Model Study on Core-Mantle Boundary Structure

By

Hideki SHIMAMURA

*Geophysical Institute, Faculty of Science,
The University of Tokyo*

Abstract

Various approaches have been explored to elucidate the nature of the core-mantle boundary, such as its position, physical shape, and the distribution of physical constants. In this study these investigations are reviewed and the possibility is examined to study the unknown factors of the core-mantle boundary characteristics when more abundant seismic data become available in the future.

In order to determine the physical constants of the lower mantle close to the core-mantle boundary, it is quite important to investigate the effect of diffraction due to the core. An attempt to obtain Q distributions in the mantle by taking spectral ratio of direct waves at different stations has been made. If the effect of diffraction is unknown, however, it is impossible to make correction for observed spectral amplitudes to find correct Q values in the lower mantle. To elucidate the nature of the core-mantle boundary, other method for obtaining the attenuation function along the boundary by taking spectral ratio of the core-diffracted waves has been developed. For this case, it is necessary to know the attenuation function for a wide variety of the core-mantle boundary structures which are suggested from other evidences.

To eliminate the effect of diffraction, it is promising to analyze the spectra of waves reflected at the core boundary. But even for this case, it would be required to solve the problems of determining the reflection coefficients when spherical waves are incident onto spherical boundary having the gradual change of physical constants.

In the present paper, the model studies of these bottlenecks are planned. First, the precise diffraction pattern by an impenetrable sphere is obtained from sonic wave experiments. Second, the difference between the diffraction patterns due to the various structures of the core-mantle boundary is studied by use of two-dimensional models which have core-mantle boundary structures with continuous velocity change, as well as irregular boundary structures. The difference between the diffraction patterns is found to be too large to make the *diffraction compensation* to the spectral ratio of the direct waves possible with an aim of getting Q values in the lower mantle.

The attenuation functions of the diffracted waves in the shadow zone are also obtained by ultrasonic model experiments and compared with seismic data. The results show that a kind of *irregular* core-mantle boundary structure having a thickness of 30 to 100 km accounts for the seismic data.

PcP spectra are generally insensitive to the structure with continuously changing velocities and other physical constants, while they are sensitive to the layered structure having discontinuous boundaries. This result suggests the possibility to discriminate whether or not there is a layered structure at the core-mantle boundary by use of the spectra of PcP .

§1. Introduction

(1) *Historical review on the studies of core-mantle boundary*

Desire to know the nature of the core-mantle boundary emerged with the discovery of the liquid-core in the solid earth. But progress in the study about the parameters

of the boundary such as its position, physical shape, distribution of physical constants, and its evolution was not remarkable, mainly because of the limited quantity and the poor quality of seismic records. Even the exact size of the core is still a debated question.

At the early stage of the study of core-

mantle boundary direct P waves were usually used in the time domain. The studies by Gutenberg (1914), Jeffreys (1939), Dahm (1936) and Macelwane (1951) were intended to determine core size by use of the Herglotz-Wiechert solution. Lehmann (1953) and Dahm (1936) proposed a low velocity layer at the base of the mantle to explain their observational or computational results. But their interpretations were rather speculative because of the limitations in the theory of diffraction and the inadequate observations.

Sacks (1966) examined the amplitudes of short period direct waves near the shadow boundary and discovered a striking fact that the amplitudes of P waves with the periods of 2.5 and 0.7 seconds decay sharply from epicentral distance of 96° . The epicentral distance obtained by Sacks is 8° (Gutenberg and Richter) to 9° (Jeffreys) shorter than the generally supported values for the shadow boundary.

The studies of the core-mantle boundary by use of direct P waves in the time domain have their own inherent limitations, because P waves are very sensitive to the local geology under observing stations, source mechanism, source propagation, scattering of waves, geometrical spreading and instruments, and also the measurement of $dT/d\Delta$ from travel time is difficult in the vicinity of the shadow boundary (Herrin et al., 1968).

Core-reflected waves were also used to study the nature of the core-mantle boundary. Among the studies by this method, the determination of core size by use of travel times of the core-reflected waves were most popular (Vogel, 1960; Lucke, 1961), which was greatly promoted by nuclear explosions because of their accurate origin times and hypocenters (Carder and Bailey, 1958; Kogan, 1960; Jeffreys, 1962; Carder, 1964; Buchbinder, 1965; Carder et al., 1967; UED, 1966; Taggart and Engdahl, 1968).

In these studies observed travel times are generally smaller than J-B tables. This means the increase of the core size when velocity structure of Jeffreys-Bullen is assumed. But this result critically depends on the velocity

model adopted in the computation of the core size. The difficulty to obtain a precise and correct velocity structure throughout the mantle limits the effectiveness of the PcP travel times studies to know the core size and to study the fine structure of the core-mantle boundary.

No investigation to estimate the sharpness of the core-mantle boundary quantitatively had been made before Kanamori (1967b). His results show that when thickness of the transition zone where V_p , V_s and density are linearly changing exceeds 1 km, theoretically convolved PcP waveform is too much distorted compared with the waveform observed by TFO short period seismometers. These results, however, do not reject the possibility that minor discontinuity or some kinds of core-mantle boundary structures are superposing upon the major discontinuity supported by him.

A study of free oscillations is also useful to investigate the nature of the core-mantle boundary. Dorman et al. (1965) intended to interpret the discrepancy in the periods of free oscillations discovered by Alsop (1963) where the observed periods of spheroidal oscillations of order numbers 3 to 9 and those of the torsional oscillations of orders 3 and 4 were longer than computed ones for Jeffreys-Bullen A model. Dorman et al. obtained "a family of" models which account for seismic observational data. The most preferred one has a "soft" layer with a thickness of 30 km at the base of the mantle where V_p and V_s are reduced by about 25%.

Bullen and Haddon (1967b) also studied the periods of the free oscillations and constructed a model of the earth. They did not mention about the core-mantle boundary structure, though the model has a larger core size by 15 to 20 km than that of Bullen A' model (Bullen, 1965). Jeffreys suggested that the postulated increment of core size proposed by Dorman et al. and Bullen and Haddon would be reduced by introducing the effect of anelasticity of the mantle. For the structural study of the core-mantle boundary, however, this method of utilizing free

oscillations is not fully helpful, because of its limited capability of resolution for the fine structure.

The travel times of PKP were studied by Ergin (1967) and a model of the core-mantle boundary structure was proposed in order to explain observed travel time differences from the J-B tables. His model has a low velocity layer at the base of mantle. One of problems in Ergin's research open for future investigation is the travel time observation of PKP_2 , because $dT/d\Delta$ of PKP_2 branch is very steep and PKP_2 is not an initial phase on seismograms, which makes travel time study of PKP_2 difficult. To put a low velocity layer at the base of mantle causes the shadow boundaries of P and PKP stretched to a large extent. But the determination of shadow boundary from amplitude observation cannot be easy because of the effects of the source, the crust, geometrical spreading and diffraction.

In general, the study of core-mantle boundary in the time domain demands precise knowledge about velocity structure over the whole part of the mantle. This is a severe restriction for the progress of the study. On the other hand the study of the boundary in frequency domain promises the possibility of finding the nature of the core-mantle boundary without strictly knowing source function, transmission function along the ray paths or crustal transfer functions. Meanwhile an approach in frequency domain often lacks in quantification or uniqueness of the solutions.

Teng (1966, 1968) intended to study the Q distribution in the mantle by taking spectral ratio between Fourier amplitude spectra of direct P waves observed at different stations. Although this method for studying the detailed Q structure is fancy, it is difficult to obtain the Q structure near the bottom part of the mantle by this method. One of the reasons is that the problem of diffraction due to the core in illuminated zone has not been solved. The diffraction possibly changes the spectra of the rays penetrating the deeper part of the mantle. Another reason comes from the nature of the attenuation coefficient

Q . As the contribution of Q in the deeper part to the total attenuation is small, precise observations are necessary in order to study the Q structure in the lower mantle.

Waves diffracted by the core were also utilized for the study of the core-mantle boundary. Alexander and Phinney (1966) and Phinney and Alexander (1966) studied the amplitude decay of core-diffracted waves in the shadow zone in the frequency domain intensively. The attenuation functions along the core-mantle boundary calculated from spectral ratio of diffracted waves obtained by Alexander and Phinney bore a general resemblance to those calculated theoretically by Scholte (1956) for a perfect liquid core. But the functions from seismic records have many peaks and troughs, being different one another among the functions obtained from different regions of the core-mantle boundary.

Phinney and Alexander intended to interpret peaks and troughs in the attenuation functions by computing frequency responses due to core-mantle boundary structures which have a single homogeneous layer intervening between perfectly elastic homogeneous mantle and homogeneous liquid core. But they could not find a decisive model because of the limitation in theoretical computation.

(2) Method of the present study

Three methods are selected as most effective to investigate the core-mantle boundary after the various methods have been reviewed and the possibility is examined to clarify the unknown factors when more abundant seismic data becomes available. It is necessary to subdue the bottlenecks of each method.

The first method is aimed to study Q values in the bottom part of the mantle by taking spectral ratio of the direct waves observed at different stations. This method was used by Teng (1966, 1968) first and then by Hirasawa and Takano (1966) and by Mikumo and Kurita (1968). The possibility of the existence of a low Q zone at the bottom of the mantle was suggested by Teng, and Mikumo and Kurita, but this possibility cannot be confirmed before quantitative study of diffraction due to the core in the illuminated zone.

Thus it is urgently necessary to solve the diffraction problem due to a sphere over a period range from about 5 to 100 seconds and over a spatial range from illuminated zone to shadow zone, and of course with the accuracy being good enough to compute the correction term due to the diffraction on the spectral ratio obtained from seismic data. This diffraction problem has not completely been solved theoretically as yet.

The second method which is considered to be effective for studying the problem on the core-mantle boundary is to investigate the attenuation function along the core-mantle boundary by taking spectral ratio of the core-diffracted waves. To study the nature of the boundary structure by use of the attenuation functions obtained from seismic data is one of inversion problems. In order to solve this inversion problem of the core-mantle boundary structure, however, it would be necessary to know the attenuation functions expected for a wide variety of the core-mantle boundary structures which were suggested to be possible by other geophysical evidences. To compute the attenuation functions theoretically for these various core-mantle boundary structures is too complicated.

The third method is to study the spectra of core-reflected waves. This method also has weakpoints which have not yet been solved theoretically. The problem is to determine reflection coefficients when spherical waves are incident onto a spherical boundary which, in some cases, has a gradual change of physical constants.

In the first stage of the present study, an experimental investigation of diffraction due to a sphere is made. This diffraction experiment has to satisfy the condition in the accuracy on the amplitudes of diffracted waves and to cover a certain frequency range which corresponds to the range in the practical observations. This experimental study would give suggestion to the first and second methods of investigations mentioned above to know the unknown factors of the core-mantle boundary.

In the next stage, various models of core-mantle boundary structures are designed to study the waves which are transmitted, reflected or diffracted by the structures. In the models an emphasis is placed on the probable core-mantle boundary structures, which had not yet been handled because of the limitation of the resolution of seismic data due to their quality or the limitation of ability of mathematical computations.

Mathematical models used in numerical computations of the core-mantle boundary structures have been oversimplified in almost all cases, such as homogeneous single layer or horizontally stratified homogeneous layers. Therefore there remain some problems open to question in the effectiveness of these mathematical treatments.

In the present study two-dimensional ultrasonic models are used to study the waves propagating through the model structures. Model structures are those which have continuous distribution of physical constants and those having some kinds of irregularities which delineate some probable models of intrusion of core material into the bottom of the mantle. All these models are too complicated to treat mathematically, but at the same time it is an urgent requirement to know the characteristics of wave transmission for these models.

§ 2. Experimental setup of the model study

(1) *Apparatus of three-dimensional diffraction experiment*

The development and the diffusion of digital computers have been brought pressure to bear upon model studies. The area of field in which model study is decidedly effective has been rapidly becoming smaller. Model study in seismology is not an exceptional case though the restriction of scale factors is not severe compared with that of other fields such as in geological field in which many physical constants including breaking strength are demanded to be proportional to those of real materials. In the field of seismological model study the research workers should possess an ingenious strategy in order to

subdue the difficulties which have been thought inherent for model study, if they want to make the model study a powerful tool in seismology.

The merits of model study in seismology are the capability to obtain complete solutions without any approximation, simplicity in experiment, visual monitoring of records, capability to use any shape of model or any structure and to choose any recording place and so on. On the other hand model study has many weakpoints such as the lack of accuracy concerning amplitudes or wave forms, the difficulty of analogy in the scale factors, the limitation in choosing physical constants and the latent danger of contamination due to undesirable factors in the experimental processes.

In the first stage of the present study, the experiment of diffraction due to a sphere is planned. The accuracy within 1% in measuring the amplitudes is necessary in this experiment. This restriction is hard to be attained in ordinary two-dimensional ultrasonic model experiments. The frequency range of the experiment has to cover those of seismic waves after multiplication of scale factor, in other words the value of ha (h is wave number of compressional wave and a is a radius of diffraction sphere) has to cover from 15 to 150.

After preliminary experiments, three dimensional experiment by use of acoustic waves in air is found to be best. Because the problem of coupling between model and receiver which is the most troublesome problem of amplitude study can be avoided by using acoustic waves in the air. Small attenuation in air for acoustic waves also favors this experiment.

The acoustic impedance of air is so small that almost any solid material will serve as a rigid body for acoustic waves in air. A terrestrial globe with the diameter of 30 inches (76.2 cm) is chosen as a diffraction obstacle. The scale factors of this kind of experiment would be properly determined by the equality of the value ha .

The block diagram of this experiment is

Fig. 1. Block diagram of the three-dimensional model experiment.

shown in Fig. 1. Source transmitter is an audio tweeter with an aperture in front of it. Horn type tweeter is best suited over the necessary frequency range in the viewpoint of transmission efficiency from electrical to acoustical energy, electrical impedance, and wave distortion. The aperture is a circular hole whose diameter is 5 mm cut in styrofoam plate with the thickness of 6 cm. The directionality and the efficiency are examined to be satisfactory. The examination of amplitude-distance relation shows that recorded amplitudes are inversely proportional to the distance between transmitter and receiver. This result means that spherical waves are emitted from the source.

So-called tone burst waves are transmitted, which improve the accuracy of the recorded amplitudes and the enhancement of signal-to-noise ratio. The length of the wave train does not play an important role in the result, because travel time difference between P and PcP are shorter than half period of the highest frequency wave used in this experiment over the entire receiver positions concerned.

Receiving transducer must have a wide directionality, small dimension and high conversion efficiency from acoustical to electrical energy. Therefore an audio crystal receiver

which is modified to have directionality more than 90° is chosen as the receiving transducer. Filters with narrow pass band are used. To coincide central frequency of the filter with the frequency of the finite length sinusoidal wave emitted from the source transducer enables the direct measurement of amplitudes on the oscilloscope with high precision.

(2) *A model having velocity gradients*

Two dimensional model experiments are designed to elucidate the difference of diffraction patterns due to various core-mantle boundary structures and to examine the spectra of waves which are transmitted, reflected or diffracted by core-mantle boundary structures.

As stated previously one of the weakpoints of model study is the limited selection of physical constants which is restricted by the materials of model. Some ideas have been proposed by various authors to control physical constants, especially to obtain increment or reduction of elastic wave velocities. Healy and Press (1960) made a laminated plate of aluminum and plastic. Elastic wave velocities in the laminated plate are a kind of the weighted mean values of the velocities of aluminum and plastic, for waves having large wave lengths compared with thickness of the plate. Therefore by varying the thickness of each material a wide range of elastic wave velocities can be obtained. But this method has not been widely used thereafter because of the scattering of waves, the separation of short period waves which pass through the plate with elastic wave velocity of each material forming the model, or the great difficulty in making the model with high precision of thickness and with tight and homogeneous adhesion.

On the other hand in USSR the reduction of elastic wave velocities by making many holes in two-dimensional model was explored (Ivakin, 1960; Ivakin and Vasil'ev, 1963; Ivakin and Aver'yanov, 1963). Ivakin and Vasil'ev suggested the possibility of reduction of V_p and V_s to the extent of 50% when total area of small holes was 64% of the model. Ivakin and Aver'yanov made crustal

model having velocity gradient by varying the density of holes.

But this method also has weakpoints such as the scattering of waves, occurrence of peaks and troughs in spectra of waves caused by resonance due to the structure, or unapplicability to the short period waves whose wavelengths are shorter than 8 to 10 times of spacing of holes.

Another attempt to reduce the velocities of elastic waves is heating of model material. Although velocities in metals are hardly changed by heating except in the vicinity of melting point, velocities in high polymer chemicals generally show large change with temperature because high polymer chemicals have many kinds of bonds of chain molecules which possess various releasing temperatures of each bond. In the field of chemical engineering temperature dependence of physical constants of high polymers has extensively been investigated to know the nature and mechanism of high polymer chemicals.

In this study velocity gradients are necessary in order to make structures of the core-mantle boundary. Various methods are tested and at length method of velocity reduction of high polymer chemicals by heating is chosen. The points of excellence of this method are ability of velocity reduction over wide frequency range compared with other methods, which is much important in data processing in the frequency domain, and capability to form many kinds of models from an original one by varying velocity gradients by the variety of heating method. Moreover, this capability to change velocity structure makes the experiment possible in which source and receiving transducer are fixed to the model while the velocity structure is changed. Therefore we can overcome the inevitable bottleneck in seismic model study on amplitudes and spectra provoked by the problem of contact between transducer and model.

On the other hand this method of controlling physical constants cannot avoid having its own demerits. One of them is inherent for this method, which is mutual dependence

among physical constants. For instance to determine elastic wave velocity means to determine density, elastic constants, attenuation coefficients and so on. Another one is the difficulty in reproducing the experiment accurately after long time elapsed, say 1 month, mainly because of difference in room temperature. But these demerits are not fatal in the present research. Moreover we can even make use of these weakpoints, for example, we can monitor all physical constants in a model by monitoring only one physical constant or the temperature. For another example the wide variety of models are able to be made by the utilization of variation of temperature circumferential to the model.

This example suggests moreover the possibility to make use of the temperature field which has been made as a means of making a certain model and to add a new field of temperature, which means the utilization of transient state, after all.

Selection of the material of model is an important problem. The criteria of the choice of the material in this experiment are in the first place the range of the variation in physical constants especially in elastic wave velocities and the magnitude of temperature derivatives of physical constants. It is necessary that the attenuation of waves is as small as possible, because high polymer chemicals has generally large attenuation coefficients compared with metals. Other criteria are specific heat, water absorption constant (because of the stability of the experiment in long experimental run), homogeneity (high polymer chemicals are apt to be inhomogeneous because of the differences of polymerization grade) and so on. Various measurements on the physical constants of high polymer chemicals had been made mainly in the field of applied chemistries, but even if the chemical names are identical, their physical constants are usually not the same because of the difference in the polymerization grade or in production processes.

The plates of polymethyl methacrylate (plexiglass) are adopted finally. Although

the derivatives dV_p/dT are larger in 6-nylon, polyvinyl chloride and celluloid than that of plexiglass, all of these higher polymer chemicals have large attenuation coefficients. On the other hand polystyrene and phenol resin possess smaller attenuation coefficients than plexiglass, but their dV_p/dT are small. These materials may be suitable for other type of model experiment. For instance with increasing temperature the attenuation coefficient of polyethylene decreases while in almost all other high polymer chemicals the attenuation increases. Therefore positive velocity gradient with positive attenuation gradient is attained by use of polyethylene as a model.

A testpiece is cut out of the model material of polymethyl methacrylate plate and measurements of elastic wave velocities and attenuation in plate are examined in thermostat. In the testpiece source and receiver transducer made of lead-zirconate-titanate whose curie point is more than 200°C and a thermister are buried. The small size

Fig. 2. Temperature dependence of compressional wave velocity, density, amplitude and Q value in the model material.

options are intended, which consist of time process controller, pulse generator, pulse amplifier, source and receiver transducers, signal amplifiers, filters, integrators, oscilloscope and photographic equipments, as well as the calibrators of amplitude and timing and attenuators. These apparatus are designed to perform their functions for data processings in the frequency domain. For instance the signal preamplifier is designed to be put emphasis on wide frequency range (1 kc to 2 Mc), high input impedance (80 M Ω at 1 kc), low noise (15 μ V_{p-p}), high gain (47 dB) and wide dynamic range (50 dB) as well as sufficient phase characteristics and small distortions, which are attained by use of FET (field effect transistor) and IC (integrated circuit).

In this study two kinds of models are used, one is circular model (referred to as type-1 model) and the other is rectangular (referred to as type-2 model) (Fig. 3). The circular model is comparable directly with the real earth, while rectangular model is superior to the circular model in the enhancement of the signal-to-noise ratio of the diffracted waves or in separation of effects due to the core-mantle boundary which will be stated later, and also in easiness to make up the models. The models of the core are hollow in both models because of complication of experimental results due to the increase of parameters and of limitation in the variety of model materials. Therefore in this experiment stresses are free at the core-mantle boundary.

The size of the model is determined by the factors of physical constants, frequency characteristics of transducers, dimension of transducers and so on. The source transducers buried in the models are non-directional cylindrical transducers specially made for this model experiment (Shimamura and Sato, 1965), whose diameter is 5 mm. Small receiving transducers are put on the edge of models and one transducer is buried in each model as a monitor. The influence on elastic waves propagated in models caused by solid-solid contact of receiving transducer which was indicated by Schwab (1967) is examined

and found that the receiving transducer does not have bad influence on the experimental results in this research. The directionality of the receivers is wide enough to be neglected in the frequency range under the present study.

Type-1 model is made for studying standard diffraction pattern caused by homogeneous mantle without any structure at the core-mantle boundary. On the other hand type-2 models are aimed at making the models of core-mantle boundary structures. Only the models of the structures which have possibilities to exist in the real earth are adopted, in other words, which have been suggested hitherto or are thought to be probable as the models of the intrusion of core material into the mantle. Seven type-2 models are made. First model is referred as *OA MODEL*, which has homogeneous mantle. Experiments are doubly done in this model before and after the Ni-Cr heating ribbon is attached to the outer edge of the model core boundary. The results make it clear that there is no detectable difference between these two experiments. The name *OA* comes from the heating current in Ni-Cr ribbon (for this case, zero ampere).

Second, third and fourth models are *2A*, *3A* and *MB MODELS*, respectively, whose names indicate also the quantity of heating current in ampere. All these models are experimented in stationary states, which are judged as stationary when V_p values determined from measurements of thermister array are settled within 0.5% of fluctuation. The stationary state in this sense usually continues from about 20 minutes after switching on the current in the Ni-Cr ribbon to more than 2 hours. The distributions of the physical constants in these models are shown in Fig. 4, which indicate the velocity gradients and the gradients of Q value toward the bottom of the model mantle. The values of V_p and Q in these models are continuous for more than second order derivatives.

Remaining three models are models of partial melting or the intrusion of core material into base of the mantle, and at the same time these structures fill the role of effective high

Fig. 4. Distributions of temperature, compressional wave velocity and Q value in the type-2 models. 4A, 3A and 2A models are illustrated.

attenuation zone (Fig. 5). *MB MODEL* has many slits whose length is 30 mm that is equivalent to 5.3% of the core radius and whose width is 1 mm along all over the core-mantle boundary with intervals of 60 mm. *CB MODEL* has core-mantle boundary structure consisted of rectangular irregularities whose height is also 30 mm and whose wavelength is 120 mm, that is 3.4% of the circumference of the core. In order to research the relation between standard model (*OA MODEL*) and *MB MODEL* or *CB MODEL*, *AB MODEL* is made which has single slit whose length is 30 mm and whose width is 1 mm on the middle of the grazing ray on the core. (*MB MODEL* is based on *AB MODEL*, and *CB MODEL* is made from *MB MODEL*.)

About 300 photographic records of P waves (or diffracted P waves) were enlarged and digitized for computations of Fourier spec-

Fig. 5. Configurations of *MB MODEL*, *CB MODEL* and *AB MODEL*.

trums. In the vicinity of geometrical shadow boundary in the illuminated zone, the discrimination between P and PcP waves is essentially impossible and at the same time the interference of P and PcP plays an important role for the diffraction in the illuminated zone. On the other hand the elongation of apparent period due to the diffraction and the attenuation of the model material is covered by the data window, while no later phase arrives in this interval. Because of these reasons, the length of the data window is kept constant throughout the records. In type-1 model the window is $83.33 \mu\text{sec}$ in length and the interval of digitizing is $0.333 \mu\text{sec}$. In type-2 models the former is $81.97 \mu\text{sec}$ and the latter is $2.05 \mu\text{sec}$. The high cut filter to prevent

aliasing is not necessary since the model material has the nature of low pass filter.

The data window applied to the digitized records is

$$D(\tau) = 0.54 + 0.46 \cos \frac{\pi \tau}{T_m},$$

which is determined through the comparison of the results of the various types of windows. Fourier spectra are computed and at the same time plottings and punch out on FORTRAN data cards is accomplished.

A set of records is obtained using the type-2 model where temperature gradient in the model is varied while source and receiver transducers are fixed to the model. The records of this experiment are digitized and the same data processings as stated above are applied to them.

PcP experiments are also accomplished on slightly modified type-2 model having the

thermister array to measure the distribution of temperature. The structures are given by heating or cooling the model which included the transient state on heating up or on cooling down processes that are monitored simultaneously by use of the thermister array.

The *PcP* records obtained from the models of 28 different structures are photographed, and among those records the records of seven typical structures are picked up for spectral analysis. Fig. 7 illustrates these seven velocity structures. The structure *E* has large velocity gradient caused by large temperature gradient because this model is obtained in the acceleration process from *D* to *F*. On the contrary the structure *G* has quite a different shape in velocity structure compared with others because of the nature of exponentially decaying temperature in the cooling process.

Fig. 6. Examples of Fourier amplitude spectra computed from the records of 0.850 model. Each computer plotting in which linear scale in amplitude is taken is normalized at the maximum value. Numerals indicate epicentral distances.

Fig. 7. Velocity structures of seven models used in *PcP* experiments. Abscissa is distance from the core-mantle boundary. Model A has homogeneous mantle. Model B is the structure obtained after 4 minutes from switching on of 1 Ampere current in the Ni-Cr heating ribbon. Model C is the state 6 min. after starting of 2A current from stationary state of 1A heating. D indicates the state got 6 min. after increase of current from 2A to 3A. E and F is obtained after 2 and 9 min. respectively on heating up by 4A current from the stationary state of 3A. G is the structure corresponds 2 min. after the 4A current is cut from the stationary state of 4A heating.

§ 3. Results and discussion

(1) Results of the experiments on diffraction by use of acoustic waves

In the experiment on the diffraction around

Fig. 8. Diffraction patterns obtained through three-dimensional model experiment.

a sphere, the source transmitter, the top of the diffraction sphere and the origin of coordinates of receiver position are placed on a horizontal plane. The receiver is moved along a vertical line. The receiver position is indicated by distance from the origin of coordinates in cm. Positive values in the coordinates indicate that the position of receiver is in the shadow zone.

The amplitudes are normalized to the value observed at the origin of the coordinates when the sphere is removed, therefore amplitude at the origin when the sphere exists is not unity. The distance from source transmitter to the top of the sphere is 58.0 cm, and that from the top of the sphere to the origin of coordinates of the receiver position is also 58.0 cm. This geometry corresponds to the model earth with homogeneous mantle whose radius of the core is 0.545 times the radius of the supposed surface, when the receiver is at the origin of coordinates.

Precise diffraction patterns are obtained from this experiment, which are shown in Fig. 8. The values of ha , 16.6, 34.6, 66.5 and 133 correspond to the experimental frequencies of 2.5, 5.2, 10.0 and 20.0 kc respectively, and also correspond to the periods of 96.3, 46.3, 24.1 and 12.0 sec in the earth respectively, assuming V_p in mantle to be 13.6 km/sec. Errors and fluctuations in measured

amplitudes are very small as is expected.

These results confirm the existence of diffraction in illuminated zone and suggest that to determine shadow boundary by amplitude observation is very difficult in these frequency range. It is found that as the frequency becomes higher, amplitudes begin to decrease at the points nearer to the geometrical shadow boundary and the decrease of the amplitude becomes faster either in illuminated zone or in shadow zone. The normalized amplitudes exceeding 1.0 is the results of interference of P and PcP waves.

The diffraction patterns obtained from the model experiments are converted to the values expected on the supposed spherical surface of the model earth by computing geometrical spreading factors. Also the coordinates of the receiver position are converted in the differences $[A - A_s]$ (degree). The epicentral distance of the geometrical shadow boundary (A_s) is 113.5° . Results are shown in Fig. 9.

The curve fitting of these diffraction patterns is done by the experimental formula,

$$U = F \exp [-G(A - A_s)] \quad (1)$$

where A is the epicentral distance of the observing station (A and A_s are in radian). F relates to the diffraction amplitudes at the geometrical shadow boundary, while G relates

Fig. 9. Diffraction patterns converted to the values on the spherical surface of the model earth.

to the inclination of diffraction curves. The values of G are obtained in the illuminated zone and in the shadow zone separately, and computed through weighted mean of $1/|A - A_s|$ by use of the values of diffraction amplitudes within the range where the amplitudes do not depart largely from linear curves in Fig. 9. The idea of the weighted mean comes from the importance of the behavior of the diffraction amplitudes in the vicinity of the geometrical shadow boundary for this study.

Fig. 10 and Fig. 11 are the plottings of the values of F and G against ha , respectively. The F values obtained in the present study

are accurately expressed by

$$F = 1.094 (ha)^{-0.098} \quad (2)$$

The values of G in the shadow zone are also expressed by

$$G = 0.466 (ha)^{0.477} \quad (3)$$

G values in the illuminated zone, which are obtained from linear part of the amplitude curves in Fig. 9, yield

$$G = 0.306 (ha)^{0.597} \quad (4)$$

though these values of $\log G$ are not lined

Fig. 10. Coefficient F in the experimental formula (1).

Fig. 11. Coefficient G in formula (1). Sato (shadow zone) is $G = 0.5452(ha)^{0.4782}$ determined by the least squares over ranges from $ha = 20$ to 200 . Sato (illuminated zone) is $G = 0.306(ha)^{0.597}$.

up well as $\log G$ values in shadow zone or $\log F$ values.

There remains possibility that the logarithm of diffraction amplitude patterns in illuminated zone are asymptotic to the horizontal line with the increase of $|A - A_s|$, on account of the shapes of diffraction patterns and the larger scatters of G value in illuminated zone than that in shadow zone in Fig. 11. In any case, the amplitude decay in the shadow zone is faster than that in the illuminated zone. The decaying rate of the diffraction amplitudes in deep shadow becomes small, but this phenomenon is not thoroughly investigated since it is not an object interested in the present study.

As is shown in Fig. 10 and Fig. 11, $\log F(ha)$ and $\log G(ha)$ obtained from the experiments generally stand well on lines. This fact suggests that these experimental formulae is applicable to wider frequency range than in the present experiment.

(2) Comparison with other investigations

The diffraction due to a spherical obstacle has been a matter of concern for a long time and many studies have been attempted to solve the problem. But there have been no complete solution in the problem on the diffraction due to a sphere for all over the spatial zones across the geometrical shadow boundary and over frequency range which is wide enough to cover necessary frequencies in seismology. (Watson, 1918; Bremmer, 1949; Takeuchi, 1950; Scholte, 1956; Nagase, 1956; Duwalo and Jacobs, 1956; Knopoff and Gilbert, 1961; Rubinow and Keller, 1961; Nussenzweig, 1965; Phinney and Alexander, 1966; etc.)

Recently extensive studies were intended by several authors. Among these studies, Sato (1968b) and Sato (1969) made a remarkable progress in the computation of the diffraction problem. Phinney and Cathles (in press) also succeeded in obtaining diffraction patterns over the entire zones of interest by computing directly the diffraction integral by use of numerical integration. Two-dimensional diffraction experiment was done by Teng and Wu (1968) on aluminum model with a hollow core whose experimental

method was similar to that of Knopoff and Gilbert and diffraction patterns of P and S waves were obtained.

But in Sato's study the computation was made in the frequency domain, therefore phases separable in the time domain could not be independently identified, such as diffracted P wave and core phases which passed through the core (PKP , $PKKP$, PKS , etc.), or diffracted P and diffracted P which had gone round the core. For this reason diffraction amplitudes in the case of liquid core obtained by him showed rugged curves which seemed to be unnatural. Fig. 10 and Fig. 11 also indicate Sato's results of F and G values for the case of a hollow core, though definitions of F and G are slightly different from this study.

The computation of Phinney and Cathles is also not so complete since it needed some kinds of short period approximation and can not eliminate diffracted waves travelled on the opposite side of the core.

(3) Diffraction compensation to elucidate Q structure

Average Q values along the ray are obtainable by taking spectral ratio of direct P waves. Fourier amplitude spectra $A(\omega)$ of a seismic record are expressed as

$$A(\omega) = A_0(\omega, \theta, \phi) A_{\text{crust}}(\omega) A_{\text{inst.}}(\omega) D_d D_f(\omega) \exp \left[- \int k(\omega, s) ds \right] \quad (5)$$

where

- $A_0(\omega, \theta, \phi)$; source function,
- $A_{\text{crust}}(\omega)$; crustal response at a recording station (equal to unity in this model study),
- $A_{\text{inst.}}(\omega)$; instrumental response (in this model study this is a constant of amplifier gain setting),
- D_d ; divergence factor due to geometrical spreading (independent of frequency),
- $D_f(\omega)$; diffraction coefficient in illuminated zone,
- $k(\omega, s)$; attenuation coefficient along a ray.

The integral in exponent of formula (5) is rewritten under the assumption that $k(\omega, s)$

is a linear function of frequency over the frequency range used in the analysis and that specific dissipation factor Q and elastic wave velocities are spherically symmetric in the earth, as

$$\int k(\omega, s) ds = \frac{\omega T}{2\bar{Q}}, \quad (6)$$

where T is the travel time of the wave and $\bar{Q}$ is the average Q value along the ray path.

In the case of model study, source function $A_0(\omega, \theta, \phi)$ is known, whereas in the seismic analysis unknown $A_0(\omega, \theta, \phi)$ is usually divided into spatial part $A_s(\theta, \phi)$ and temporal part $A_t(\omega)$. (Teng, 1966; Teng and Ben-Menahem, 1965.) Therefore by taking spectral ratio between observed $A_j(\omega)$ and reference $A_s(\omega)$,

$$\frac{A_j(\omega)}{A_s(\omega)} = \frac{[A_s(\theta, \phi)]_j [D_d]_j}{[A_s(\theta, \phi)]_s [D_d]_s} \left\{ \frac{[A_{crust}(\omega)]_j}{[A_{crust}(\omega)]_s} \right. \\ \left. \frac{[A_{inst.}(\omega)]_j}{[A_{inst.}(\omega)]_s} \right\} \left\{ \exp \left[-\frac{\omega}{2} \left(\frac{T_j}{\bar{Q}_j} - \frac{T_s}{\bar{Q}_s} \right) \right] \right\} \\ \left\{ \frac{D_f(\omega)_j}{D_f(\omega)_s} \right\} \quad (7)$$

is obtained, where subscript j means values at the recording station and subscript s does those at the reference station.

The first term in the right-hand side of the equation (7) is independent of frequency, and the second term can be compensated if the crustal transfer functions and instrumental response are known. $A_j(\omega)/A_s(\omega)$ after compensating the second term is denoted as $[A_j(\omega)/A_s(\omega)]_{cor.}$. Then by taking natural logarithm of formula (7),

$$\ln \left[\frac{A_j(\omega)}{A_s(\omega)} \right]_{cor.} = C - \frac{\omega}{2} \left(\frac{T_j}{\bar{Q}_j} - \frac{T_s}{\bar{Q}_s} \right) + X(\omega) \quad (8)$$

where

$$C = \ln \frac{[A_s(\theta, \phi)]_j [D_d]_j}{[A_s(\theta, \phi)]_s [D_d]_s} \\ X(\omega) = \ln \{ [D_f(\omega)]_j / [D_f(\omega)]_s \}$$

is obtained.

Teng (1966, 1968) and Mikumo and Kurita (1968) treated up to the second term of the right-hand side of formula (8). When the third term is negligible, the plotting of the

values of the left-hand side in (8) against the angular frequency ω will be linear with the inclination of $(T_j/\bar{Q}_j - T_s/\bar{Q}_s)/2$. Nevertheless this assumption to neglect the third term in (8) is not valid if the diffraction term $X(\omega)$ has appreciable quantity in the vicinity of the geometrical shadow boundary.

In this research a precise diffraction pattern is obtained by sonic wave experiment, whose results are accurate enough to compute the value of $X(\omega)$. By taking reference station (with subscript s) in the illuminated zone at an epicentral distance far enough from the shadow boundary, $X(\omega)$ are computed as the value of $\ln [D_f(\omega)]_j$.

Fig. 12. Values of $X(\omega)$ computed from the experimental results. Parameters are $(\Delta - \Delta_s)$. Abscissa is expressed in ha , period in the real earth and frequency in the type-1 model.

Computed results are shown in Fig. 12, which indicates that the curve of $X(\omega)$ is not linear with respect to frequency and that in illuminated zone it is neither monotonously increasing nor decreasing. Moreover, the inclination of these curves for stations in illuminated zone yields comparable values with those obtained by Teng or Mikumo and Kurita from seismic records, and interpreted as the possible existence of high attenuation zone at the base of the mantle. This result may suggest the possibility of the

existence is ostensible. It may also be suggested that the method used by Teng and by Mikumo and Kurita probably approximate a linear curve by the least squares to a curve essentially non linear in the vicinity of the geometrical shadow boundary.

Next problem is whether or not the quantity of the diffraction due to a sphere can explain the apparent low Q values obtained from seismic analysis. An example of *diffraction compensation* is planned on Fourier amplitude spectra obtained from type-1 model whose source depth is 15% of the radius of the model earth (0.850 model). The spectra are smoothed by a moving average having a width of 4.5 kc and the compensated values

$$\ln \left[\frac{A_j(\omega)}{A_s(\omega)} \right]_{\text{cor.}} - X(\omega)$$

are computed. The values are actually obtained from the calculation of

$$\ln \frac{[A_j(\omega)]_{\text{cor.}} [D_f(\omega)]_s}{[A_s(\omega)]_{\text{cor.}} [D_f(\omega)]_j} \quad (9)$$

The computed values against frequency will

stand on a line, if the compensation of diffraction is made appropriately.

Fig. 13 is the example of these results, which shows the values computed from the formula (9) for stations from $\Delta=90^\circ$ ($\Delta-\Delta_s=-17^\circ$) to $\Delta=100^\circ$ ($\Delta-\Delta_s=-7^\circ$). It is shown that the plottings are fairly on a line except at very low frequencies where spectra are apt to be contaminated by the false assumption of DC line on record or by other factors. From these curves Q value of 50 is obtained, which coincides with the Q value of the material that composes the homogeneous mantle of this type-1 model. This result also suggests that Q value in model material is not dependent on frequency within these frequency ranges.

(4) Differences in diffraction patterns

There remains, however, a doubt in the validity of the *diffraction compensation* to apply the real earth. The doubt comes from the possibility that the diffraction patterns will be highly affected by physical conditions surrounding the core. For instance, if there is a low Q zone at the base of mantle, the change

Fig. 13. Computer plottings of the values of formula (9).

in amplitudes of PcP will make difference of the diffraction pattern in which interference of P and PcP plays a big role, compared with *standard* diffraction pattern obtained for a model with homogeneous mantle.

The velocity gradients at the bottom part of mantle also cause the deflection of rays and make difference in the diffraction pattern.

Of course there will be a possibility to differ in diffraction patterns due to difference of physical condition or physical constants of the mantle and the core even when the mantle and the core are homogeneous, respectively.

Then the problem is how much the diffraction patterns are changed by the difference in physical conditions and in physical constants of an assumed model. When the degree of difference in the diffraction patterns is small, the nature of the mantle is known by use of the *diffraction compensation* in which a sample of diffraction pattern is used. On the other hand if the diffraction patterns are sensitive to the velocity structure or to the Q structure near the core-mantle boundary, the method of *diffraction compensation* may not be effective.

Strictly speaking, what we intend to know in the practical process of *diffraction compensation* is not the diffraction patterns themselves, but the functions $X(\omega)$. The functions $X(\omega)$ are not directly observed in the diffraction experiments, which means that the determination of $X(\omega)$ is much difficult in the two-dimensional model experiment where the problem of contact between the model and transducers restricts the accuracy of observed amplitudes. To avoid this difficulty, a method is explored to examine the degree of differences in $X(\omega)$ without the extraction of each $X(\omega)$ from diffraction patterns directly obtained by experiments, since the degree of differences in $X(\omega)$ is essentially wanted to know.

The method is to subtract $\ln [A_j(\omega)/A_s(\omega)]_{\text{cor.}}$ of the model (A) from the $\ln [A_j(\omega)/A_s(\omega)]_{\text{cor.}}$ of the reference model which has a homogeneous mantle (B), where subscript j and s indicate the same parameters in both models respectively.

This method is formulated as

$$\begin{aligned} & \left\{ \ln \left[\frac{A_j(\omega)}{A_s(\omega)} \right]_{\text{cor.}} \right\}_A - \left\{ \ln \left[\frac{A_j(\omega)}{A_s(\omega)} \right]_{\text{cor.}} \right\}_B \\ &= (C_A - C_B) \\ & - \frac{\omega}{2} \left\{ \left[\left(\frac{T_j}{\bar{Q}_j} \right)_A - \left(\frac{T_j}{\bar{Q}_j} \right)_B \right] - \left[\left(\frac{T_s}{\bar{Q}_s} \right)_A - \left(\frac{T_s}{\bar{Q}_s} \right)_B \right] \right\} \\ & + [X_A(\omega) - X_B(\omega)]. \end{aligned} \quad (10)$$

If the reference station (s) is sufficiently distant from the geometrical shadow boundary in the illuminated zone, the ray which arrives at station s is not affected by the core-mantle boundary structure and the value $[(T_s/\bar{Q}_s)_A - (T_s/\bar{Q}_s)_B]\omega/2$ in formula (10) is zero.

The experimental scheme is that if the difference between $X_A(\omega)$ of a model under consideration and $X_B(\omega)$ of the reference model is small, the values of the left-hand side of formula (10), which are obtained from model experiments, is linear function of frequency. Moreover, the average Q value along the ray which arrives at station j will be obtained from the inclination of the line, since $(T_j)_A$ and $(T_j)_B$ are obtainable from seismic observation and $(\bar{Q}_j)_B$ is Q value of the model material at room temperature.

On the contrary if the values of left-hand side of formula (10) against frequency deviate largely from a line, the difference in $X(\omega)$ should be attributed to the cause of deviation, if the errors in data processing or signal-to-noise ratio of records are not to blame for that points. When this is the case, the *diffraction compensation* will become very difficult.

Model experiments are accomplished on the type-2 models which have rectangular shape in their outline. Formulae (8), (9) and (10) are also valid for the type-2 model because the difference in length of ray path is expressed as the difference of travel time T . Besides the rectangular shape of type-2 model is superior to the ordinary circular model in extracting the values of $X(\omega)$ accurately when they become necessary. In reference model (OA model) $\bar{Q}_j$ and $\bar{Q}_s$ is equal to each other, and the value $-(T_j/\bar{Q}_j - T_s/\bar{Q}_s)\omega/2$ in formula (8) is very small in the type-2 model because of the small difference of travel time $(T_j - T_s)$,

which increases the precision of $X(\omega)$ in (8) relatively.

For instance, the value $-\frac{1}{2}(T_j/\bar{Q}_j - T_s/\bar{Q}_s)$ is 1.3×10^{-2} μsec when the station s is at -10 cm and j is at 0 cm, whereas $dX(\omega)/d\omega$ is an order of $1 \mu\text{sec}$ ($0.1/100$ kc). Therefore the left-hand side value of (8) calculated from observation becomes a precise approximation of $C+X(\omega)$, where C is a known constant. On the other hand in a circular model, the value $-\frac{1}{2}(T_j/\bar{Q}_j - T_s/\bar{Q}_s)$ is a comparable order with $dX(\omega)/d\omega$, which is the same situation as in the real earth.

(5) Experimental results on function $X(\omega)$

Experiments using seven kinds of type-2 models are made in order to know if the values computed by formula (10) go well with the experimental results. At first 4A MODEL is analysed. Fourier amplitude spectra at stations spreading over from $d = -15$ cm to $d = +2$ cm and at the reference station at $d = -25$ cm are smoothed by a moving average with the width of 4.5 kc. The gain of the signal amplifiers and the effect when waves are incident obliquely onto the free surface

are compensated, though both corrections are not indispensable for the examination of the formula (10) but they make it possible to reduce $(C_A - C_B)$ of the constant term in the formula (10) to the effects of divergence factor isolatedly. The plottings of the difference of spectral ratios against frequency and linear approximations within several

Fig. 14. Examples of the computed values of formula (10) on the 4A MODEL. Indicated inclination is the theoretical value when $X_A(\omega) = X_B(\omega)$ for $d = -1.0$ cm.

Fig. 15. Computed values of formula (10) on 2A MODEL.

portions of frequency ranges by the least square method are made also by a computer.

The results are shown in Fig. 14, which indicates a considerable deviation from linear lines at every station. The inclinations of the plottings for this model, when $X_A(\omega) = X_B(\omega)$, are obtainable from the distributions of the Q value (Fig. 4) and the travel times by the computation of the value on the right-hand side of the formula (10), whose result for $d = -1$ cm is indicated in Fig. 14. The discrepancy with observed value of the difference of spectral ratio is shown, too.

Computations for *3A MODEL* and *2A MODEL* also show that the plottings deviate largely from linear lines, though these models have smaller velocity gradients and larger values of Q at the bottom part of the model mantle than *4A MODEL* (Fig. 15). In these figures smaller values of receiver position d have a tendency to accompany larger irregularities in the plottings, which probably means that difference between $(T_j/\bar{Q}_j)_A$ and $(T_j/\bar{Q}_j)_B$ and the difference between $X_A(\omega)$ and $X_B(\omega)$ become larger when the ray becomes close to the core-mantle boundary structure.

The results of the computations of the left-hand side values of formula (10) on remaining models, *MB MODEL*, *CB MODEL* and *AB MODEL* are similar to *4A*, *3A* or *2A MODEL* (Fig. 16).

Another kind of experiment is accomplished to know the difference in the diffraction patterns caused by the difference of core-mantle boundary structures. In this experiment the structure is changed by heating of Ni-Cr heating ribbon while the receiver transducer is fixed at $d = -1$ cm. The records are taken on a non heat source model with homogeneous mantle (reference model) and on the models with heat source. The elastic waves passing in the latter model are recorded at 1, 2, 3, 4 and 5 minutes after switching on of the current of 4 Amperes in the heating ribbon.

Spectra of these models are divided by the spectrum of the reference model and natural logarithm, such as

$$\ln \frac{[A(\omega)]_A}{[A(\omega)]_B} = \ln \frac{(D_d)_A}{(D_d)_B} - \frac{\omega}{2} \left(\frac{T_A}{\bar{Q}_A} - \frac{T_B}{\bar{Q}_B} \right) + \ln \frac{[D_f(\omega)]_A}{[D_f(\omega)]_B}, \quad (11)$$

Fig. 16. Computed values of formula (10) on MB MODEL.

is calculated. If the third term of the right-hand side, which expresses the difference in diffraction patterns between the two models, is small compared with the first or the second term, then the plottings of the observed spectral ratio will be a linear function of frequency, whose constant term is expressed by the first term and whose inclination is indicated by the second term. One of the merits of this experiment is the capability to avoid fluctuations in the observed amplitudes and the spectra caused by the change in the coupling between the model and the transducers which have been troublesome problem in seismic model study, by means of fixing the transducers throughout the experiment. Therefore the reliability of spectral ratio, especially that of the constant term in it is believed to be improved.

The results of this experiment are shown in Fig. 17, which show clearly that the difference in diffraction patterns is too large to be ignored. Neither troughs near the frequency

of 25 kc nor peaks near 100 kc are explained without the third term in formula (11) which expresses the difference of the diffraction patterns. The relative abundance of higher frequency component in the models which are heated strongly by Ni-Cr ribbon may be due to the extension of the shadow boundary caused by the negative velocity gradient at the base of the model mantle. In addition, precisions in the experiment and in the data processing are proved by the coincidence of over-all level in the spectral ratios, and the continuous disposition of the results attended with the magnitude of the scale of the core-mantle boundary structures.

As was stated previously, the magnitude of $-[T_j/\bar{Q}_j - T_s/\bar{Q}_s]\omega/2$ and that of $X(\omega)$ are comparable order either in the circular model or in the real earth. Therefore the results of these model studies show that the difference of diffraction patterns in the illuminated zone is too large to apply the method of the *diffraction compensation* possible, which has

Fig. 17. Computed spectral ratios of formula (11). Among the spectra, "5 min(2)" is the exercise on the accuracy of data processing, which is a spectrum computed from the data that is digitized over again on the same record as "5 min."

been thought to be a powerful tool to extrude the distribution of Q values near the base of the mantle. The core-mantle boundary structures which are used in these experiments have no exaggeration compared with any possible structure in the earth. For instance, if there is such a structure in the real earth as *2A MODEL* or *1 min model*, the existence of the structure is scarcely discerned with the structures of homogeneous mantle by seismic observations.

The possibility of the frequency dependence of the specific dissipation factor Q has to be elucidated when we apply the method of the spectral ratio. As a special example, a structure such as *MB MODEL* or *CB MODEL* may have *effective* Q values which has frequency dependence partly due to the resonance caused by the structures. Even about homogeneous materials, there has been no extensive research on the Q value-frequency relation under high pressure and high temperature, and with partial melting which may occur at the deep part of the mantle, though for rather shallow part in the earth any contradictory phenomenon to the frequency independence of Q has not been observed (Knopoff, 1964; Attewell and Ramana, 1966; Sato, 1967). Only a few studies such as Mizutani and Kamamori (1964) and Walsh (1968) were made about this problem. The researches on ice and glacier often treat the attenuation coefficient of the elastic waves near the melting point, but these results are not directly applicable to the study of the core-mantle boundary. Though no definite conclusion has been obtained in the investigations on the problem of the frequency dependence of Q at deeper part of the mantle, the possibility that Q is a function of frequency has not been denied by any investigation.

If the Q values in the deeper part of the mantle are frequency dependent, the method of spectral ratio to extrude Q structure is not effective. The method is also ineffective in the cases where effective Q values are probably frequency dependent such as *MB MODEL* or *CB MODEL*. For these cases it is unable

to obtain Q structures by the method of the spectral ratio, or this method leads to a false conclusion on Q values. For example, when a part of the ray passes in the zone where Q is proportional to frequency, the spectral ratio $\ln[A_j(\omega)/As(\omega)]_{\text{cor.}}$ stands on a line too, but the Q value obtained from the inclination of the line is not dependent on the intrinsic Q value in that zone, and is larger than the Q value of the mantle where Q value has no frequency dependence. It is to be noted that the method of the spectral ratio is riskful when the value of Q has a possibility to be frequency dependent.

The results of the present study show consequently that the method of taking spectral ratio is not efficient in order to know the nature of the core-mantle boundary. Even if the diffraction patterns due to any model of the earth could be calculated, the method of the *diffraction compensation* on the spectral ratio to obtain the Q structure in the deeper part of the mantle would still be an ineffective method, because the consistency of the compensated results with an assumed model structure does not lead to a unique solution.

(6) *Diffacted waves in the shadow zone*

Diffacted waves in the shadow zone are also promising tool to study the nature of the core-mantle boundary. Because of their long ray paths along the core-mantle boundary, they provide plenty of information for this study. Alexander and Phinney (1966) and Phinney and Cathles (in press) obtained the attenuation function of the diffracted waves caused by the core-mantle boundary by use of seismic records. As previously stated, this method demands the records of excellent quality and the data processings with high precision because of large attenuation of the core-diffracted waves. But it is essentially important to solve the inversion problem in which to extract an actual core-mantle boundary structure from attenuation functions obtained from seismic data is needed, though the attenuation functions will be made clearer in the future studies. It is immediately necessary to know the attenuation functions for a wide variety of structures which have been

suggested by geophysical data.

It is one of the merits of model study that we can study on complicated models as easily as on simple models. The model study is one of the best methods for studying the detailed structure of the core-mantle boundary, in spite of the difficulty in selecting parameters of physical constants. In the study of the diffracted waves seven two-dimensional models are used, which are OA , $2A$, $3A$, $4A$, MB , CB and AB models.

The shape of models is chosen to be type-2. The ray paths in the type-2 model become shorter than those of circular models according to the deep penetration of the waves in the shadow zone. The signal-to-noise ratio of the diffracted waves is increased in the type-2 models because of the reduction of attenuation due to geometrical spreading, the cut down of Q type attenuation in the model material, and the development of signal amplitudes by the reduction of the angle of incidence to the receiver transducer. For instance, the amplitude of the wave of 100 kc which arrives at $d=45$ cm is enlarged 1.4 times by the geometrical spreading and 1.98 times by intrinsic attenuation Q than the amplitude obtained in the circular model which has the same length of ray at the shadow boundary. The amplitudes expected at the surface of the circular scale model of the earth are computed by use of a computer from observed amplitudes on the type-2 models. Although this kind of method to enhance the accuracy of results has not been attempted as yet, this method is highly powerful in the present study, in which materials having rather high attenuation are used in order to make continuous velocity gradients over a wide frequency range of waves.

Attenuation functions are obtained by formulae which are essentially the same as those of Phinney and Alexander (1966), as follows.

$$A(\omega, \phi, \theta) = S(\omega, \phi) M1(\omega, \phi) C(\omega, \phi, \theta) M2(\omega, \phi, \theta) A_{\text{inst.}}(\omega), \quad (12)$$

where

ϕ ; azimuthal angle from source to station,

θ ; angular distance ($\Delta - \Delta_s$),

$M1$; transfer function along the ray path from source to the bottom of mantle,

C ; transfer function of the ray along the core-mantle boundary,

$M2$; transfer function of the ray from the bottom of the mantle to the receiving station.

By taking spectral ratio among stations located on the same azimuth, ignoring the difference in $M2$ and compensating the instrumental response, function C is isolatedly obtained. Attenuation function $\gamma(\omega)$ is assumed to be linearly dependent on the distance θ , then

$$\begin{aligned} \ln \left[\frac{A_j(\omega, \theta_j)}{A_s(\omega, \theta_s)} \right] &= \ln \frac{C_s(\omega, \theta_j)}{C_s(\omega, \theta_s)} \\ &= [\gamma(\omega, \theta_s) - \gamma(\omega, \theta_j)] + \ln \sqrt{\frac{\sin \theta_s}{\sin \theta_j}} \\ &= \gamma(\omega)(\theta_s - \theta_j) + \frac{1}{2} \ln \left(\frac{\sin \theta_s}{\sin \theta_j} \right), \quad (13) \end{aligned}$$

where $\frac{1}{2} \ln \frac{\sin \theta_s}{\sin \theta_j}$ is geometrical spreading factor along the surface of the core. Ratio of the amplitudes after correction of the geometrical spreading factor is

$$\ln \left[\frac{A_j(\omega, \theta_j)}{A_s(\omega, \theta_s)} \right]_{\text{cor.}} = \gamma(\omega)(\theta_s - \theta_j). \quad (14)$$

The attenuation function $\gamma(\omega)$ is obtained by dividing the left-hand side of formula (14) by $(\theta_s - \theta_j)$. Therefore plural values of $\gamma(\omega)$ at one frequency for every recording station are calculated. The final value of $\gamma(\omega)$ is obtained by computing the weighted mean value of $\gamma(\omega)$ whose weight is $|\theta_s - \theta_j|$ by the reason of quantity of informations about the core-mantle boundary. This method to determine $\gamma(\omega)$ is suitable for model study compared with that of Alexander and Phinney (1966), since in model experiments records of high quality are available even in deep shadow, and it is desirable for model study to economize recording stations. Also an excellence of this method is the capability to monitor the accuracy in the determination of $\gamma(\omega)$ by comparing $\gamma(\omega)$ obtained from a station with $\gamma(\omega)$ from other station, while

the method explored by Alexander and Phinney is not good for checking the accuracy.

(7) Attenuation functions of diffracted waves

The functions $\gamma(\omega)$ from various stations generally coincide with one another. Slight discrepancies come mainly from the difference in the contact between the model and the receiver which results in the difference in over all level of $\gamma(\omega)$ and not in spectral structures. The spectral distortion due to the contact is negligible in the present study.

The curves of $\gamma(\omega)$, including even the minute peaks and troughs, obtained in the experiment are considered to be reliable because of the good coincidence of every $\gamma(\omega)$, except in extreme high frequencies where the signal-to-noise ratio goes worse.

Fig. 18. Attenuation functions of the core-diffracted waves converted from results of various investigations.

Figure 18 illustrates the various values of $\gamma(\omega)$ which are converted from the diffraction data obtained by various authors. The unit of $\gamma(\omega)$ is $(\text{deg})^{-1}$ and the abscissa is non dimensional numeral, ha . By taking these units, the difference in the scale factors is erased. In this figure Phinney-(numeral) means the cases treated by Phinney and Alexander (1966). Phinney-1 is a reference earth model having homogenous mantle and core. The reductions of Vp and density in the core of Phin-

ney-1 model mean the approach to Phinney-4 model, which has a hollow core. On the other hand the increases of Vp and density in the core mean the approach to Phinney-5 model (mixed boundary condition), whose values of $\gamma(\omega)$ coincide nearly with those of Phinney-3. These results obtained by Phinney and Alexander suggested that the difference in the assumed physical constants does not produce a large difference in $\gamma(\omega)$, if unnatural conditions are not assumed.

Sato-P means $\gamma(\omega)$ converted from the results of Sato (1969) in the case of P -wave diffraction for the model with a homogeneous elastic mantle and a hollow core.

The results of the diffraction experiment in this study by use of acoustic waves are indicated as three dimensional experiment in this figure. Though physical conditions in this model are the same as Phinney-2 on the points of acoustic mantle and rigid core, the result shows a large discrepancy. This discrepancy remains to be explained.

Indicated as two dimensional experiment is the result of *OA MODEL* having homogeneous mantle. This model corresponds to Phinney-4 and Sato-P with respects of the elastic mantle and hollow core, though the

Fig. 19. The function $\gamma(\omega)$ obtained from seismic data are superposed upon Fig. 18. Observational $\gamma(\omega)$ acquired by Alexander and Phinney are expressed by hatching.

mantle of is model is appreciably anelastic. Whether or not the discrepancies among these three results are explainable by differences in the assumed physical constants, the degree of approximation in the theoretical results, and/or the other factors is open for a future study.

Figure 19 shows the superposition of $\gamma(\omega)$ obtained from seismic data by Alexander and Phinney (1966) on Fig. 18. Observational $\gamma(\omega)$ are treated in the present study as the zone hatched in Fig. 19. Small consideration is taken on the regional variation in $\gamma(\omega)$ suggested by Alexander and Phinney, because of the poor quality of their seismic records and of no experience of the repetition at the same region. But the upper boundary of the hatched zone is nearly the same as the $\gamma(\omega)$ which are obtained from regions KD1, PB2 and SC, by their notation. The lower boundary also coincides approximately $\gamma(\omega)$ of region PB1.

The hatched zone of observed $\gamma(\omega)$ exists at the region of higher attenuation than the reference curve computed by Phinney and Alexander (Phinney-1 model) under plausible assumptions. The peak in this zone can not be explained by models which have homogeneous mantle and homogeneous core. There-

$\gamma(\omega)$

Fig. 20. Obtained $\gamma(\omega)$ from the model experiment.

fore it can be concluded that there is a kind of core-mantle boundary structure at least in some regions.

Attenuation functions $\gamma(\omega)$ obtained from model experiments are shown in Fig. 20. MB MODEL and CB MODEL produce behaviors of $\gamma(\omega)$ resembling with each other, especially in frequencies lower than 45 kc. Whereas AB MODEL shows quite a different value from the other models. This means small attenuation in the diffracted waves.

Models having the velocity gradient and the gradient of attenuation coefficient at the base of the mantle give the attenuation functions shown in Fig. 21. These functions have

$\gamma(\omega)$

Fig. 21. Obtained $\gamma(\omega)$ from the model experiment.

in general higher attenuation than the model without a core-mantle boundary structure. The effects of attenuation due to the material have to be invoked in the interpretation of these results. Though any discussion has not been made on the existence of this effects, the idea that the attenuation in the medium does not play any role on the wave propagation along the core-mantle boundary is not reasonable. For instance, the experimental results obtained by Rykunov (1957, 1959) on model, which had the mantle of paraffin-polyethylene and the core of gelatin ($\mu=10^7$ after converted by the model ratio),

shown in Fig. 18 showed discrepancy with the results of the theoretical computation by Phinney and Alexander (1966). The result of Rykunov on model of $\mu=10^7$ coincided with the model of $\mu=10^{11}$ by the computation. Although Phinney and Alexander thought that the discrepancy was caused by the inaccuracy in Rykunov's experiment, there remains the probability for explaining this contradiction by the effects of the attenuation due to the model material, because paraffin-polyethylene has very low melting temperatures and large anelasticity.

Among various $\gamma(\omega)$ shown in Fig. 18, $\gamma(\omega)$ which includes the effect of anelasticity are only the model experimental results. This fact gives out a question on the model fitting done by Phinney and Alexander, who have compared the function $\gamma(\omega)$ computed from the model having perfect elastic mantle with observational $\gamma(\omega)$ which are influenced by anelasticity.

But this problem is not thoroughly studied in the present study because of the complexity of the problems, such as the problems of the validity of the application of Q type attenuation, the effective attenuation coefficient when the Q value in mantle and that in core are different, and the effective attenuation coefficient when there is a gradient in the Q values, and so on. If the diffracted wave is attenuated by the Q type attenuation, and assume that the effective Q value in the Q type attenuation is the Q value at the bottom of the mantle, we can compute the contribution of the anelasticity to the function $\gamma(\omega)$. Fig. 21 shows the contributions, which are linear. The values of Q of 3.5, 9, 18 and 60 are used in the computation for the models 4A, 3A, 2A and 0A, respectively. If the assumptions described above are correct, real $\gamma(\omega)$ compensated after the contribution of the attenuation expressed by subtracting the contributions from the observed $\gamma(\omega)$. But these Q values, which are the minimum values in each model, are clearly too small, if we assume the Q type attenuation on the diffracted waves.

We can conjecture, if we take the hypo-

thesis of the Q type attenuation, that the Q values adopted are too small because the diffracted waves are propagated not only along the very core-mantle boundary, but also along the part of lower mantle with a considerable thickness where Q is larger than that on the boundary. The complexity of the problem comes from the probability that the waves of lower frequencies are related to a thicker range of mantle and core, and vice versa.

Even in the material having homogeneous Q value, the effective Q values are different from the Q value of the material because of the sphericity of the core-mantle boundary which causes differences in the effective ray length.

8) Comparison of the experimental results with the seismic data

No reliable Q structure has been obtained on the bottom part of the mantle. The lack of the knowledge on the Q structure makes it impossible to compensate the effect of Q on the attenuation function $\gamma(\omega)$. The existence of high frequency components such as 20 cps in PKP waves observed by Sumner (1967) puts a limit on the Q value at the base of the mantle, since Q values of some tens seem to be too low to have such high frequency waves in PKP . However, if the Q values at a deep part of the mantle is proportional to frequency as suggested by Walsh (1968), such Q values at the frequencies which are utilized in diffraction studies are compatible with the high frequency PKP observations.

Mizutani and Kanamori (1964) obtained an experimental result that the Q value in Woods metal decreases by an order of magnitude with melting, but they failed to clarify the frequency dependence of Q in the vicinity of the melting point. It may be a risky speculation to conceive that the melting of the mantle at the core-mantle boundary has the same mechanism and behavior of Q as those of Q of Woods metal, whose melting temperature is 72°C and Q value at room temperature is about 20. The problem of Q structure at the base of the mantle still remains to be solved in the future studies. The Q value

at the base of the mantle of the real earth probably is not extremely smaller than Q value in *OA MODEL*, if we consider the values of $\gamma(\omega)$ in Fig. 19.

Fig. 22. The function $\gamma(\omega)$ of MB, CB, AB and OA models. Results of the reference model of Phinney and Alexander and observational $\gamma(\omega)$ from seismic data are superposed.

Figure 22 shows the attenuation functions obtained from the model experiments, the seismic observation by Alexander and Phinney and the theoretical computation of reference model with plausible conditions given by Phinney and Alexander. The direct comparison of the experimental results with seismic observational results require some assumptions. But these assumptions are justified to an appreciable extent by considerable evidences. One of the evidences is the coincidence between the results of Phinney-1 model and *OA MODEL*, another is a probable parallelism in the relative disposition on the $\gamma(\omega)$ -frequency diagram among the experimental models, even when there is the difference in the attenuation coefficients or in the nature of the core material compared with the present study.

AB MODEL is clearly not good for being compatible with the observational $\gamma(\omega)$ be-

cause of the smallness of the values. The decrease of the slit length in *AB MODEL* will draw the result toward the result of *OA MODEL*. The increase of the slit length possibly make proximity to the diffraction of a half plane where the frequency dependence of the attenuation coefficients is far weaker than the spherical diffraction (Knopoff and Gilbert, 1961).

On the other hand *MB MODEL* and *CB MODEL* have probabilities which can account for the observed $\gamma(\omega)$ in the real earth. The values of the attenuation functions are reduced toward the values of the attenuation functions in *OA MODEL* by decreasing the dimension of the irregularities in these models.

The peaks and troughs in $\gamma(\omega)$ due to the fundamental and harmonic resonances of core-mantle boundary structures, whose fundamental tone is about 20 kc, are not dominant. A wide variety of the attenuation functions, including the function which has desired peaks and troughs, is likely obtainable by changing both thickness and extension of the irregularities.

The results of *4A MODEL*, *3A MODEL* and *2A MODEL* are still open to discussion on the direct comparison with $\gamma(\omega)$ in the real earth. The resolution of the results of these models is limited by such inherent weakpoint of this model study as the impossibility of separating the effect due to the velocity gradients from that due to the attenuation of the model material. We can safely say that among these models *3A MODEL* is most suitable to explain observed attenuation functions in the real earth. There is no conflict except one between this model and the geophysical data up to the present. The incompatibility is the observation of higher frequency component in *PKP*, but it becomes reconcilable if we take the attenuation type of $Q \propto \omega$.

In general the velocity gradient at the base of the mantle is able to cause the peaks and troughs in curves of $\gamma(\omega)$. On the other hand the effect of Q structure on $\gamma(\omega)$ is open to question as yet, and this limits a further investigation on the models having velocity gradients and/or gradients of the attenuation

coefficient. These models probably possess the same $\gamma(\omega)$ as that in the earth. In order to verify the actual existence of these structures, the estimation of attenuation coefficients at the bottom of the mantle over a wide frequency range is indispensable. The estimation will be attained by means of observations of *PKP*, *PcP* and other phases by the instruments which cover a wide frequency range and a wide dynamic range. Of course the precise knowledge about physics under high temperature and high pressure will be greatly helpful for this study.

(9) On *PcP*

Seven types of waveforms are picked up for analysis acquired from seven different core-mantle boundary structures shown in Fig. 7. Unexpectedly, these waveforms resemble each other in spite of the difference of the velocity structures. Remaining 21 records of 21 kinds of structures also have nearly the same appearances. Since the treatments in the time domain are not powerful for this kind of study, researches in the frequency domain are necessary. The spectral ratios of each record to the reference record, taken from the reference model which has homogeneous mantle (model A), are calculated

and the natural logarithms of them are taken as ordinate.

These logarithms are expressed as the products of transfer functions of the core-mantle boundary structures and the attenuation coefficients of the material, since the differences in the angle of incidence at the core-mantle boundary is negligibly small.

The spectral ratios calculated from experimental results are shown in Fig. 23. The symbols from A to G in Fig. 23 correspond to the velocity structures A to G, respectively, which are illustrated in Fig. 7. The parallelism of the results from similar velocity structures is shown in Fig. 23. For example, the results from model E and those from model F have the values close to each other until more than 300 kc. This means that the contaminations in the values of the spectral ratio due to the various noises are not serious at least up to the frequency of about 300 kc. Another example is that the values of model E located generally between D and F. Among the velocity structures in Fig. 7, model E is the transition state from model D to model F. These facts show that the results of the spectral ratios correspond to the velocity structures very well.

Fig. 23. Natural logarithm of *PcP* spectral ratios. Symbols A to G have the same meanings as in Fig. 7.

The parallelism of the results of model D and those of model G in Fig. 23 is interesting, since the velocity structures D and G are much different in velocity gradients, though the absolute values of the velocities are similar in these models.

In order to make a comparison with these experimental results, transfer functions by use of Haskell's matrix method are computed, where horizontally stratified structures are adopted to approximate the actual velocity structures.

For model F which has the largest low velocity zone, six kinds of computations on transfer functions are made, in which the real velocity structures is approximated by 1, 2, 5, 10, 20 and 40 steps of stratified structures.

The obtained transfer functions for model F and model A are illustrated in Fig. 24. It

Fig. 24. The transfer functions of PcP reflections computed by approximations of 2, 5 and 40 homogeneous stratified layers. The transfer function of the model A which has a homogeneous mantle is also shown.

is shown that the transfer function of F become smoother according to the increase in number of steps.

The comparison of experimental results with computational results by Haskell's method is made through the computation of a function, $\ln \{[\text{spectral ratio (model F/ model A)}]/[(\text{transfer function F})/(\text{transfer function A})]\}$. This function which is plotted against frequency will be a line which passes the origin of coordinates, if computed transfer functions are

correct. Downward inclination of the line will be caused by the attenuation in the model material which is not taken into consideration in the computation of transfer functions. However, the calculated values do not show such a simple trend (Fig. 25). There is no

Fig. 25. Comparison of PcP experimental results with computational results.

downward linear portion except from 70 to 150 kc. Nevertheless even the portion has too large an inclination which leads to too low a value of Q at the base part of mantle in model F. Real Q values are obtained from temperature of the model. Moreover, the crossing point of the portion of the line at ordinate is not zero but about $\exp(0.3)$. This value is far beyond the possible error in the present experiment, during which source and receiver transducers are fixed. Only when the amplitude fluctuation more than 30% is observed, the value of $\exp(0.3)$ is obtained.

In the frequency range lower than 70 kc and higher than 150 kc the discrepancies between the experimental results and the computational results are shown. This problem on the contradiction remains to be solved in the future. What should be investigated first is the validity of extending the Haskell's method to the problem where spherical waves are incident upon spherical boundaries whose radii are not greatly larger than the wave lengths.

The quantity of P to S conversion caused by the discontinuities of velocity derivatives in the layers has to be also clarified because there is little P to S conversion in the present experiment.

(10) *Possible models on the core-mantle boundary structure*

The poverty of knowledge about physics and chemistry of solid earth under high pressure as well as high temperature put a serious restriction in the study of core-mantle boundary structure. To study the constitution and the evolution of the earth's core is undoubtedly a decisive key to ascertain the actual boundary structure, though little has been known about these unknown factors.

The density of the core turned out to be 10% smaller than liquid iron (Takeuchi and Kanamori, 1966; and so on). The small density has been interpreted as the solution of Si, S or O into iron (MacDonald and Knopoff, 1958; Ringwood, 1959a; and others), or as the mixture of Ni (Takahashi and Bassett, 1965). But the latter interpretation was most likely denied by the high pressure experiments by Al'tshuler *et al.* (1968). Alder (1966) estimated the solubility of various materials into pure liquid iron by use of the solubility theory under the condition expected at the core-mantle boundary. Kawai and Inokuchi (1967) suggested the decrease of the melting point according to the increase of pressure in most metals. They made an interesting working hypothesis which could explain the mantle, outer core and inner core by introducing the Fe-Ni-Si alloy system.

There is therefore very little knowledge on the core-mantle boundary structure from studies of high pressure physics. The diffusion of core material into mantle is only a hypothesis imagined by geophysicists. But this hypothesis has neither strongly supported nor decisively denied, because of scarcity of the study as stated above. In this hypothesis it has been generally assumed that the diffusion results in the reduction of the elastic wave velocities at the bottom of the mantle as well as in the increase of the density.

Various models on the core-mantle boundary structure which have been proposed from seismic study have mostly their foundation on this assumption. Almost all the models treated are radially symmetrical.

In regard to the type of the core-mantle boundary structure, however, there remains the other probability. That is the model which has many small irregularities at the core-mantle boundary, in a word, like the *MB MODEL* or *CB MODEL* in the present study. The model is possible if we take either the hypotheses generally accepted for the core-mantle system or the hypothesis of Kawai and Inokuchi. Moreover, this type of core-mantle boundary structure goes well with almost all the seismic data and the widely supported hypotheses about the evolution and the constitution of the core-mantle system.

As stated previously, the models with irregular core-mantle boundary structure, whose thickness is smaller than that of the *MB MODEL* or the *CB MODEL*, may be suitable for interpreting the seismic diffraction data obtained so far. The thickness of the structure is estimated to be 1 to 3% of the radius of the core, for either models of *MB* type or those of *CB* type.

The observed results on short period *PcP* waves (Kanamori, 1967b) do not show any discrepancy with these new models. Little attenuation of short period *PcP* waves is expected for these structures, if the size of each irregularity is assumed to be smaller than the wave length, which is about 10 km in the Kanamori's study, and the spacing of the irregularities is assumed to be larger than that. These structures scarcely affect long period *PcP* waves.

On the other hand the irregularities of *MB* type are superior to *CB* type, since the travel time anomalies of *PcP* waves from nuclear explosions studied by Taggart and Engdahl (1968) are within ± 1 sec throughout the wide area of the world.

High frequency component in *PKP* waves observed by Sumner (1967) is likely compatible with these structures also.

These kinds of irregular core-mantle bound-

ary structures can reconcile the discrepancy of the free oscillation of the earth indicated by Dorman *et al.* (1965). These structures undoubtedly elongate the periods of the spheroidal free oscillations over order numbers 3 to 9, without enforcing a serious change upon the periods of spheroidal oscillations of order numbers other than 3 to 9, or torsional oscillations as well as other geophysical data such as the moment of inertia of the earth or travel times of seismic waves. *MB* type structures are likely a *soft layer* for the spheroidal oscillations.

The increase of the core size proposed by Sacks (1966) through the amplitude study of extremely short period direct waves comes up to about 80 km. Since the quantity of shift in the shadow boundary caused by diffraction in the illuminated zone is not exceed 1 degree for the short period waves used in the Sacks's analysis (ha is 640 for the 2.5 sec waves and 2300 for 0.7 sec waves), the proposed position of the shadow boundary at $\Delta=96^\circ$ is still smaller than the generally supported ones which had been determined by use of long period waves.

To interpret this phenomenon the present author adopted a hypothesis that the *diffraction sphere* for these high frequency waves is different from and larger than the *diffraction sphere* for the lower frequency waves. It is most preferable for the difference between the both spheres to be 2% of the core radius, if the shadow boundary is assumed to be 96° and the velocity structure of Jeffreys is adopted. The hypothesis of dual diffraction sphere is favorable for the *MB* type irregularities. Because the outer boundary of the *MB* type structure may fill the role of the diffraction sphere for shorter period waves, whereas the inner boundary does for longer period waves. The results of the present study on the attenuation of diffracted waves indicate that the difference of the radii of the dual diffraction sphere is 1 to 3% of the core radius, in terms of the hypothesis. The correspondence in the values of the difference of the radii between the Sacks's result and the result of the present study favors a pro-

bability to interpret the small value of the shadow boundary observed by Sacks by the existence of *MB* type model stated above.

§ 4. Concluding remarks

The diffraction pattern due to a sphere, which has been regarded as a bottleneck in the study of the core-mantle boundary by means of *P* wave spectral study, is obtained from three dimensional model experiment. The coverage of the spatial region over illuminated zone and shadow zone is perfectly attained. Also the coverage of frequency range which is necessary in seismic studies is attained.

The compensation term of diffraction in the illuminated zone, which has been required in order to deduce *Q* structure from spectral ratio of the direct *P* waves, is computed from the experimental results. The term of diffraction proved to have the same magnitude as the value of the spectral ratio which has been only a basis of arguments of the possible existence of the low *Q* zone at the bottom of the mantle.

The differences in the diffraction patterns caused by various core-mantle boundary structures are examined by two dimensional model experiments. The structures include velocity gradients, the gradients of attenuation and some kinds of irregular boundaries. It is concluded that the influences upon the diffraction in the illuminated zone due to various core-mantle boundary structures are too large to be ignored. Some models having very slight velocity gradients and the attenuation gradients are also experimented. These models are hardly distinguished from the model with homogeneous mantle, by means of the observations of seismic travel times on the earth's surface. But these models show the diffraction considerably different from the diffraction of the model with the homogeneous mantle. This fact is the case when the core is not hollow, since in the diffraction in the illuminated zone the interference of *P* and *PcP* plays a great role. This result indicates that it may be very difficult to success in extracting *Q* structure at the base

part of the mantle from the spectral ratios of seismic body waves.

Diffracted waves in the shadow zone are also investigated. The study of the core-mantle boundary structure through the spectral ratios of the core-diffracted waves is an inversion problem, to obtain a unique solution of which is very difficult. It is required in such a problem to know the responses caused by various kinds of boundary structures. Two-dimensional model experiments on seven kinds of models are performed to get the spectral responses of the diffracted waves.

The existence of the influences on the spectra of diffracted waves caused by the internal friction of the materials which constitute mantle and core is discovered. The influences have not been treated in the discussions relating to the diffracted waves, nevertheless they are likely to affect the results in the fitting of models to investigate the actual structures in the earth.

Whether the attenuation of the diffracted waves is Q type or not, or whether the Q values in the deeper part in the earth are independent of frequency is still open to question. But at any rate the consideration on the internal friction has become necessary for precise studies on the diffracted waves hereafter.

The attenuation functions obtained from the model study are compared with seismic data acquired by the work of Alexander and Phinney (1966). Some of the models are found to be able to account for the seismic data. One of them is the model having the velocity gradient and the attenuation gradient at the base of the mantle toward the core-mantle boundary, and the others are models which have irregular core-mantle boundary structures.

The comparison of these models with other geophysical evidences are greatly obstructed by the ambiguities of the geophysical data. The accumulation of the data in the future will doubtlessly enable us to fit a more precise model to the core-mantle boundary structures. Among the suitable models se-

lected in the present study, the models with gradients of velocity and attenuation needs an assumption on the frequency dependency of Q values in order to account for the observations of short period PKP and PcP waves. Nevertheless the assumption has been neither confirmed nor denied as yet.

On the other hand the models with irregularities are consistent with almost all the geophysical evidences, especially MB type model which has many thorny irregularities along the surface of the core-mantle boundary is suitable.

The thickness of the thorny irregularities is estimated to be 1 to 3% of the core radius (30 to 100 km thick in the real earth). The estimation is done through the consideration on the seismic diffraction data and the effect of the attenuation of material, which constitutes the experimental model, on the diffraction spectra. The thickness is able to account for the *large* core size proposed from the amplitude study of the short period direct P waves (Sacks, 1966) without the revision of the *real* core size.

PcP waves are also experimented on the two-dimensional models. The results show that the differences of spectra among the models which have various core-mantle boundary structures with continuous velocity distributions are not large.

The computation of the transfer functions by use of the Haskell's matrix method is made to compare the results with the experimental results. The results suggest that the computational results are greatly different from the experimental results. If this discrepancy does not come from the difference of the type of the incident wave (the former is a plane wave, whereas the latter is a spherical wave), the spectra for horizontally stratified structures, which are used in the Haskell's method as an approximation of gradient of physical constants, are greatly different from those for structures with the continuous distribution of the physical constants. It is further suggested that there is a possibility to discern whether or not there really are a few discontinuities of the physical con-

stants at the base of the mantle by analyzing the spectra of the core reflected waves.

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