Attenuation of Short-period P-waves and Q in the Mantle

By

Tuneto KURITA

Department of Transportation Engineering, Faculty of Engineering, Kyoto University

Abstract

Q for short-period P-waves in the mantle has been investigated, based on the spectral analysis of the initial P phases of two earthquakes. Amplitude spectrums in the epicentral range between 21 and 103 degrees have been calculated, compensating for the propagation effects through the total seismic path. Amplitude-distance relations over the frequency range between 0.3 and 1.3 cps show that the rate of diminution of amplitude with epicentral distance is independent of frequency. From this observational result it follows that the intrinsic Q value becomes greater with depth between about 700 and 2500 km, and also with increasing frequency. The frequency independence of Q theory is inappropriate within the frequency range between 0.3 and 1.3 cps. It is also shown that the source asymmetry is not masked by the propagation effects except the stations near the nodal line for these frequencies.

§ 1. Introduction

Recently, for long-period P-waves, Q distribution with depth has been discussed by some investigators (TENG (1966), KURITA (1966), HIRASAWA and TAKANO (1966), MIKUMO and KURITA (1968)), on the assumption that Q is not dependent on frequency at least within the narrow frequency range considered. The results are considerably divergent mainly due to the inherent incapability of the method for determining Q in the upper mantle shallower than about 1000 km.

On the other hand, for short-period P-waves, the same attempt has not yet been succeeded, since it is considerably difficult to exclude various effects on the amplitude spectrum of the signal. Alternatively, the average Q values for the whole mantle, and for the upper and lower mantle have been estimated by ASADA and TAKANO (1963), and KANAMORI (1967 a, b, c).

Needless to say, it is most desirable for estimating a Q-depth relation to obtain the amplitude-distance relation from observations. In the time domain this relation was obtained by some authors (WADATI and HIRONO (1956), ULOMOV (1962), FEDOTOV (1963)) for short epicentral distances, and the dependence of the attenuation coefficient k with depth in the upper mantle was estimated, where \( k = \omega / 2QV \) (V; wave velocity, \( \omega \); circular frequency), as reviewed by KURITA (1966).

In the present study, variations of amplitude spectrum with the epicentral distance are estimated for different frequencies. This method is straightforward, but involves the well-known difficulties in evaluating the combined effects of focal mechanism, geometrical spreading and minute geological features near the focus and recording stations. Attempts will, however, be made to take into account all these effects in the following data processing.

The resultant amplitude-distance curves are interpreted on the standpoint that Q is dependent both on frequency and depth, and frequency only. The Q-depth and Q-frequency relations estimated will be discussed in some detail.

§ 2. Data

Table 1 lists information of the earthquakes used in this study, which is referred to as No. 1 for the shallow event and No. 2 for the intermediate one. Tables 2 and 3 indicate the relevant information of the stations and also the fault plane solutions of the earthquakes,
which will be referred to in the later discussions. Fault plane solutions in Table 3 are those determined by Dr. J. H. Hodgson. For earthquake No. 1, two fault plane solutions are given. The choice of the two will be discussed in § 4. In Figs. 5 and 12, epicentral distances to the stations and azimuths as seen from the epicenter are shown, together with the P-wave radiation patterns of the earthquakes. Data analyzed are the vertical component of short-period seismograms registered at the USCGS worldwide standardized stations.

**Table 1. Information of the earthquakes used for the present study (USCGS).**

<table>
<thead>
<tr>
<th>Shock No.</th>
<th>Origin Time</th>
<th>Location</th>
<th>Depth (km)</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>May 11, 1962 14h11=51.9</td>
<td>17°0N, 99°7W Near Coast of Mexico</td>
<td>25</td>
<td>7(Pas) 7-7 1/4(Pal)</td>
</tr>
<tr>
<td>2</td>
<td>Aug. 28, 1962 10°59=59.0</td>
<td>38°0N, 23°1E Greece</td>
<td>120</td>
<td>6 3/4(Pas)</td>
</tr>
</tbody>
</table>

**Table 2. List of stations and their pertinent information.**

(1) Shock No. 1

<table>
<thead>
<tr>
<th>Station No.</th>
<th>Station Code</th>
<th>Epicentral Distance (deg)</th>
<th>Azimuth (deg)</th>
<th>Magnification</th>
<th>Quality of Records</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>DAL</td>
<td>16.0</td>
<td>8.9</td>
<td>25,000</td>
<td>scratchy</td>
</tr>
<tr>
<td>2</td>
<td>LUB</td>
<td>16.6</td>
<td>353.7</td>
<td>25,000</td>
<td>scratchy</td>
</tr>
<tr>
<td>3</td>
<td>SHA</td>
<td>17.2</td>
<td>35.6</td>
<td>6,250</td>
<td>scratchy</td>
</tr>
<tr>
<td>4</td>
<td>ALQ</td>
<td>18.9</td>
<td>342.6</td>
<td>400,000</td>
<td>scratchy</td>
</tr>
<tr>
<td>5</td>
<td>BHP</td>
<td>21.2</td>
<td>109.7</td>
<td>12,500</td>
<td>good, spectral analyzed</td>
</tr>
<tr>
<td>6</td>
<td>PLM</td>
<td>22.5</td>
<td>319.7</td>
<td>50,000</td>
<td>scratchy</td>
</tr>
<tr>
<td>7</td>
<td>GOL</td>
<td>23.2</td>
<td>348.8</td>
<td>400,000</td>
<td>scratchy</td>
</tr>
<tr>
<td>8</td>
<td>FLO</td>
<td>23.2</td>
<td>18.8</td>
<td>50,000</td>
<td>scratchy</td>
</tr>
<tr>
<td>9</td>
<td>MD S</td>
<td>27.6</td>
<td>15.8</td>
<td>100,000</td>
<td>scratchy</td>
</tr>
<tr>
<td>10</td>
<td>BOG</td>
<td>28.0</td>
<td>113.2</td>
<td>12,500</td>
<td>scratchy</td>
</tr>
<tr>
<td>11</td>
<td>GEO</td>
<td>29.4</td>
<td>37.7</td>
<td>50,000</td>
<td>good, spectral analyzed</td>
</tr>
<tr>
<td>12</td>
<td>SCP</td>
<td>30.3</td>
<td>34.0</td>
<td>50,000</td>
<td>scratchy</td>
</tr>
<tr>
<td>13</td>
<td>CAR</td>
<td>32.5</td>
<td>97.2</td>
<td>25,000</td>
<td>good, spectral analyzed</td>
</tr>
<tr>
<td>14</td>
<td>WES</td>
<td>35.0</td>
<td>37.9</td>
<td>50,000</td>
<td>scratchy</td>
</tr>
<tr>
<td>15</td>
<td>BEC</td>
<td>35.1</td>
<td>57.6</td>
<td>12,500</td>
<td>good, spectral analyzed</td>
</tr>
<tr>
<td>16</td>
<td>TRN</td>
<td>37.7</td>
<td>94.6</td>
<td>25,000</td>
<td>good, spectral analyzed</td>
</tr>
<tr>
<td>17</td>
<td>ARE</td>
<td>43.3</td>
<td>138.6</td>
<td>50,000</td>
<td>good, spectral analyzed</td>
</tr>
<tr>
<td>18</td>
<td>LBP</td>
<td>45.6</td>
<td>135.3</td>
<td>50,000</td>
<td>good, spectral analyzed</td>
</tr>
<tr>
<td>19</td>
<td>TOL</td>
<td>83.5</td>
<td>50.4</td>
<td>50,000</td>
<td>good, spectral analyzed</td>
</tr>
<tr>
<td>20</td>
<td>STU</td>
<td>89.3</td>
<td>38.7</td>
<td>25,000</td>
<td>good, spectral analyzed</td>
</tr>
<tr>
<td>21</td>
<td>AQU</td>
<td>94.8</td>
<td>43.2</td>
<td>100,000</td>
<td>good, spectral analyzed</td>
</tr>
<tr>
<td>22</td>
<td>HNR</td>
<td>102.5</td>
<td>263.9</td>
<td>12,500</td>
<td>good, signal undetected</td>
</tr>
<tr>
<td>23</td>
<td>ATU</td>
<td>103.8</td>
<td>42.8</td>
<td>12,500</td>
<td>good, signal undetected</td>
</tr>
<tr>
<td>24</td>
<td>IST</td>
<td>105.2</td>
<td>37.7</td>
<td>25,000</td>
<td>good, signal undetected</td>
</tr>
</tbody>
</table>
where
\[ c; \text{ constant} \]
\[ a; \text{ spatial amplitude factor due to focal mechanism} \]
\[ S(\omega); \text{ amplitude spectrum at the focus} \]
\[ R(\omega); \text{ effect of transmission around the source} \]
\[ C(\omega); \text{ crustal response under the station} \]
\[ I(\omega); \text{ instrumental response of the seismograph} \]
\[ D; \text{ divergence factor due to geometrical spreading} \]
\[ Q(\omega, r); \text{ specific dissipation factor, which generally may depend on } \omega \text{ and } r \]
\[ r; \text{ distance between the ray and the center of the earth} \]
\[ V(r); \text{ velocity distribution as a function of distance from the center of the earth} \]
\[ s; \text{ ray parameter}. \]

The integration in expression (1) can be written as
Table 3. Fault plane solutions of the earthquakes and information of the stations whose records were used for analysis.

<table>
<thead>
<tr>
<th>1962.5.11 MEXICO D=25KM</th>
<th>FAULT PLANE SOLUTION I</th>
<th>FAULT PLANE SOLUTION II</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLANE A</td>
<td>PLANE B</td>
<td>PLANE A</td>
</tr>
<tr>
<td>STRIKE</td>
<td>DIP</td>
<td>STRIKE</td>
</tr>
<tr>
<td>(DEG)</td>
<td>(DEG)</td>
<td>(DEG)</td>
</tr>
<tr>
<td>BHP</td>
<td>21.2</td>
<td>109.7</td>
</tr>
<tr>
<td>GEO</td>
<td>29.2</td>
<td>07.7</td>
</tr>
<tr>
<td>CAR</td>
<td>62.5</td>
<td>97.2</td>
</tr>
<tr>
<td>BNL</td>
<td>55.1</td>
<td>57.6</td>
</tr>
<tr>
<td>TRN</td>
<td>37.7</td>
<td>64.6</td>
</tr>
<tr>
<td>ANE</td>
<td>40.7</td>
<td>138.6</td>
</tr>
<tr>
<td>LBP</td>
<td>45.6</td>
<td>135.3</td>
</tr>
<tr>
<td>TOL</td>
<td>47.7</td>
<td>50.0</td>
</tr>
<tr>
<td>STU</td>
<td>38.3</td>
<td>48.3</td>
</tr>
<tr>
<td>RNN</td>
<td>47.8</td>
<td>45.2</td>
</tr>
<tr>
<td>KFU</td>
<td>48.8</td>
<td>45.0</td>
</tr>
<tr>
<td>IST</td>
<td>159.2</td>
<td>37.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1962.5.26 GEFEC D=322KM</th>
<th>FAULT PLANE SOLUTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLANE A</td>
<td>PLANE B</td>
</tr>
<tr>
<td>STRIKE</td>
<td>DIP</td>
</tr>
<tr>
<td>(DEG)</td>
<td>(DEG)</td>
</tr>
<tr>
<td>BHP</td>
<td>21.2</td>
</tr>
<tr>
<td>GEO</td>
<td>29.2</td>
</tr>
<tr>
<td>CAR</td>
<td>62.5</td>
</tr>
<tr>
<td>BNL</td>
<td>55.1</td>
</tr>
<tr>
<td>TRN</td>
<td>37.7</td>
</tr>
<tr>
<td>ANE</td>
<td>40.7</td>
</tr>
<tr>
<td>LBP</td>
<td>45.6</td>
</tr>
<tr>
<td>TOL</td>
<td>47.7</td>
</tr>
<tr>
<td>STU</td>
<td>38.3</td>
</tr>
<tr>
<td>RNN</td>
<td>47.8</td>
</tr>
<tr>
<td>KFU</td>
<td>48.8</td>
</tr>
<tr>
<td>IST</td>
<td>159.2</td>
</tr>
</tbody>
</table>

\[
\int_{\gamma_{\omega}} \frac{ds}{Q(\omega, r) V(r)} = \frac{T}{Q(\omega)}, \quad \frac{\tilde{A}(\omega) = \tilde{A}(\omega) dD}{c \exp \left\{ -\frac{\omega}{2} \frac{t}{Q(\omega)} \right\}} \tag{3}
\]

where

- \( T \): travel-time from the source to the station
- \( Q(\omega) \): average dissipation factor along the ray

Defining \( K(\omega) \) to be the ratio of the logarithmic amplitude at two stations subscripted with \( i \) and \( j \), we then have

\[
K(\omega) = \ln \frac{\tilde{A}_{\omega}(\omega)}{\tilde{A}_{\omega}(\omega)} = \frac{\omega}{2} \left( \frac{T_{ij}}{Q(\omega)} \right) \tag{4}
\]

If we put

\[
\tilde{A}(\omega) = A(\omega) / S(\omega) R(\omega) C(\omega) \tag{2}
\]

\[
= c a D \exp \left\{ -\frac{\omega}{2} \frac{T}{Q(\omega)} \right\}, \tag{5}
\]

then

\[
\tilde{A}(\omega) = \tilde{A}(\omega) a D = c \exp \left\{ -\frac{\omega}{2} \frac{T}{Q(\omega)} \right\} \tag{6}
\]

In expression (2), \( A(\omega) \) can be calculated by
Fig. 1. Vertical component of P phases reproduced from the seismograms, with the station code, epicentral distance and magnification of seismograph. Records on the left are for earthquake No. 1 and those on the right for earthquake No. 2.
performing the Fourier transform of the waveform observed at each station and the denominator will be carefully taken into account from physical considerations of each factor. In expression (3), the denominator is common to all frequencies and will be evaluated for each ray. Then expression (4) becomes a known quantity. Fixing the \(i\)-th station and varying the \(j\)-th one, one can obtain a \(Q\) distribution with depth by solving the integral equation (6).

However, we have no sufficient data to do this because of the lack of the stations with good record over the epicentral range from about 45 to 70 degrees, then an alternative procedure is adopted. At first, the amplitude spectrums of \(P\)-waves recorded at each station are calculated, and then amplitude-distance curves are plotted for various frequencies. Compensating various propagation effects step by step, we compute \(\tilde{A}(\omega)\) and hence \(\tilde{A}(\omega)\) for the records of each station. Then, by averaging the logarithmic amplitude \(\ln \tilde{A}(\omega)\)-distance curve, we estimate \(K(\omega)\) in expression (4) and interpret \(Q\) from the standpoint that \(Q\) is dependent both on depth and frequency, and only on frequency. The former will give the general features of \(Q\) distribution in the mantle for each frequency, and the latter, though not necessarily justified, will result in the average \(Q\) values in the mantle.

In the following, the procedure for spectral analysis of the initial part of \(P\)-waves is given.

(a) Choice of the time interval of analysis, \(T_m\) and data window, \(D(t)\)

The initial part of \(P\)-waves is truncated just before a conspicuous later phase \(pP\), and the time window \(D(t) = 1/2T_m \cdot (1+\cos \pi t/T_m)\) for \(0 \leq t \leq T_m\) and otherwise \(D(t) = 0\), centered at the arrival of the initial \(P\) phases, is applied.

(b) Instrumental response \(I(\omega)\)

To compensate the frequency characteristics of seismograph, the standard magnification curves shown in Fig. 2 are used.

(c) Crustal response under the station, \(C(\omega)\)

The correction of the crustal response due to Haskell-Thomson matrix formulation, based on the standard crustal structure, may be considered to be a first approximation over the lower frequency range as treated in KURITA (1966), but this is not the case for higher frequency range. HANNON (1964) showed that thin surface layers introduce large long-period undulations into the amplitude response curve, while thick layers small short-period ones. In Fig. 3, the crustal amplitude responses of the vertical component of \(P\)-waves for the incident angle of 33 degrees are shown, which is reproduced from HANNON (1964). CAO and CATN are two and three layered crustal models, respectively, and CAME and CATK are six layered ones containing four thin surface layers, the thicknesses of which are less than 1 km. It may be conjectured from this figure that the crustal structure under the recording stations may not cause the amplitude variation much greater than several times, which corresponds to 2 in logarithmic scale among stations at any frequencies now in consideration. So long as the minute crustal structure is not known precisely, we cannot totally exclude this effect. In the present case, the amplitude spectrums are smoothed out, on the assump-
Attenuation of Short-period P-waves and Q in the Mantle

Fig. 3. Crustal amplitude response of vertical component of P-waves for an incident angle of 33 degrees to four crustal models, reproduced from HANNON (1964). CAO and CATN are two and three layered models, respectively. CAME and CATK are six layered models containing four thin surface layers, the thicknesses of which are less than 1 km.

Fig. 1. a) Comparison of crustal and mantle amplitudes at a focus of 300 km depth for a signal with a period of 0.001 sec and an amplitude of 1 m/s. The mantle response is normalized to the crustal response. The mantle response is significantly lower than the crustal response for periods greater than 0.001 sec.

The spatial amplitude factor is corrected for the P-wave radiation pattern based on the fault plane solution after HODGSON. Now, we consider a coordinate system in which the x-axis is directed north, the y-axis east and the z-axis upward. When a fault plane solution is given whose nodal planes are denoted by A and B, the source amplitude of P-waves, a can be expressed as

\[ a = \prod_{i=1}^{2} \left( l_i \sin \theta \cos \phi + m_i \sin \theta \sin \phi + n_i \cos \theta \right) \]

where \( \theta \) is the take-off angle of a seismic ray at the focus, and \( \phi \) is the azimuth of a station relating to the epicenter, which is measured clockwise from northward, and

\[ l_i = \sin (\text{dip} A) \sin (\text{strike} A), \]

\[ m_i = \pm \sin (\text{dip} A) \cos (\text{strike} A), \]

\[ n_i = \cos (\text{dip} A). \]

\( l_i, m_i \) and \( n_i \) are their corresponding values to \( l_1, m_1 \) and \( n_1 \), respectively, when A is replaced by B (MIKUMO (1963)).

(g) Divergence factor, \( D \)

If we assume that the medium has no sharp discontinuities, the divergence factor \( D' \) is expressed as

\[ D' = \left( \frac{\rho_2 V_2}{\rho_0 V_0} \right)^{1/2} D \]

\[ D = \frac{1}{r_o} \left( \frac{\sin i_o}{\sin d \cos i_o} \right)^{1/2} \left( \frac{d i_o}{d} \right)^{1/2}, \]

where

\[ r_o; \text{ radius of the earth} \]

\[ \rho_2, \rho_0; \text{ density around the focus and at the surface, respectively} \]
\( V_0, V_0; \) P-wave velocity around the focus and at the surface, respectively
\( \Delta; \) epicentral distance
\( i_0; \) take-off angle at the focus.

On assuming that \( V_0 \) and \( \rho_0 \) have the same values for all the stations, we use \( D \) instead of \( D' \). \( D \) is calculated from \((i_0 - \Delta)\) curve of RITSEMA (1958), which is tabulated in Table 3.

\section*{§ 4. Analysis}

(A) Logarithmic amplitude spectrums of the initial part of P-waves compensated only for the instrumental response \( I(\omega) \), are shown in Fig. 4 for earthquake No. 1, together with noise spectrums (dotted line) before the signals. Time interval of analysis \( T_m \) is taken to be 10.6 sec (solid line) for all the stations, and to be the same with the \( pP-P \) time (chained line) shown in Table 3, which is obtained by JEFFREYS and SHIMSHONI (1964). Difference in the spectrums between these two cases is negligible. From this figure, it
Fig. 4. Logarithmic amplitude spectrums as a function of frequency at each station, for earthquake No. 1. Solid and chained lines correspond to the signal, of which the time interval of analysis is 10.6 sec, and $pP-P$ time as tabulated in Table 3, respectively. Dotted line corresponds to noises prior to the signal.
Fig. 5. Radiation patterns of the fault plane solutions I and II, together with epicentral distances of the stations for earthquake No. 1. The amplitude of the radiation patterns are plotted in arbitrary scale for the take-off angles at the focus shown in the figure.

may be regarded that signal to noise ratio is high over the frequency range at least between 0.3 and 1.3 cps. These spectrums are smoothed over this frequency range, and the logarithmic amplitude $\ln A(\omega)$-distance curves are plotted as in Fig. 6, in which the effects due to the free surface are not taken into account. In Fig. 5, the radiation patterns $A1$ and $A2$ corresponding to the fault plane solutions I and II are shown, together with the epicentral distances of the stations. The take-off angle at the focus is taken to be 30 degrees, which corresponds to about 60 degrees of epicentral distance. With the variation in the take-off angle, the general features of the radiation pattern do not change greatly, as seen in Fig. 5(a). When the spectral amplitudes in $\tilde{A}(\omega)$ are corrected further for the spatial amplitude factor $a$, they rather scatter in the distance range, for any of four radiation patterns, compared with the case when the focal mechanism is not taken into account. As could be seen in Fig. 5, the stations cluster in the areas near the nodal lines and apart from them. We divide the stations into two groups as in Fig. 7, for the case of the radiation pattern $A1$ of the fault plane solution I. The amplitude-distance curves thus obtained generally show a gradually descending trend for each frequency. This procedure might be justified because seismic waves radiated from the source nearly along the nodal lines do not seem to obey the geometrical radiation pattern faithfully. Fig.
Attenuation of Short-period P-waves and Q in the Mantle

Fig. 6. Logarithmic amplitude $\ln A(\omega)$-distance curve for earthquake No. 1. The effects due to the free surface are not yet taken into account.

Fig. 7. Logarithmic amplitude-distance curves, compensated for spatial amplitude factor $\alpha$ for earthquake No. 1. The upper and lower curves connect the stations near and distant from the nodal line, respectively.

Fig. 8. Logarithmic amplitude $\ln A(\omega)$-distance curves, compensated further for the divergence factor $D$ and the effect due to the free surface for earthquake No. 1, for the incident angle according to Ritsema (1958). The upper and lower curves connect the stations near and distant from the nodal line, respectively. For 0.3 cps, the cases when the incident angle according to Carpenter (1966) (middle curve) and when the effect due to the incident angle is not compensated (lower curve) are also shown.

Fig. 9. Logarithmic amplitude $\ln A(\omega)$-distance curves for earthquake No. 1. Upper curves which connect the stations near the nodal line are lowered by 3 in logarithmic scale to adjust the lower curves which connect the stations distant from the nodal line.
8 shows the logarithmic amplitude \( \ln A(\omega) \)-distance curves compensated further for the divergence factor \( D \) and the effect due to the free surface for the incident angle after Ritsema (1958). For comparison, the amplitude-distance curve compensated for the effect due to the free surface for the incident angle after Carpenter (1966) (middle curve) and that not compensated for the effect due to the free surface (lower curve) are also shown for 0.3 cps. The effect due to the difference in incident angle is small. In Fig. 9, the

(a) Fault plane solution I, radiation pattern \( A_2 \).

(b) Fault plane solution II, radiation pattern \( A_1 \).

(c) Fault plane solution II, radiation pattern \( A_2 \).

Fig. 10. Logarithmic amplitude \( \ln \overline{A}(\omega) \)-distance curves for other radiation patterns of earthquake No. 1.

Fig. 11. Logarithmic amplitude \( \ln \overline{A}(\omega) \)-distance curves for earthquake No. 2. The effects due to the free surface are not taken into account.
amplitudes at stations near the nodal lines are reduced by 3 in logarithmic scale which corresponds to 20.1. This is based on the plausible assumption that near the nodal line much greater energy than expected from the geometrical radiation pattern may be emitted. Thus we obtain a rather smoothly descending amplitude-distance curve. The logarithmic amplitude \( \ln A(f) \)-distance curves corrected for the other three radiation patterns are shown in Fig. 10, all of which can be properly excluded, for amplitude-distance curves vary with epicentral distance much greater than could be compensated by the effect of crustal response as discussed in \( \text{c)} \) of § 3. Thus we see that the asymmetrical features of the radiation of seismic wave from the focus are not masked by the propagation effects, which contradicts with the results of Sutton et al. (1967) and rather favours with those of Utsu (1966) which show that the greatest part of the station difference in spectral amplitude at lower frequency results from the source mechanism.

The initial motions of the long-period records observed at the stations in Table 3 are compression except CAR and HNR, where the direction of initial motion is not known. Naturally the radiation patterns I-A2 and II-A2 are excluded, for these radiation patterns have many stations where the direction of initial motion is rarefaction. The fact that GEO is compression favours with I-A1 rather than II-A1. This fact agrees with the above results.

(B) For earthquake No. 2, the logarithmic amplitude \( \ln A(f) \)-distance curves are shown in Fig. 11, in which the effects due to the free surface are not taken into account. This figure shows that the decrease in amplitude with epicentral distance is recognizable if we take into consideration the stations beyond 90 degrees, where the effects of the core may be remarkable as will be discussed in § 5. Two radiation patterns of a fault plane solution, together with epicentral distance of the station are shown in Fig. 12. For all azimuths the initial motion of \( P \)-waves is compression, and the amplitude factors due to the radiation pattern, \( a \)'s are not small for all the stations. Hence it could not be determined which of the two radiation patterns is correct, although the logarithmic amplitude \( \ln A(f) \)-distance curves are plotted.
As may be seen in Figs. 6 or 9 and 11, the amplitude-distance curves for each frequency have almost the same shape, and especially the diminution of amplitude with the increase in epicentral distance is nearly the same amount as far as around 90 degrees. UTSU (1966) analyzed P-waves recorded by the long- and short-period seismographs, and drew the average amplitude-distance curves for high and low frequencies over the epicentral range from about 15 to 45 degrees, by the spectral analysis of the records from many earthquakes. It was noted there that the amplitude-distance curve obtained is almost the same for high (0.5-1.0 cps) and low (0.02-0.06 cps) frequencies. Recently CARPENTER et al. (1967) and CLEARY (1967) computed the amplitude-distance curve for short-period P-waves from explosions and earthquakes over the epicentral distance from about 30 to 100 degrees, and compared with GUTENBERG and RICHTER’S curve (1956) with satisfactory coincidence. If these curves are compared with ours in Figs. 6 and 11, it is interesting to see that there seems to be a similar trend, though it should be noted that for shock No. 2, there are some stations where the signals are small or scratchy before 100 degrees.

Referring to Table 2 and Fig. 1, we see that for earthquake No. 1, the signal disappears between AQU (94.8°) and HNR (102.5°), and for earthquake No. 2, it becomes small with considerably fast around 93 degrees, with sufficient amplitudes at LUB (92.3°), COR (92.4°) and ALQ (93.7°), though the magnification of the seismograph is high at AQU and ALQ. This fact survives even if the spatial amplitude factor a of the radiation pattern in Table 3 is taken into consideration. The rapid decrease of amplitude around 93 degrees may be caused by the diffraction effect and/or the phase change from the solid to liquid state around the core.

§5. Estimation of Q

We write expression (5) in the form,

\[
\frac{T_i}{Q_i(\omega)} = \frac{T_o}{Q_o(\omega)} + \frac{2K(\omega)}{\omega},
\]

where \(Q\) would be frequency independent only if \(K(\omega)\) is found from observations to be zero or proportional to \(\omega\). In Fig. 13, the logarithmic amplitude-distance curves corrected for the effects other than the spatial amplitude factor \(a\) of the radiation pattern are shown. It may be noted there that \(K(\omega)\) is approximately equal to zero and then \(T/Q(\omega)\) is nearly constant over the epicentral distance over the frequency range from 0.3 to 1.3 cps. This results in that \(Q\) is not dependent on frequency at least within this epicentral range, if the radiation pattern is not taken into account. However, as seen in Figs. 6 or 9 and 11, the diminution of the corrected logarithmic amplitude spectrum \(\ln A(\omega)\) or \(\ln A'\) with epicentral distance is almost the same as far as about 90 degrees over the frequency range now considered. This means that \(K(\omega)\) is not zero and positive constant. Thus expression (8) shows that \(Q\) must not be frequency independent. If the observational results that \(K(\omega)\) is independent on frequency can be extrapolated to the high frequency, \(T/Q(\omega)\) becomes nearly constant regardless of the value of \(K(\omega)\). This causes the interesting result that the intrinsic \(Q\) value is independent on frequency and dependent only on depth at higher frequencies. (A) Now we consider the case that \(Q\) is a function both of frequency and depth.

When the \(i\)-th station is taken fix and \(j\)-th one is transferred distant from \(i\)-th one, and then \(T_i\) and \(T_j\) are replaced with \(T_0\) and \(T\) respectively, expression (8) reduces to

\[
\frac{T}{Q(\omega)} = \frac{T_o}{Q_o(\omega)} + \frac{2b(T-T_0)}{\omega},
\]

in which \(K(\omega)\) is replaced by \(b(T-T_0)\), where \(b\) is the absolute value of the gradient of the smoothed \(\ln A\) curve and now is about 0.0037. The range where expression (8) stands may be extended over 80 degrees in epicentral distance, despite of possible diffraction effects around the core within this epicentral range, as discussed in MIKUMO and KURITA (1968), for we now consider rather shorter-period waves. Expression (9) represents that \(T/Q\) becomes greater with travel-
time, and that as frequency becomes high, \( T/Q \) has small values. From the \( Q \cdot \delta \) curve thus obtained, we can estimate a \( Q \)-depth relation for each frequency as an inverse problem. However, it may be premature to calculate this relation exactly, for we have no sufficient data. Now we only tentatively evaluate \( Q \), following above expression. We take a reference station around at 30°-40° degrees, and there \( Q_0 \) to be about 200 from Figs. 10 or 11 in MIKUMO and KURITA (1968) for each frequency now considered. At the high frequency side, \( T/Q \) has slightly ascending trend with epicentral distance, and the lower the frequency is, the more remarkable is the ascending trend. For stations around 80 degrees, \( Q \) takes values from about 250 to 400 with the increase in frequency from 0.3 to 1.3 cps. Detailed description of the values estimated is reserved.

CARPENTER (1966) found that \( T/Q \) is nearly equal to 1 for the signals recorded at teleseismic distances. This situation generally agrees with KANAMORI's (1967c) and MIKUMO and KURITA's (1968) results, which show rather descending \( T/Q \)-\( \delta \) curves. The former shows that \( T/Q \) is almost constant over the epicentral range between about 60 and 100 degrees, and takes the values between 1.0 and 1.5 over almost the same frequency range as treated in this report. The latter represents that \( T/Q \)-\( \delta \) curve is nearly constant over the epicentral range between about 40 and 80 degrees over the frequency range between 0.03 and 0.13 cps.

The fact that \( T/Q \) is nearly constant over the epicentral distances means that the intrinsic \( Q \) value becomes larger with depth, some example of which will be found in MIKUMO and KURITA (1968). Range of epicentral distance between about 45 and 80 degrees corresponds to the depth range between about 1000 and 2500 km, where the seismic ray bottoms.

The results obtained show that \( Q \) increases with epicentral distance slightly slowly, compared with the case when \( T/Q \) is constant, and that \( Q \) takes a large value with the increase in frequency. This implies that \( Q \)-depth relation in the mantle depends on frequency and that the intrinsic \( Q \) value becomes greater with depth between about 700 and 2500 km, and also with increasing frequency over the frequency range now in consideration.

If we assume that \( \bar{Q}_{i}(\omega) = \bar{Q}_{j}(\omega) = Q(\omega) \) in expression (8), it means that the average \( Q \) values along two rays to the stations subscripted with \( i \) and \( j \) are equal. This assumption is substantiated by the results obtained by MIKUMO and KURITA (1968), in which \( \bar{Q} \) at the stations around 40 and 80 degrees is identical. The above assumption is equivalent to the assumption that \( Q \) is a function only of frequency and independent of depth, which may be apart from the condition in the real earth. Under this assumption, we obtain from expressions (8) or (9),

\[
\bar{Q}(\omega) = \frac{\omega(T_{j} - T_{i})}{2K(\omega)}
\]

or

\[
\bar{Q}(\omega) = \frac{\omega(T - T_{0})}{2bK(\omega)} = \frac{\omega}{85}f \text{,}
\]

where \( f \) is frequency.

If the \( Q \)-frequency relation obtained is tentatively compared with the theoretical curve calculated by LOMNITZ (1962) from a logarithmic creep function of the form that \( \phi(t) = q \ln (1 + at) \), where \( q \) and \( a \) are constants and \( t \) is time, it becomes apparent that the general tendency of the obtained results agrees with his curve.

Although \( Q \) is generally considered to be frequency independent at least over a narrow frequency range, as reviewed by some authors (WESTON (1963), KNOPOFF (1964), ATTEWELL and RAMANA (1966)), the frequency dependence of \( Q \) has been discussed by some authors. FRANTI (1965) performed the spectral analysis of diffracted \( P_{s} \) waves from explosions observed in the distance range from about 200 to 350 km, and obtained the results that \( Q \) is nearly proportional to frequency, being equal to about 50\( f \), over the frequency range from about 1.25 to 20 cps. From the energy spectrum of records of underground nuclear explosions and small-magnitude earthquakes, SUTTON et al. (1967) deduced that \( Q \) needs to increase by about 180 with in-
creasing frequency from 1 to 2 cps, on assuming a linear frequency dependence of $Q$ to explain the observed results. They found that the contours for the total energy and the ratio of higher-to-lower frequency energy show a similar correlation with tectonic provinces. This indicates that the asymmetrical propagation of the short-period seismic energy may not be frequency dependent over the frequency range from 0.5 to 2.5 cps. SUMNER (1967) examined the relative changes with distance in amplitude spectrums (1 to 30 cps) observed at near stations shorter than about 200 km from the source, along the western flank of the Andes, and found the frequency dependence of $Q$, which is nearly equal to $100f$ within the range between 1 to 12 cps and decreases over about 15 cps. He states that a possible explanation of decreasing of $Q$ over 15 cps may be scattering rather than anelasticity. SATO (1967) performed the spectral analysis of shear waves multiply reflected from the core. On assuming $Q$ to be dependent on frequency, he showed that $Q$ increases with frequency, being from about 230 at 0.01 cps to 720 at 0.03 cps. The possible partial melting in the upper mantle or the phase change from the solid to liquid state around the core might cause the frequency dependence of $Q$. However, the possibility may remain that these results might be caused by a simplified assumption for calculation formula, impropriety of data or analytical procedure, unknown property of the constitution of the earth, local surface geology, or the impropriety of the $Q$-type loss mechanism.

§ 6. Concluding remarks

If there are some uncertainties for the results obtained from this analysis, they are as follows.

(A) In the above procedure in estimating the $Q$ values in the mantle, two smoothing steps are involved,

(i) in obtaining $K(\omega)$ by averaging the values calculated for each frequency, on the assumption that $K(\omega)$ is not a complex function of $\omega$, at least within such a narrow frequency range between 0.3 and 1.3 cps as now considered.

Thus, some subjectivity will enter in both steps.

(B) Spectral structure at the focus has been assumed to be the same for all directions. The anisotropy of it cannot be ruled out, since earthquake No. 1 is a shallow event occurred near the coast of Mexico.

(C) The fact that half of the stations for earthquake No. 1 locate near the nodal line, and some arbitrariness of the amount of the parallel reduction of amplitude spectrum at stations near the nodal line, may affect the estimation of $K(\omega)$.

(D) The divergence factor $D$ may decrease much faster than the values in Table 3, calculated from the $(i_d-\Delta)$ curve of RITSEMA (1958). However, it may not amount to such a large value that $\ln A(\omega)$-distance curve becomes nearly constant or decrease with epicentral distance, causing $K(\omega) \leq 0$.

It becomes apparent from the above analysis that the rate of diminution of amplitude with the increase in epicentral distance between about 20 and 90 degrees, does not differ for frequencies between 0.03 and 0.13 cps. This implies that short-period $P$-waves attenuate by almost the same amount with long-period ones and the diminution of amplitude does not amount to such a large value that $Q$-type attenuation theory predicts over the frequency range now considered. On assuming the $Q$-type attenuation theory to be correct, we have tentatively estimated the $Q$ values of the short-period $P$-waves in the mantle, and obtained the results that the intrinsic $Q$ value becomes greater with depth between about 700 and 2500 km, and with increasing frequency.
Thus, it may be concluded that the frequency independence of Q theory is not appropriate within the frequency range between 0.3 and 1.3 cps. The study to improve the uncertainty of the amplitude reduction, particularly for the radiation pattern, and much more data analysis will give the more detailed Q-depth and Q-frequency relations.

§ 7. Acknowledgement

The author is grateful to Dr. T. MIKUMO for the kind advice and stimulating discussions through the study. He is also indebted to Dr. J. H. HODGSON, for informing me of the fault plane solutions of the earthquakes used for the present analysis.

Seismograms were made available by the United States Coast and Geodetic Survey, and the computation was carried out on a HITAC-5020 at Kyoto University Computation Center and a HITAC-5020E at the Computer Center, University of Tokyo, to which my thanks are due.

This report was presented at the annual meeting of the Seismological Society of Japan on May, 1967.

References

ASADA, T., and K. TAKANO:
ATTEWELL, P. B., and Y. V. RAMANA:
1966 Wave attenuation and internal friction as functions of frequency in rocks, Geophysics, 31, 1049-1056.
CARPENTER, E. W.:
CARPENTER, E. W., P. D. MARSHALL, and A. DOUGLAS:
CLEARY, J.:
1967 Analysis of the amplitudes of short-period P waves recorded by long range seismic measurements stations in the distance range 30° to 102°, J. Geophys. Res., 72, 4705-4712.
FEDOTOV, S. A.:
FRANTTI, G. E.:
GUTENBERG, B., and C. F. RHICHTER:
HANNON, W. J.:
HIRASAWA, T., and K. TAKANO:
JEFFREYS, H., and M. SHIMSHONI:
KANAMORI, H.:
KANAMORI, H.:
1967b Spectrum of short-period core phases in relation to the attenuation in the mantle, J. Geophys. Res., 72, 2181-2186.
KANAMORI, H.:
KNOPOFF, L.:
KNOPOFF, L., R. W. FREDRICKS, A. F. GAGNI, and L. D. PORTER:
1957 Surface amplitude of reflected body waves, Geophysics, 22, 842-847.
KURITA, T.:
LOMNITZ, C.:
MIKUMO, T.:
1963 Mechanism of local earthquakes in Kwanto region, Japan, derived from the amplitude relations of P and S waves, Geophysical papers dedicated to Professor Kenzo Sassa.
Mikumo, T., and T. Kurita:  

Ritsema, A. R.:  
1958 $(i-d)$-curves for bodily seismic waves of any focal depths, Verhandelingen, 54, 1-10.

Sato, R., and A. F. Espinosa:  

Sumner, R. D.:  

Sutton, G. H., W. Mitronovas, and P. W. Pomeroy:  

Teng, T. L.:  

Uloomov, V. I.:  

UtSU, T.:  

Wadati, K., and T. HiroMo:  

Weston, D. E.:  
(Received May 10, 1968)