Relation between Elastic Anisotropy and Preferred Orientation of Marble

By

Karel KLÍMA and Ota KULHÁNEK
Geophysical Institute, Czechoslovak Acad. of Sci.
Prague, Czechoslovakia

The properties of rocks are determined by the properties of grains consisting the investigated sample. The majority of rocks, the elastic parameters of which were studied, showed elastic anisotropy. Our main objective here is to illustrate the relation between the preferred orientation of optical axes of calcite grains and the elastic anisotropy of marble, which was considered to be a typical monomineral rock.

The results, we are discussing, are based only on the space distribution of optical axes of 200 grains and velocities of longitudinal waves measured in 133 directions on spherical samples. The initial experimental material was taken from Babuška [1].

Diagrams (a) and (b) in Fig. 1 show the initial measured data of sample of marble S6/1 from the West Sudetes region. Diagrams were plotted by using the Schmidt's projection and the isoline system. Both diagrams show that regions of maximum preferred orientation density function correspond to regions of minimum velocity and vice versa. Diagrams (c) and (d) will be discussed later. The disadvantage of the visual comparison of both quantities in graphical representation is the impossibility of quantitatively expressing the assumed similarity. In this respect, by using the cross-correlation approach, we have improved on this.

The cross-correlation function $k(\alpha, \beta, \gamma)$ is given by

$$k(\alpha, \beta, \gamma) = \int_S \rho(\alpha, \beta, \gamma) \cdot v^{-1}(x_i) \cdot dS \quad (1)$$

where angles $\alpha, \beta, \gamma$ define the rotation of the preferred orientation coordinate system against the system of measured velocities; $v^{-1}(x_i)$ is the measured reciprocal velocity in the $x_i$ direction; $g_{ij}$ are components of transformation tensor, which are function of $\alpha, \beta, \gamma$; $S$ is the area of the reference hemisphere and $dS$ its element. The preferred orientation density function $\rho(x_i)$ was calculated from

$$\rho(x_i) = \sum_{n=1}^{N} F_n(x_i), \quad (2)$$

Fig. 1. Sample S6/1. (a)–the measured function of the preferred orientation density of calcite optical axes. Blackened parts correspond to the highest density value. (b)–spatial representation of measured longitudinal waves velocity. Velocity maxima are in the middle of the diagram, velocity minima are on the left and right side of the diagram. (c)–spatial distribution of computed velocities according to Voigt's approximation, (d)–velocities computed according to Reuss' approximation. Diagrams (a) and (b) after Babuška [1].
Fig. 2. Graphical representation of the cross-correlation function \( k(\alpha, \beta, \gamma) \), where \( \alpha \) and \( \beta \) are plotted from the centre of a circle to its edge and from the marked 0 in a clockwise direction, respectively. The system parameter \( \gamma = 0^\circ \). The area of the highest cross-correlation function value is blackened.

where

\[
F_n(x_i) = \begin{cases} 
1 & \text{if } |x_i \cdot x_i| \geq 0.84085 \\
0 & \text{if } |x_i \cdot x_i| < 0.84085.
\end{cases} \quad (3)
\]

We have defined function \( \rho(x_i) \) as a number of grains whose optical axes lie inside the cone with the \( x_i \) axis and with the spherical cap area equal to 1. According to the limited number of velocity measuring points, in Eq. (1) we must limit ourselves to summation.

Figure 2 shows the graphical representation of the cross-correlation function \( k(\alpha, \beta, \gamma) \), computed for the sample S6/1.

The problem of determining elastic constants and moduli of the aggregate on the basis of the properties of individual grains has been formulated. However, up to the present time, exact solution of the problem is still outstanding. We used the modified methods of Voigt and Reuss [2, 3]. The basic formulae are given by following expressions

\[
\star c_{ijkl} = \frac{1}{N} \sum_{i=1}^{N} c_{ijkl}
\]

\[
= \frac{1}{N} \sum_{p=1}^{N} \sum_{r=1}^{N} g_{ip} \cdot g_{jr} \cdot g_{sr} \cdot g_{it}
\]

(4)

Elastic parameters \( \star c_{ijkl} \) computed by Voigt's approximation are determined as a mean value of particular grain elastic constants \( \star c_{ijkl} \). Elastic constants of grains are constants of calcite \( c_{prst} \), which are transformed according to the position of grain in the sample.

The same procedure has been applied for elastic moduli computation by using Reuss' approximation. Voigt's and Reuss' approximation determine the upper and the lower limit of real elastic parameters, respectively. The comparison of measured and computed velocities according to both approximations
Fig. 4. The cross-correlation function $k(\alpha, \beta, \gamma)$ for the sample M38a. Before oil saturation (a)-for $\gamma=0^\circ$, (b)-for $\gamma=180^\circ$. After oil saturation (c)-for $\gamma=0^\circ$, (d)-for $\gamma=180^\circ$.

is shown in Fig. 1. It is obvious, that the course of isolines in diagrams (c) and (d) is simpler, than in diagram (b), but it indicates the basic character of the velocity spatial distribution.

In comparison with the measured velocities the values of the computed velocities are in average 1.4% higher for Voigt's approximation and 5.9% lower for Reuss' approximation.

The velocity spatial distribution in the sample of marble M38a from Kutná Hora crystalline complex was considerably influenced by small pores and intergranular spaces. To avoid this effect the sample was saturated by oil in vacuum. Measured and computed data of the sample M38a are shown in diagrams in Fig. 3. The mean velocity increases by oil saturation from 4.14 km/sec to 6.38 km/sec and the coefficient of anisotropy decreases from 28.8% to 9.1%.

In comparison with velocities measured after the oil saturation the Voigt's method yields results in average of 4.6% higher and the Reuss' method of 2.9% lower. The computed coefficients of anisotropy are 8.1% and 8.5%.

As shown in Fig. 4 the course of the cross-correlation function of the preferred orientation density function and measured velocities changes in details only.

By the procedure described, further three samples were computed, with analogous results. Thus we suppose, that the procedure can be used for determination of parameters of monomineral rocks.


References


(Received Nov. 16, 1968)