient was calculated as shown in Fig. 6. In Fig. 7 are shown the variation of $Q$ with temperature thus obtained. Except in the narrow range around the melting point the variation of $Q$ with temperature is so small as to be masked by the experimental errors. The sharp decrease of $Q$ which occurs on melting is about one order of magnitude. This implies that if there is a layer in the deeper part of the earth in which the attenuation of seismic waves is one order of magnitude as large as in the neighbouring layers, the temperature in the layer is very likely to be close to the melting temperature unless the material is drastically different.

§ 6. Conclusion

Conclusion of this study can be listed as follows:
1) The change of $P$ wave velocity is about 2.4% for the temperature change of 60°C before melting.
2) The change of $S$ wave velocity is about 9.3% for the temperature change of 60°C before melting.
3) $P$ wave velocity drops about 20% on melting.
4) Lamé constant $\lambda$ does not change significantly on melting.
5) The value of $Q$ drops one order of magnitude on melting.
6) The sharp change of elastic properties occurs only in a narrow range of temperature around the melting point.

Fig. 6. Logarithmic decrement of amplitude of compressional wave at various temperatures.
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Fig. 7. Variation with temperature of quality factor Q for compressional wave velocity.

References


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