Q Distribution for Long-Period P Waves in the Mantle

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Abstract

The Q -depth relation in the earth's mantle has been estimated for long-period P waves from two deep-focus and three intermediate earthquakes. The amplitude spectrums of the first P waves from 114 seismograms at recording stations in the distance range between 15 and 103 degrees, have been corrected for the crustal structure and the instrument and used to measure the effects of attenuation over the whole mantle. Probable Q models are presented on the basis of the spectral amplitude ratios between these stations. The low Q between 120 and 280 in the upper mantle shows a rapid increase at depths around 900 km to several thousands or greater in the lower mantle, and there is an apparent drop of Q near the base of the mantle. The frequency dependence of the spectral ratios against epicentral distance over about 80 degrees differs for the deep-focus and intermediate earthquakes, suggesting the possible diffraction effects due to the core.

§1. Introduction

The attenuation of seismic energy in the earth, which can be specified by the specific dissipation factor Q , may be closely related to the internal composition, state and temperature field of the mantle. A number of studies have been made by various methods to evaluate Q of seismic waves for different frequencies, but it is only recent that attempts have been focused to estimate the intrinsic Q distribution with depth over almost the whole mantle.

A method has been developed for this purpose on the basis of a theory and observations for the attenuation of surface waves and the decay of the free oscillations of the earth (ANDERSON and ARCHAMBEAU, 1964; ANDERSON *et al.*, 1965). The second approach was made by the use of spectral amplitude ratios of body waves at many stations, correlating their variations with epicentral distance (TENG, 1966). Another approach was based on the observed results that the peak of the amplitude spectrum

of P waves moves towards longer period side with the increase in epicentral distance (KURITA, 1966). (This paper will be referred to as paper I.)

All of these methods reduce to an inverse problem for determining a Q -depth relation from observations at the earth's surface. Surface waves are more sensitive to the Q distribution in the upper mantle than that in the deeper portion, while body waves provide an average Q along a ray path. More direct evidence on the dissipative properties over the whole mantle should therefore be obtained from body wave observations at successively increased epicentral distances than by the surface wave method, if the effects of source, crustal structure and instrument could be removed by proper techniques.

The method employed in the present study is essentially the same as the second one. Data from 2 deep-focus and 3 intermediate earthquakes are analyzed concurrently, which make possible to extract more reliable infor-

mation on the Q structure particularly for the lower mantle, by superposing results from different earthquakes with nearly the same focal depths.

Some effects appropriate to recording stations are reduced by taking differences in the logarithmic amplitude ratios between different frequencies. A step is introduced to estimate the general features of the Q distribution from an experimental relation between apparent Q 's and epicentral distances. Theoretical dependences of the amplitude ratios on distance expected from possible Q structures are then computed and compared with observations. The average Q values along the vertical paths in the upper and lower mantle, which have been determined from previous studies using multiply reflected core phases, are also taken into our consideration as constraints imposed upon the estimates of possible Q -depth relations. In relation to the evaluation of Q for the deeper mantle, the diffraction of P waves around the core is discussed in some detail.

§ 2. Method of Analysis

The Fourier amplitude spectrum $A(\omega)$ of P waves that should be observed at any recording station may be written in the form,

$$A(\omega) = a D S(\omega) C_m(\omega) C(\omega) I(\omega) \cdot \exp \left[- \int k(\omega, s) ds \right] \quad (1)$$

where we assume that the waves are radiated from the source to all directions with the same spectral structure. The above symbols are,

- a ; spatial amplitude factor relating to the focal mechanism
- D ; divergence factor for the effect of geometrical spreading
- $S(\omega)$; source spectrum
- $C_m(\omega)$; crust-mantle response around the source
- $C(\omega)$; crustal response for the recording station
- $I(\omega)$; instrumental response
- $k(\omega, s)$; attenuation coefficient along a ray

ω ; circular frequency

s ; ray element

$k(\omega, s)$ is assumed to be a linear function of frequency within the frequency range under consideration, and is related to $k(\omega, s) = \omega/2Q(r)v(r)$, where $Q(r)$, the specific dissipation factor, and $v(r)$, the velocity, both being a function of the distance r from the center of the earth. The integration in eq. (1) is taken from the source to station and may be rewritten as

$$\int k(\omega, s) ds = \frac{\omega}{2} \int \frac{ds}{Q(r)v(r)} = \frac{\omega}{2} \frac{T}{\bar{Q}} \quad (2)$$

T is the travel time, and $\bar{Q}$ the average dissipation along the ray now considered. If we take the ratio of the amplitude spectrum at any station (j) to that at a reference station (s), we have,

$$\frac{A_j(\omega)}{A_s(\omega)} = \frac{a_j D_j C_j(\omega) I_j(\omega)}{a_s D_s C_s(\omega) I_s(\omega)} \exp \left[- \frac{\omega}{2} \left(\frac{T_j}{\bar{Q}_j} - \frac{T_s}{\bar{Q}_s} \right) \right] \quad (3)$$

or

$$\ln \frac{\bar{A}_j(\omega)}{\bar{A}_s(\omega)} = \ln \frac{a_j D_j}{a_s D_s} - \frac{\omega}{2} \left(\frac{T_j}{\bar{Q}_j} - \frac{T_s}{\bar{Q}_s} \right) \quad (4)$$

where

$$\bar{A}_j(\omega) = \frac{A_j(\omega)}{C_j(\omega) I_j(\omega)} \quad (j=1, 2, \dots, n)$$

n being the number of stations. The common source effect $S(\omega)$ has thus been removed from our considerations, and the difference in $C_m(\omega)$ may be neglected because of small difference of the take-off angles at the focus. Expressions similar to eqs. (3) and (4) have been derived by TENG (1966), and used to estimate the effects of attenuation from the dependence of the left-hand side on wave frequency. It is noticed, however, that the dependence is likely to be rather sensitive to station effects such as uncertainties of local crustal structure, instrumental responses, and other factors not explicitly given in (1).

To reduce these effects to a minimum extent, we take the difference between eq. (4) for any frequency subscripted with k and that for a standard frequency ω_0 ,

$$\begin{aligned}
F &\equiv \ln \frac{\bar{A}_j(\omega_k)}{\bar{A}_s(\omega_k)} - \ln \frac{\bar{A}_j(\omega_0)}{\bar{A}_s(\omega_0)} \\
&= \ln \left[\frac{\bar{A}_j(\omega_k)}{\bar{A}_j(\omega_0)} \cdot \frac{\bar{A}_s(\omega_0)}{\bar{A}_s(\omega_k)} \right] \\
&= -\frac{\omega_k - \omega_0}{2} \left(\frac{T_j}{\bar{Q}_j} - \frac{T_s}{\bar{Q}_s} \right) \quad (5)
\end{aligned}$$

The left-hand side F may be obtained from observations with known crustal and instrumental responses without any knowledge about the source and geometrical spreading. The gradient $F/(\omega_k - \omega_0)$ may be determined by the least squares, fitting data to a straight line through the origin for any station pairs. Now we define the determined gradient to be $dF/d\omega$, then we obtain,

$$\left(\frac{dF}{d\omega} \right)_j = -\frac{1}{2} \left(\frac{T_j}{\bar{Q}_j} - \frac{T_s}{\bar{Q}_s} \right) \quad (6)$$

For a number of observations from an earthquake, we have the corresponding number of $dF/d\omega$ as a function of epicentral distance or of travel time. Once a functional form $f(\Delta)$ for $dF/d\omega$ was determined from the empirical relation, the average Q may be expressed in term of epicentral distance,

$$\bar{Q} = T(\Delta) / [T_s / \bar{Q}_s - 2f(\Delta)] \quad (7)$$

This relation will give some ideas of the general trend of the Q structure.

The next step is to derive an intrinsic Q distribution with depth over the whole mantle. $T/\bar{Q}$ in eq. (2) can also be written by

$$\frac{T}{\bar{Q}} = \int \frac{ds}{Q(r)v(r)} = \int \frac{r^2}{\sqrt{\gamma^2 - p^2}} \frac{dr}{rQ(r)} \quad (8)$$

$$= \int \frac{d(\sqrt{\gamma^2 - p^2})}{Q(r)(1-\zeta)} \quad (8')$$

and

$$\eta = r/v(r), \quad p = \eta \sin i_a = dT/d\Delta, \quad \zeta = \eta dv/dr,$$

where i_a is the take-off angle at the focus. A procedure for obtaining the solution of the integral equation similar to (8) has been presented recently by SATO and ESPINOSA (1967a). There would be some problems to be solved, however, before a direct application of the technique to our case, in view of scattering of $dF/d\omega$ values. The present method is to compute many sets of theoretical curves for the right-hand side of expression (6), by performing numerical integrations through eq. (8') for a given velocity profile and the possible Q structures estimated before, and to compare with the empirical values. If a specific theoretical curve is found to match the experimental values, the corresponding Q structure would be inferred as most probable.

§ 3. Data Analysis

(1) Data

Table 1 lists all the earthquakes used in this study, together with necessary information referred to in later discussions. Data analyzed are the long-period seismograms recorded at the world-wide standardized station network (WWSSN) operated by the USCGS. Information of the stations pertinent to all the earthquakes, including the epicentral distances, azimuths, and emergent angles, are summarized in Table 2 at the end of this paper. Fig. 1 shows the fault-plane solution on a Wulf net projection for the first P waves recorded during earthquake No. 1. Stations in

Table 1. Information of the earthquakes used for the present study.

Shock No.	Origin Time	Location	Depth (km)	M	Number of Stations	Reference Station	Frequency Range
1	Nov. 3, 1965 01 h 39 m 02.5 s	9°1 S, 71°4 W Peru-Brazil Border	583	6.2	43	LPA $\Delta = 28^\circ 5$	0.024 ~0.136
2	March 17, 1966 15 h 50 m 32.2 s	21°1 S, 179°2 W Fiji Islands	626	6.2	20	CTA $\Delta = 32^\circ 3$	0.030 ~0.130
3	Aug. 28, 1962 10 h 59 m 59.0 s	38°0 N, 23°1 E Greece	120	6.8	22	LAH $\Delta = 42^\circ 9$	0.040 ~0.124
4	Jan. 20, 1964 17 h 08 m 37.4 s	20°7 S, 169°9 E New Hebrides Isl.	141	6.8	13	WEL $\Delta = 20^\circ 9$	0.030 ~0.130
5	April 24, 1964 05 h 56 m 10.1 s	5°1 S, 144°2 E New Guinea	106	6.3	16	RIV $\Delta = 29^\circ 3$	0.030 ~0.130

Fig. 2. P waveforms reproduced from the vertical component seismograms and their corrected amplitude spectra $\hat{A}(\omega)$, for earthquake No. 1 ($h=583$ km, $M=6.2$).

Fig. 2.

the compression zone (open circles) are omitted from later analysis, since there might be some distortion of the spectrum in relation to the focal mechanism, and those in the distances shorter than 14 degrees are also excluded

Fig. 1. Fault-plane solution for earthquake No. 1.

because of unsimilarity of their waveforms. The *P* waveforms on the vertical component record are shown in the order of increasing epicentral distances on the left side of Figs. 2 and 3, for earthquakes No. 1 and 3 respectively.

(2) Spectral analysis of *P* waves

Since earthquakes No. 1 and 2 are of deep-focus, the direct *P* waves are well isolated from later arrivals. Only conspicuous later phase is *PcP*, which accesses by 40 sec to the *P* waves at an epicentral distance of 60 degrees and overlaps on them at about 85 degrees. In the case of the intermediate earthquakes No. 3, 4 and 5, a sharp *pP* phase arrives about 25 to 30 sec after the first *P* waves over all the distances. Fig. 4 shows the time differences between various later phases and the first *P* waves for focal depths of 120 km and 583 km, corresponding to earthquakes No. 3 and 1 respectively. Although *PcP* should arrive closely to the direct *P* waves for distances greater than about 70 degrees, according to Fig. 4, the phase is not clearly recorded in the former earthquakes.

Fig. 3. *P* waveforms reproduced from the vertical component seismograms and their corrected amplitude spectra $\hat{A}(\omega)$, for earthquake No. 3 ($h=120$ km, $M=6.8$).

Fig. 4. Time differences between the first P waves and various later phases.

To avoid the effects of contamination by later phases, the signal tail of P waves has been truncated before the arrival T_L of the closest later phase. The time length taken on each seismogram is given in Table 2. The signals were digitized at about 0.7 sec interval.

A time window, which is of the same type described in paper I, was also applied to reduce the distortion of the resulted spectrum introduced by the truncation. The time window, $D(t) = 1 - (t/T_m)^n$ for $0 \leq t \leq T_m$, and otherwise $D(t) = 0$, is again illustrated in Fig. 5 for the cases of $n = 20$ and ∞ , together with the corresponding spectral window. T_m was chosen for all the stations as about 40 sec for the deep-focus earthquakes and about 28 sec for the intermediate earthquakes. The amplitude spectrum $A(\omega)$ of the windowed signals was then computed by the conventional Fourier transform technique.

Fig. 5. Data windowing used for the present analysis and the corresponding spectral window.

(3) Instrumental and crustal responses

The instrumental response $I(\omega)$ of the long-period seismograph-galvanometer system used at the WWSSN is shown in Fig. 6. Two amplitude response curves have been calculated for two different systems having free-periods of seismograph of 15 sec for most of the stations and of 30 sec for some other stations, with a galvanometer period of 100 sec, both being critically damped. The peak magnification ranges from 375 to 3000.

Fig. 6. Calculated amplitude response of the long-period seismograph system for the WWSSN.

The effects of crustal layering beneath a recording station can be expressed by the crustal transfer function. The amplitude response $C(\omega)$ for some typical crustal models has been given in paper I, where no great differences are found in the frequency range from 0.03 to 0.13 c/s with which we are concerned. For this reason, Haskell's unlayered model was consequently applied as a first approximation to all the stations, to compensate only the difference in the response for different incident angles to the Moho-discontinuity.

(4) Corrected amplitude spectrums and their ratios

The amplitude spectrums $\bar{A}_j(\omega)$, which have been corrected for the instrumental and crustal responses described above, are presented on the right side of Figs. 2 and 3 for earthquakes No. 1 and 3, in comparison with the recorded waveforms. The time length of analysis limits the lower side of the frequency range to be investigated, while

the higher side is limited by microseismic noise and the uncertainty of local underground structure. For further discussion, therefore, we confine ourselves to use the amplitude spectrum in the range from 0.03 to 0.13 c/s (33 to 7.7 sec). It may be seen in the figure that the general pattern of the corrected spectrum shows a gradual transition with the increase in epicentral distance, although there are small spectral undulations that may probably be due to uncertainty of crustal structures, deviations from the symmetry of focal mechanism and various noises.

Computations follow for obtaining the ratio of the amplitude spectrums between two stations, and hence the difference of logarithmic amplitude ratios, F , which is defined in eq. (5). Stations indicated in Table 1 and a frequency around 0.04 c/s were used as references. The computed values of F have been smoothed by applying a moving average with a width of 0.024 c/s, to detect the general trend of their variation against frequency. The least squares yielded $(dF/d\omega)_j$ on the left side of (6), from the slope of a straight line fitted to the plots of smoothed F .

(5) Dependence of $dF/d\omega$ on epicentral distance

$dF/d\omega$ values obtained for all the stations are plotted in Fig. 7 against epicentral distances with respect to a reference station LPA ($\Delta=28.5^\circ$) for earthquake No. 1. Solid circles give the results from the windowed ($n=20$) signals, while open circles show the corresponding values to which a data window has not been applied. The probable errors of these points determined from least squares range from 0.02 to 0.10 sec. If we assign a standard frequency to 0.08 c/s instead of 0.04 c/s, these points move to an order of 0.1 sec. The maximum variations due to these reasons are estimated not to exceed 0.3 sec. Similar diagram has been obtained with a reference at OXF ($\Delta=46.6^\circ$), but the computed values displace only by less than 0.1 sec, so that their trend does not differ from the former case. A few points deviating to the upper side in Fig. 7 may be those subjected to spectral contaminations by PcP or microseisms, and will be omitted from later discussions.

It can be seen that there is an upward-warping trend in the plotted points, although they scatter in a rather wide range. If the

Fig. 7. $dF/d\omega$ against epicentral distance for earthquake No. 1. Broken lines indicate the relation for constant dissipation models.

dissipation of seismic energy was constant over the whole mantle, eq. (6) would reduce to $(dF/d\omega)_j = -(T_j - T_0)/2Q_0$ ($\bar{Q}_j = \bar{Q} \equiv Q_0$), so that the relation should be represented by a straight line with a negative slope on the $dF/d\omega \sim T$ diagram. In Fig. 7, the corresponding curves are drawn for reference sake for three assumed values of Q_0 . A comparison of the above trend with the assumed curves suggests that the real earth may be far from a constant dissipation model.

§4. Evaluation of the Q distribution in the Mantle

Since it has been found that the general dependence of $dF/d\omega$ on epicentral distance is not much affected by the choice of a reference station, we may be allowed to superpose the results from different earthquakes with almost the same focal depth, as far as the reference is assigned to nearly equal epicentral distances. Fig. 8 shows the results computed from 2 deep-focus earthquakes No. 1 and 2, while those from 3 intermediate earthquakes No. 3, 4 and 5 are grouped into Fig. 9. All of the computed values are from the analysis of windowed signals, and their probable variations are of the same order of magnitude as estimated in the foregoing section. It is noticed that each of the earthquakes gives a similar pattern for $dF/d\omega$, and that the distance dependence shows appreciable difference between the deep-focus and intermediate earthquake groups, for great distances. The difference will be discussed in the last section. Dotted lines in the figures are drawn tentatively, by taking average of the computed values over every 10 degrees of distance and smoothing the averages.

Before a direct comparison of the experimental curve with theoretical prediction for a Q structure, an estimate is made for the general trend of possible Q-depth relations. The experimental curve has been transformed by eq. (7) into the relations between $T/\bar{Q}$ or $\bar{Q}$ and epicentral distance, as illustrated in Figs. 10 and 11 respectively, assuming $\bar{Q}_0$ to be from 150 to 250. These assumptions may

be justified for reference stations around 28 degrees, from the notion of low Q models for the upper mantle, which have been suggested by recent studies. The rapid increase of $\bar{Q}$ with increasing epicentral distance up to about 80 degrees strongly suggests that the intrinsic Q value also increases with depth down to the lower mantle. It may also be inferred from the figures that Q in the upper mantle is still lower than that assumed above. For distances greater than 80 degrees, $\bar{Q}$ drops rapidly. This corresponds to the decrease of $dF/d\omega$ values, towards the great distances. If the reason for this is attributed only to the attenuation, intrinsic Q would take very low values near the base of the lower mantle.

It is to be noted that the present method of using $dF/d\omega$ values does not yield a unique solution for the Q structure but only its relative distribution. It therefore requires a boundary condition or an appropriate assumption of an absolute Q at a depth, to determine the intrinsic Q distribution in the mantle. Reliable Q values for shear waves in the upper mantle appear to have been obtained mainly from surface wave studies by ANDERSON *et al.* (1965), but the corresponding ones for longitudinal waves do not yet, although they found that a relation $Q_\alpha/Q_\beta = 2.25$ seems to fit the experimental attenuation of Love and Rayleigh waves.

For this reason, we adopted the models of ANDERSON and ARCHAMBEAU (1964), ANDERSON *et al.* (1965), and of TENG (1966), irrespective of the type of waves, as starting Q values for the upper mantle in our study. About 30 Q models have been constructed on the basis of the general trend as inferred from $\bar{Q}-\Delta$ relations in Figs. 10 and 11, together with variations of the adopted Q's in the upper mantle. Theoretical dependence of $dF/d\omega$ with epicentral distance was then computed through eqs. (8') and (6) from each of the models with the Jeffreys-Bullen velocity profile, and compared with the experimental relation in the two earthquake groups. There seems to be a consistency between the theoretical and experimental relations in both the groups for Models 5, 10, 11 and 14, as may

Fig. 8. dF/dw against epicentral distance for deep-focus earthquakes Nos. 1 and 2, and the theoretical relation from the proposed models.

Fig. 9. dF/dw against epicentral distance for intermediate earthquakes Nos. 3, 4 and 5, and the theoretical relation from the proposed models,

Fig 10. $T/\bar{Q}$ and $\bar{Q}$ against epicentral distance for the deep-focus earthquakes, and the theoretical relation from the proposed models.

Fig. 11. $T/\bar{Q}$ and $\bar{Q}$ against epicentral distance for the intermediate earthquakes, and the theoretical relation from the proposed models.

be seen in Figs. 8 and 9. The corresponding theoretical curves of $T/\bar{Q}$ and $\bar{Q}$ from the models are drawn in Figs. 10 and 11 respectively. Fig. 12 shows the Q models mentioned, in which Model 5 has the same structure as the MM8- Q_β model of ANDERSON *et al.* (1965) for S waves above 600 km, while Model 14 as the MM8- Q_α model for P waves above 300 km. The structure of Models 10 and 11 in the upper mantle lies between Models 5 and 14. Definite Q values between depths of 1000 and 2500 km are rather hard to evaluate. Although a tentative value of 10,000 is assigned to the above models, an alternative of 6,000 or 12,000 did not make much difference in the theoretical curves. The Q distribution in the lower mantle deeper than 2500 km has been tentatively given to explain the rapid drop of $dF/d\omega$ for distances beyond 80 degrees, on the assumption that this phenomenon would be a manifestation only of the attenuation near the core-mantle boundary. Another possibility is discussed in the next section.

§ 5. Discussion

(a) Comparison of average Q with previously evaluated values

The common features of the proposed models are the low Q values between 100 and 300 increasing gradually in the upper mantle above 800 km, their abrupt increase at depths around 900 km, high values reaching several thousands or even greater in the lower mantle, and possible rapid drop of Q towards the core-mantle boundary. The above models including those similar to MM8- Q_α and MM8- Q_β yield only small differences in the $dF/d\omega \sim \Delta$ relation, although the Q 's in the upper mantle differ by a factor of more than 2. All of the four models are compatible with our data. It is, therefore, not able to determine, solely from the obtained results, which of the four models is the most probable Q structure for P waves.

One estimate of these models might be obtained from a comparison of the average Q values in each model with those evaluated from multiply reflected core phases such as ScS , $sScS$ and PcP . Table 3 gives the average

Fig. 12. Probable Q models for P waves.

Table 3. Average Q values for the proposed models.

Case	Depths (km)	Model 5	Model 10	Model 11	Model 14
	0—800	120	166	195	272
1	800—2500	1650	2050	3650	3500
	2500—2900	106	122	122	122
	0—2900	222	300	329	412
2	800—2900	1690	2310	4060	4900
	0—2900	292	417	508	702

Q's of the presented models for the upper, lower and whole mantle, which are calculated from a relation $T/\bar{Q} = \sum_k T_k/Q_k$.

Since the averages for the lower mantle and hence over the whole mantle are considerably affected by the apparent low Q zone near the core-mantle boundary, two cases are given: case 1 includes the zone; and case 2 does not, assuming high Q values in the lower mantle to reach its base. The former case is based on the assumption that the drop of Q towards the core-mantle boundary would be due only to the attenuation effects, while the latter would correspond to the standpoint that the apparent low Q results solely from the diffraction of waves around the core.

The previous results that may be compared with the present case for periods from 8 to 33 sec are those mainly from ScS and its related phases. The average $\bar{Q}_\beta$ for shear waves evaluated for the whole mantle ranges from 350 to 600 (PRESS, 1956; OTSUKA, 1963; ANDERSON and KOVACH, 1964; KOVACH and ANDERSON, 1964; STEINHART *et al.*, 1964; SATO and ESPINOSA, 1967b), and that for the upper mantle shallower than 600 km has been estimated in the above three articles in 1964 to be from 150 to 200. The corresponding averages for longitudinal waves have been obtained by KANAMORI (1967a, b) from short period P, PcP and some other phases. His results show the probable range of average values in the upper, lower and whole mantle as, $180 < \bar{Q}_\alpha' < 240$, $1600 < \bar{Q}_\alpha'' < 6000$, and $410 < \bar{Q}_\alpha < 630$. Our Q models that seem to satisfy all of the above conditions are Models 10 and 11 in case 2. However, if an allowance of 10% is made to the obtained values as well as to

the above-mentioned bounds, Models 10, 11 and 14 in case 1 would also survive. Model 5 and TENG's model do not seem consistent with these estimates. Any conclusion may not be reached from this side for the problems which model is most probable and whether a substantial low Q zone exists near the core-mantle boundary or not.

(b) *Effects of the diffraction around the core*

The apparent low Q at the base of the mantle results from a rapid diminution of amplitudes for higher frequencies. It has also been pointed out by GUTENBERG (1960), SACKS (1966) and TENG (1966) from the observational sides that the amplitudes of shorter-period waves decay more rapidly than those with longer-periods near the core-mantle boundary. The possible explanations for this phenomenon would be the existence of a low Q layer, the diffraction of waves around the core, and/or the effects of a layered velocity structure near the core-mantle boundary.

GUTENBERG (1960) attributed the above phenomenon to the diffraction around the core, based on a comparison of his observations in the time domain with the experimental results obtained by RYKUNOV. RYKUNOV (1957) has investigated the diminution of P wave amplitudes in the shadow zone by using ultrasonic modeling, and showed that the decrease in amplitudes is faster for shorter-period waves than for longer ones, depending on the rigidity of a model core near the shadow boundary. The periods used in his model, however, do not cover requirements of similarity for diffracted P waves observed on the earth's surface.

SACKS (1966) also considered the above phenomenon to be due to the diffraction by

the earth's core, since FRIEDLANDER's theory favors his observational results. FRIEDLANDER (1958) has discussed the diffracted field of incident plane pulses due to a rigid sphere, obtaining an approximate expression for the pressure pulse near the front. From his expression, SACKS derived theoretical amplitude-distance curves of P arrivals with several periods within a range of the shadow zone, and found a satisfactory agreement between the theory and observations in the sense that the amplitude of shorter-period diffracted waves decreases more rapidly than for longer waves. It is to be remarked, however, that FRIEDLANDER's theory is not valid near the shadow boundary, so that the decrease of $dF/d\omega$ values beginning from a distance around 80 degrees would not be explained by this mechanism.

TENG (1966) came to the conclusion, by applying NUSSENZVEIG's theory to his observations, that a low Q region for P waves appears to exist at the bottom 200 km of the mantle. NUSSENZVEIG (1965) has given a thorough discussion for the diffraction of a plane wave by a totally reflecting sphere, and obtained an approximate expression for the shift of the shadow boundary, $s=1.36 (aV^2/\omega^2)^{1/3}$ (a : the radius of the sphere, V : the wave velocity, ω : the circular frequency). His expression implies that the lower the frequency is, the apparent radius of the core becomes large. This would lead to the results for our case as well as for TENG's that the slope of the amplitude spectrum against wave frequency should increase, at least, in a range before the geometrical shadow boundary. This is contrary to the case of the attenuation for which higher frequency energy decays faster. NUSSENZVEIG's expression holds, however, only under the condition $(ka)^{1/3} \gg 1$ with two other restrictions, so that the frequency would have to be limited to higher than several cycles/sec in the case of the real earth. It requires an exact solution to extend his theory to our results for frequencies from 0.03 to 0.13 c/s.

PHINNEY and ALEXANDER (1966) presented a theory for the diffraction of elastic waves

around the core, developing NUSSENZVEIG's results. They have obtained a theoretical attenuation function for diffracted P waves in the shadow zone, in which a spherical geometry and the response of a single-layer core-mantle boundary are included. Recently, SHIMAMURA (1967) made a two-dimensional model experiment by applying ultrasonic waves ($f=1\sim 500$ kc/s) to a plastic disk ($a=210$ cm, $V_p=2.42$ km/sec), to investigate the diffraction effects due to the core on the amplitudes of direct and diffracted waves near the shadow boundary. Frequencies between $7\sim 40$ kc/s would correspond to our case, if we take similar values for the scaling factor $ka=40\sim 220$. His preliminary results appear to show that the amplitudes of lower frequency waves decay from about 80 degrees in epicentral distance shorter by 20 degrees than that for the geometrical shadow boundary.

Although some other theoretical studies, including those of NAGASE (1956), DUWALO and JACOBS (1959), KNOPOFF and GILBERT (1961) and RUBINOW and KELLER (1961), have been made relating to the diffraction problem, we have, at the present stage, no complete theory and model experiment of diffraction for the zone before the shadow boundary that could fully account for our results.

It is to be again noted here that the experimental $dF/d\omega \sim A$ curve for the intermediate earthquake group drops more steeply than that for the deep-focus earthquake group for distances beyond 80 degrees, as has been seen from Figs. 8 and 9. It has also been confirmed that this discrepancy does not result from differences in the analyzed time length and the frequency range investigated between the two earthquake groups. The proposed Q models including a low Q zone near the core-mantle boundary do not yield such a large discrepancy between the two earthquake groups in the corresponding theoretical curves, as can be seen from a comparison of the two figures. For distances greater than 80 degrees PcP overlaps on the direct P waves. The possibility that a layered structure near the core-mantle boundary might contribute

to the present problem may be ruled out, by recent theoretical studies of TENG (1967), which indicate only a weak frequency-dependence of the reflection from the layered boundary, for the case of great incident angles nearly equal to 90 degrees, within the frequency range now considered.

Another possible explanation for the observed discrepancy may be the diffraction of waves due to the core before the shadow boundary, although this effect is not involved in expression (1). Fig. 13 illustrates two

Fig. 13. Seismic rays to an epicentral distance of 90 degrees, in the case of intermediate and deep-focus earthquakes. Foci, $I: h=120$ km, $D: h=583$ km.

seismic rays to a station at a distance of 90 degrees, in the case of the intermediate and deep-focus earthquakes respectively. The deepest point of the ray is about 2700 km for the former earthquake, which is shallower by about 50 km than that for the latter. This difference suggests that *P* waves from the deep-focus earthquakes will become subjected to the diffraction effects from a slightly shorter epicentral distance, compared to the case of the intermediate earthquakes.

Since the problem of a frequency-dependence of the diffraction effects before the shadow boundary has not yet been fully solved, we consider three different cases. If the diffraction is independent of frequency, it should not give any effects on the $dF/d\omega \sim \Delta$ relation for the two earthquake groups. If it yields a greater decrease of amplitudes for higher frequency waves than for lower frequency ones,

the drop of the experimental curve for the intermediate earthquakes would be less than that for the deep-focus earthquakes in a distance range. This is contrary to the observed trends. If the diffraction causes a greater decay of lower frequency waves compared to higher waves, the experimental curve for the former earthquakes would drop more greatly than that for the latter earthquakes in the corresponding range. This seems to agree, at least qualitatively, with the observed trends in Figs. 8 and 9. If we adopt the last interpretation, the *Q* structure evaluated from the deep-focus earthquakes would give an upper bound of possible *Q* values for depths deeper than 2700 km. The diffraction effects should be more complicated than was discussed, particularly towards the geometrical shadow boundary.

It may be suggested, however, from the above considerations that there could be a substantial low *Q* zone near the core-mantle boundary, as far as we thus attribute the observed discrepancy in the experimental relation between the two earthquake groups to the effects of the frequency-dependent diffraction. The general features of our *Q* models are consistent with those of TENG's model (1966), and of a model (for the case of $a=5$ km) by KURITA (1966), although the absolute values of their models are considerably different from those of ours. The possible existence of a low *Q* zone might be explained by the phase change from the solid to liquid state around the core-mantle boundary. However, the present conclusion particularly for the lower mantle was based on a qualitative discussion and is only tentative. More precise determinations of the *Q*-depth relation await further studies on both theoretical and experimental sides, for the combined effects of diffraction around the core and superposition of *PcP* and *P* for distances greater than 70 degrees.

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Table 2. List of stations and their pertinent information. Δ ; epicentral distance, φ ; azimuth, T ; travel time, i_d ; emergent angle at the focus, T_L ; analysed interval, T_0 ; free period of seismograph, T_g ; free period of galvanometer.

(1) Earthquake No. 1.

Station No.	Station Code	Δ (deg.)	φ (deg.)	T (sec)	i_d (deg.)	T_L (sec)	T_0 T_g (sec)
1	ANT	14.6	176.4	184	80.5	41.2	15—100
2	BHP	19.7	335.4	233	69.5	47.7	15—100
3	CAR	20.0	13.0	236	68.5	38.0	15—100
4	GIE	20.6	292.8	241	68.0	46.1	15—100
5	PEL	23.9	178.5	271	64.5	49.8	15—100
6	SJG	27.5	10.9	302	61.5	53.5	15—100
7	LPA	28.5	156.3	312	61.0	54.6	15—100
8	NAT	36.3	86.3	377	56.5	47.6	15—100
9	CHG	38.8	43.1	397	55.5	57.6	15—100
10	BEC	41.7	8.6	420	53.5	51.5	15—100
11	SHA	42.7	338.5	428	53.0	45.7	15—100
12	SOM	43.6	178.1	435	52.5	42.1	15—100
13	ATL	44.1	344.4	440	52.0	52.2	15—100
14	OXF	46.6	339.4	458	50.5	52.5	15—100
15	GEO	48.0	354.1	468	49.5	51.5	15—100
16	OGD	50.0	356.8	483	48.0	52.3	30—100
17	SCP	50.0	353.6	483	48.0	54.2	15—100
18	FLO	50.8	340.9	489	47.5	47.7	15—100
19	WES	51.2	0.1	492	47.5	46.1	15—100
20	LUB	51.4	327.2	494	47.5	47.7	15—100
21	AAM	52.4	348.5	500	46.5	45.7	15—100
22	ALQ	54.9	324.8	518	45.0	46.6	15—100
23	TUC	55.8	319.5	524	44.5	40.0	15—100
24	GOL	57.9	329.4	538	43.5	42.3	30—100
25	RCD	60.3	334.1	555	42.0	40.5	15—100
26	PLM	60.4	316.7	556	42.0	40.2	15—100
27	BKS	66.6	318.0	594	38.5	54.6	15—100
28	COR	71.0	323.5	621	36.5	40.2	15—100
29	LON	71.4	326.0	624	36.0	40.0	15—100
30	PTO	76.2	43.8	650	33.5	46.5	15—100
31	MAL	77.5	49.2	657	33.0	40.9	15—100
32	TOL	78.9	46.4	664	32.5	40.2	15—100
33	VAL	80.1	33.4	671	32.0	47.3	15—100
34	SPA	81.0	180.0	675	31.5	40.6	30—100
35	CMC	82.8	344.7	684	30.5	45.7	15—100
36	SDB	83.0	104.0	686	30.5	43.2	15—100
37	WIN	85.2	112.0	696	29.5	41.4	15—100
38	TRI	93.1	44.4	734	27.5	46.6	15—100
39	NOR	94.1	6.9	738	27.5	40.0	15—100
40	PRE	96.0	116.9	746	27.5	41.8	15—100
41	BUL	96.2	111.5	747	27.5	40.0	15—100
42	ATU	99.6	53.0	763	27.0	40.2	15—100
43	IST	103.7	50.0	780	27.0	40.2	15—100

(2) Earthquake No. 2.

Station No.	Station Code	Δ (deg.)	φ (deg.)	T (sec)	i_d (deg.)	T_L (sec)	T_0 T_g (sec)
1	HNR	23.2	296.9	261	65.0	36.8	15-100
2	CTA	32.3	265.5	341	58.5	37.0	15-100
3	RAB	32.5	297.2	343	58.4	36.8	15-100
4	PMG	34.4	284.6	358	57.6	37.0	15-100
5	GUA	49.3	311.2	475	48.7	36.8	15-100
6	BAG	69.8	297.5	610	36.9	37.3	15-100
7	LEM	71.9	269.5	623	35.9	36.8	15-100
8	ANT	73.7	118.9	633	35.0	37.0	15-100
9	HKC	78.0	299.6	657	33.0	36.4	15-100
10	GSC	81.5	47.1	675	31.2	37.0	15-100
11	SNG	83.5	280.2	684	30.3	37.0	15-100
12	TUC	84.2	52.3	688	29.9	36.6	15-100
13	LON	85.1	35.6	692	29.6	36.8	15-100
14	ALQ	88.7	51.7	709	28.4	36.6	15-100
15	COL	89.1	12.8	711	28.2	36.4	15-100
16	CHG	89.4	290.3	713	28.2	36.6	15-100
17	SHL	97.9	294.3	751	27.2	36.8	15-100
18	CMC	100.2	20.3	762	27.0	36.8	15-100
19	LPA	101.2	134.3	766	27.0	37.0	15-100
20	OXF	101.5	57.4	768	27.0	37.0	15-100

(3) Earthquake No. 3.

Station No.	Station Code	Δ (deg.)	φ (deg.)	T (sec)	i_d (deg.)	T_L (sec)	T_0 T_g (sec)
1	QUE	36.9	88.8	414	38.0	29.8	30-100
2	LAH	42.1	82.8	458	36.5	26.4	30-100
3	WES	68.5	307.4	646	26.9	26.6	30-100
4	BEC	69.4	295.4	652	26.5	26.4	30-100
5	SCP	73.4	309.0	676	25.3	26.4	30-100
6	GEO	74.1	307.0	680	25.1	25.0	30-100
7	MDS	78.7	316.8	706	23.5	27.6	30-100
8	TRN	79.2	275.1	709	23.3	28.6	30-100
9	MNN	79.5	319.6	710	23.2	27.6	30-100
10	NHA	79.6	82.7	710	23.2	27.8	30-100
11	FLO	82.1	313.7	724	22.3	28.6	30-100
12	CAR	83.6	278.3	731	21.9	27.4	30-100
13	RCD	85.0	324.3	739	21.4	27.8	30-100
14	BAG	85.8	72.6	742	21.2	28.0	30-100
15	GOL	89.4	322.8	759	20.3	28.4	30-100
16	LON	90.0	336.7	762	20.3	28.4	30-100
17	DAL	90.0	313.1	762	20.3	27.4	30-100
18	LUB	92.3	316.8	773	19.8	26.0	30-100
19	COR	92.4	336.7	774	19.8	26.6	30-100
20	BOG	92.8	278.0	776	19.7	30.2	30-100
21	ALQ	93.7	320.6	779	19.6	28.4	30-100
22	NNA	105.0	266.5	830	19.1	29.2	30-100

(4) Earthquake No. 4.

Station No.	Station Code	Δ (deg.)	φ (deg.)	T (sec)	i_d (deg.)	T_L (sec)	$T_0 - T_v$ (sec)
1	AFI	18.7	71.8	249	51.5	23.0	30—100
2	WEL	20.9	169.7	272	48.0	27.6	30—100
3	ADE	30.9	236.0	365	41.0	27.2	30—100
4	MUN	49.0	245.2	514	35.0	25.6	30—100
5	MAN	59.5	302.2	590	31.0	24.3	30—100
6	NHA	68.0	293.2	646	28.0	26.9	30—100
7	CHG	79.8	294.5	714	23.5	29.0	30—100
8	SHL	88.6	297.9	759	21.0	33.1	30—100
9	GSC	89.0	51.6	760	21.0	25.5	30—100
10	HOW	90.4	293.7	767	20.5	23.9	30—100
11	CMO	91.3	16.7	772	20.5	29.8	30—100
12	TUC	92.3	56.4	776	20.5	25.8	30—100
13	ALQ	96.6	55.3	796	20.0	28.5	30—100

(5) Earthquake No. 5.

Station No.	Station Code	Δ (deg.)	φ (deg.)	T (sec)	i_d (deg.)	T_L (sec)	$T_0 - T_v$ (sec)
1	RIV	29.3	168.1	352	42.0	23.6	30—100
2	MAN	30.2	310.9	361	41.5	24.3	30—100
3	BAG	31.7	312.9	374	41.0	17.7	30—100
4	MUN	37.4	221.0	422	39.0	22.9	30—100
5	NHA	38.8	296.5	434	38.0	25.1	30—100
6	HKC	40.1	313.9	446	38.0	22.3	30—100
7	AFI	44.2	104.6	479	37.0	20.3	30—100
8	SEO	45.4	340.7	488	36.5	22.0	30—100
9	KIP	62.4	62.8	612	30.0	26.5	30—100
10	NDI	72.5	302.0	677	26.0	23.8	30—100
11	LAH	75.7	304.2	694	25.0	23.8	30—100
12	COL	85.5	23.4	747	21.5	33.0	30—100
13	COR	95.3	45.8	793	19.5	33.0	30—100
14	BKS	95.9	52.6	795	19.5	32.1	30—100
15	CMC	98.7	22.2	809	19.5	32.4	30—100
16	BOZ	103.5	44.4	830	19.5	40.4	30—100