Thermal History of the Earth with Consideration of the Formation of Its Core

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Abstract

Various possible ways of core formation are tested from the thermal history of the Earth. Calculation shows that the conversion of gravitational to thermal energy from a uniform to the present differentiated Earth is 1.68×10^{38} ergs. It is concluded that the release of the gravitational energy on core formation dominates the thermal state of the Earth.

Earth models in which the core has been growing slowly since 4500 My ago or 3000 My ago fail to meet the requirement of a solid mantle. In these models the present surface heat flows are in excess of the observed heat flow.

Earth models in which the core was substantially complete at 4500 My ago or 3000 My ago within a short period less than 100 My give the observed heat flow.

The gravitational energies released in reconstruction of the terrestrial planets are also calculated. Calculation gives the gravitational energies of 1.02×10^{38} ergs, 1.10×10^{36} ergs, and 1.34×10^{36} ergs for Venus, Mars, and Mercury respectively. The results show that the energies released by the formation of iron cores of the Earth and Venus play an important role for the internal heating of these planets.

§1. Introduction

It is supposed by many that the Earth's core is composed of molten iron with a possible admixture of some nickel and silicon. The known density of the core is in agreement with this supposition. The mantle is mainly composed of silicates or oxides of light elements. The core is chemically distinct from the mantle. The mass of the core is one-third of the Earth's mass. The separation of the core is thus the principal event of the terrestrial evolution. The reconstruction of the Earth from a uniform, metal-silicate phase mixture to the present state has released substantial amounts of gravitational energy. We should not ignore the gravitational energy when calculating theoretical models of thermal history of the Earth containing the iron core. However, reports of this type of study have apparently not been published to date.

In the present paper the results of some new calculations are discussed. Their main purpose is: a) to compare the thermal history

of the Earth assuming a long-enduring and an instantaneous formation of the core resulting from the differentiation of the mantle, and b) to clarify the possible thermal history of the Earth containing the gravitational energy emitted in the process of core formation by comparing theoretical results of the development of temperature within a number of Earth models with observations on the present thermal state of the Earth.

The time-scale of core accumulation is uncertain. Various hypotheses concerning on the period of core growth are classified into two groups as follows: (A) The core has been growing slowly since 3000 My ago (RUNCORN, 1962; 1965). (B) The core was substantially complete at an early stage in the Earth's history within a short period less than 100 My (STACEY, 1963; BIRCH, 1965; SHIMAZU, 1965; FUJII and UYEDA, 1966). We call A and B processes of the above core separation gradual and rapid formation of the core respectively. These two variants are critically tested with respect to their thermal history.

On the basis of a lack of data on the thermal state of the primeval Earth we shall use two models for the initial temperatures of the Earth. In one model the temperature is assumed to be 1200°K at the center. We call the model cold model. In the other model the temperature is supposed to be 4000°K at the center and to decrease outward along a Simon melting curve. We call the model hot model.

The temperature of the surface is always adopted as equal to 273°K . The age of the Earth is set equal to 4.5×10^9 years. From this moment is measured the time t used in the calculation.

We shall assume that at that time the metallic iron is preexisted and distributed among silicate phases.

§ 2. Analysis and Model of Core Formation

In this section processes of core formation are analyzed and are classified.

(A) Gradual Formation of the Core

(A-1) Initial cold Earth:

If the Earth originally formed as a gravitational collection of gases or particles at a relatively low temperature, then the initial composition and structure would be expected to be largely uniform throughout. However, an initial loss of gravitational potential energy in the process of planet formation out of particles by accretion is quite large. For instance, for a spherical body with radius R in hydrostatic equilibrium in which density is uniform, the loss of gravitational potential energy (negative), Ω , or the work done by gravitation in drawing together the material from a state of infinite diffusion is determined by

$$\Omega = -0.3GM^2/R, \quad (1)$$

where G =gravitational constant and M =total mass (EDDINGTON, 1959, p. 87). The gravitational potential energy becomes -3×10^{39} ergs for the present Earth mass and radius. Let the specific heat be $C_p = 1.3 \times 10^7$ ergs/g-deg, the average temperature rise ΔT becomes

$$\Delta T = |\Omega|/(MC_p) = 4 \times 10^4 \text{ degrees}, \quad (2)$$

which is sufficient to melt the Earth through-

out. But, for this consideration, SHIMAZU (1965) has pointed out that most of the initial loss of gravitational potential energy is quickly radiated back into the space when the accreting material hit at the surface of the condensing planets. Except when dust clouds prevented radiation, this energy would escape almost instantaneously.

From this point of view the initial temperatures within the Earth formed as a result of an accumulation of particles of the gas-dust protoplanetary cloud would be low. It seems safe to assume that in the original Earth the temperature at the center would be about 1200°K as a result of adiabatic compression (LUBIMOVA, 1958; ANDERSON and PHINNEY, 1967). We shall assume that at that time the gross distribution of the metallic phases and the long-lived isotopes are uniform. The concentration of radioactive elements throughout the Earth until core formation commences is assumed to be equal to that of chondritic meteorites. At the original state of uniform chondritic composition, we put $U = 1 \times 10^{-8}$ g/g, $K^{40} = 8 \times 10^{-8}$ g/g (reduced to the present values) and $Th/U = 4$. The heavier iron nickel phase begins to migrate toward the center as soon as the Earth becomes warm enough through radioactive decay. If the Earth accreted from chondritic material, the metallic phase would have reached the melting temperature some 1500 My after the completion of the aggregation process for the initial cold Earth (MACDONALD, 1959; IRIYAMA, 1966). It seems safe to assume that in the case the core began to form at a time 1500 My, i.e., 3000 My ago. For the first 1500 My there was no core formation. After separation of the core, the abundance of U, Th, and K^{40} in the silicate mantle is changed to satisfy the mass conservation, preserving the ratio of Th/U .

While the mantle-core separation occurs in the closed system, so that the accompanying release of gravitational energy is largely contained within the Earth. RUNCORN (1965) argues emphatically that the separation of iron toward the center has gradually occurred through the Earth's life. We assume that

Fig. 1. Diagram of a gradual growth of the Earth's core.

GH—Model GH,

GC—Model GC [after RUNCORN (1965)].

after RUNCORN the core has started its growth a little over 3000 My ago. The history of growth of the core is given in Figure 1, which is reproduced from that derived by RUNCORN. It is seen that the rate at which iron is added to the core is proportional to the surface area of the core and to the mass of iron remaining in the mantle. The gradual growth of the core provides a source of energy continuously released since 3000 My ago. The initial temperature distribution $T_0(r)$ in this case is given in Figure 3, which is reproduced from that calculated by LUBIMOVA. This core formation process is referred to as model GC.

(A-2) Initial hot Earth:

If the time between the formation of elements and the aggregation of the Earth was short compared with the half-lives of short-lived isotopes, radiogenic energy due to the isotopes should not be ignored. For instance, MACDONALD (1959) points out

that the short-lived isotopes play an important role for the initial heat of the Earth provided we assume that the time between the element formation and the aggregation was 100 My. According to his account, the important short-lived nuclides are U-236, Sm-146, Pu-244, and Cm-247. He argues that if all the heat produced by the four isotopes was retained by the material, then the mean temperature increase of the Earth was about 3000°C. On the other hand, SHIMAZU (1965) argues that the important short-lived nuclides for heating at the early stage are Cl-36 and Al-26 for the silicate phase and Fe-60 for the metal phase from the quantitative view point.

In either estimate the temperatures within the Earth at 4500 My ago attain to the melting temperature of iron in the mantle. From this point of view it is necessary to assume that the core has started to form at a time

4500 My ago. The growth curve of the core is given in Figure 1. The gradual growth of the core provides a source of energy continuously released since 4500 My ago. The initial temperature distribution $T_0(r)$ in this case is given in Figure 5. This core formation process is referred to as model GH.

(B) *Rapid Formation of the Core*

(B-1) *Initial cold Earth:*

The commencement of core differentiation would be determined by melting of the iron and by a reduction in the viscosity of silicate phase in the mantle, as pointed out by TOZER (1965) and UYEDA (1967a). The formation of the core would not proceed until the mantle is, by radioactive heating, to become soft enough to permit the iron to fall. We may take the temperature dependence of the mantle viscosity to be of the form $\exp(E/kT)$ where E is an activation energy and k the Boltzmann's constant. Hence, as TOZER (1965) has argued, the process of core formation releases a lot of energy which will cause a further fall in viscosity, so that this process will quickly establish itself or be catastrophic. The rate of the gravitational energy release will be itself dependent on the rate of core formation in a manner which encourages a catastrophic change.

From this point of view the time-scale of the core accumulation would be a geologically short time less than 100 My. Now, if we assume an initially cold Earth, the core is substantially complete at the time between 1500 to 1600 My after the end of the accretion process of the Earth. The rapid growth of the core provides a source of energy fast released at the time between 1500 and 1600 My. The initial temperature distribution $T_0(r)$ in this case is given in Figure 7. This core formation process is referred to as model RC.

(B-2) *Initial hot Earth:*

From geochronological data, the Earth has an age of about 4500 My. SHIMAZU (1965) has pointed out that this age refers to the time when the upper mantle and the crust can be regarded as a closed system with characteristic U/Pb , and the value of 4500

My is regarded as the age of the upper mantle. The separation of the core affects the U/Pb ratio of the silicate mantle, for the metal carries down significant amounts of Pb but not U . If the mantle-core separation is continued significantly during the stage of a closed system, the consistent value of 4500 My for the age of mantle cannot be obtained from rocks differentiated from the mantle at different ages. Hence, SHIMAZU (1965) argues emphatically that the Earth had separated into the core and the mantle at or before 4500 My ago.

From this point of view it is necessary to assume that the core was substantially complete at the time between 0 to 100 My and that its subsequent growth has been slight. The rapid growth of the core provides a source of energy fast released at the time between 0 to 100 My. The initial temperature distribution $T_0(r)$ in this case is given in Figure 9. This core formation process is referred to as model RH.

§ 3. Theory

Here various basic formulas are analyzed as follows:

(1) *Loss of gravitational potential energy*

(1-A) *Gradual formation of the core;*

We shall consider a whole series of Earth models, with different radii of the core. We shall assume that the Earth model has the same external radius R . We further assume that during the period of core formation the total mass of the Earth is remained constant. To construct a spherically symmetrical Earth model in hydrostatic equilibrium where the rotation is neglected, the following equations are used;

$$dP = g \rho dr, \quad (3)$$

$$g = Gm/r^2, \quad (4)$$

$$dm = 4\pi r^2 \rho dr, \quad (5)$$

where ρ is the density at distance r from the center of the Earth, P the pressure, m the mass inside the sphere of radius r , g the gravity field strength, and G the constant of gravitation.

We know, from the seismological data, the relation of ρ and P for the core. Hence, for the core, we assume an equation of state of the form (BULLEN, 1963, p. 231, model A)

$$\rho = 7.2381 + 2.0698P - 0.18828P^2, \quad (6)$$

where P is the pressure in units of 10^{12} dynes/cm².

For the mantle, we assume an equation of state of the form (MACDONALD, 1962)

$$d\rho/dr = \rho_0^3 g / (\alpha \rho^2), \quad (7)$$

where ρ_0 = surface density and α = ratio of incompressibility to density.

ρ_0 will gradually decrease with time, as the iron separates to form the core. In the uniform Earth ρ_0 and α are taken to be 4.09 g/cm³ and 29 (km/sec)² respectively. The boundary conditions at $r=0$ and R are as follows:

$$r=0: \quad dP/dr=0, \quad (8)$$

$$g=0, \quad (9)$$

$$r=R: \quad dP/dr = \rho(R)g(R). \quad (10)$$

The gravitational potential energy of the Earth, GPE , is expressible in a Protean variety of forms (BIRCH, 1965):

$$GPE = -4\pi \int_0^R g \rho r^3 dr = -12\pi \int_0^R P r^2 dr. \quad (11)$$

The release of gravitational potential energy, GE , with time or with core radius is given by

$$GE = |GPE_{t_i} - GPE_{t_{i-1}}|, \quad (12)$$

where t denotes the time. The rate of heat production, G , on core formation is given by

$$G = GE / (M \cdot \Delta t), \quad (13)$$

where M is the mass of material subjected to core formation process and Δt is a time interval ($t_i - t_{i-1}$) in the process.

(1-B) Rapid formation of the core;

For a homogeneous and the present differentiated Earth, we calculate the gravitational potential energies. The loss of gravitational energy is the difference of the two large quantities. As a result of a rapid growth of the core, we shall assume that the energy is

rapidly emitted over the whole body of the Earth within a short period less than 100 My.

(2) Basic equation of heat transfer

We shall consider the thermal history of the Earth containing the emission of gravitational energy on core formation. The equation governing the transfer of heat is given by

$$\partial T / \partial t = (1/\rho C_p) [\nabla \cdot (K \nabla T) + A + G], \quad (14)$$

where K = total thermal conductivity,

A = rate of heat production by radioactivity,

G = rate of heat production by gravitation,

C_p = specific heat = 1.26 joules/g-deg.

We shall take the thermal conductivity in the mantle to be of the form

$$K = K_{\text{photon}} + K_{\text{phonon}}. \quad (15)$$

The photon conductivity (K_{photon}) is given by

$$K_{\text{photon}} = 16n^2 \beta T^3 / 3\epsilon, \quad (16)$$

where n^2 = refractive index = 3, ϵ = opacity = 10 cm⁻¹, and β stands for the Stefan-Boltzmann constant. For the opacity value as $\epsilon = 10$ cm⁻¹, present calculations would remain valid also if the exciton conductivity is of importance in the lower mantle.

The phonon conductivity (K_{phonon}) is given by

$$K_{\text{phonon}} = B/T, \quad (17)$$

where the constant B is determined from the condition that on the earth's surface (where $T = 277^\circ\text{K}$) the thermal conductivity K_{phonon} should be equal to the conductivity of the crustal rocks $K = 0.0334$ joules/cm-sec-deg.

The thermal conductivity of the core is assumed to be due to free electron or 0.418 joules/cm-sec-deg.

§ 4. Results

(A) Gradual formation of the core

(A-1) Initial cold Earth GC;

In model GC we investigate the thermal history of an initially cold Earth with consideration of the energy emitted in a gradual growth of the core. The densities for the

Table 1. Density and pressure in the undifferentiated Earth.

Radius km	Density g/cm ³	Pressure Mb
0	6.96	2.150
300	6.95	2.139
600	6.93	2.112
900	6.90	2.074
1200	6.87	2.024
1500	6.82	1.962
1800	6.77	1.890
2100	6.70	1.808
2400	6.63	1.715
2700	6.54	1.613
3000	6.45	1.503
3300	6.34	1.385
3600	6.22	1.260
3900	6.09	1.129
4200	5.94	0.993
4500	5.77	0.854
4800	5.58	0.712
5100	5.38	0.569
5400	5.14	0.428
5700	4.87	0.289
6000	4.56	0.155
6300	4.19	0.029
6371	4.09	0.000

initial, undifferentiated Earth are listed in Table 1.

For the present Earth (BULLARD, 1964, p. 96, Table 4), the gravitational potential energy is -2.488×10^{39} ergs; for the undifferentiated Earth, -2.290×10^{39} ergs. The total loss of the gravitational energy due to the core-mantle separation becomes 1.98×10^{38} ergs. However, the lost gravitational energy is not completely converted to heat. After UREY (1952, p. 176) and BIRCH (1965) we assume that some 15% of the loss of gravitational energy remains as strain energy in the Earth's interior. Therefore, the net energy change in reconstruction of the Earth is 1.68×10^{38} ergs. This is 1.5 times greater than the total amounts of heat 1.10×10^{38} ergs liberated by radioactivity in the chondritic Earth. It is assumed that the gravitational energy has been gradually emitted over the whole body

Fig. 2. Total thermal energy released by a gradual growth of the core up to various times for model GC.

of the Earth since 3000 My ago. The total thermal energy released by the gradual growth of the core up to various times is shown in Figure 2. For the values in the model, the mean rate of the gravitational energy release is 9.37 ergs/g-year.

In the present calculations we shall use three models about the concentration of radioactive elements in the mantle. In one model, as a result of the gradual growth of the core since 3000 My ago, we assume that the radioactivity has been gradually concentrated in the upper 600 km of the Earth since 3000 My ago. In a second model we assume that the radioactivity has been gradually concentrated in the upper 200 km since 3000 My ago. Finally, for the sake of comparison, it is assumed that the radioactivity has been gradually concentrated in the whole mantle since 3000 My ago. These three models are referred to as model GC-U6, GC-U2, and GC-M respectively.

The development of temperature as a function of time within model GC-U6 is shown in Figure 3. At the stage of uniform Earth the temperature increased slowly with time. After the beginning of the core-mantle separation it began to increase rapidly, and at $t = 3.5 \times 10^9$ years this rapid increase ceased. At $t > 3.5 \times 10^9$ years the temperature in the interior increases slowly with time until the present time, while in the first few hundred kilometers below the surface it decreases

Fig. 3. The development of the temperature distribution within model GC-U6.

Table 2. Melting temperatures of diopside (MACDONALD, 1959).

Depth (km)	300	600	1000	1500	2000
Melting point (°K)	2520	2990	3870	4570	5190

slowly with time until the present time. The present temperature is 4390°K at the Earth's center, 4410°K at the core-mantle boundary, which exceeds the melting temperature of diopside at depths between 100 and 1500 km (see Table 2). The low temperature state within the core is due to a lack of radioactive heat sources within the metallic core. At the time between 1.6 to 3.0×10^9 years the build-up of radioactive heat sources near the surface raised the temperature of the material at the 600 km depth above the temperature at a greater depth within the mantle.

In models GC-U6, GC-U2, and GC-M the present temperature distributions are listed

Table 3. Present temperature distributions for GC models.

Depth km	Model GC-U6 T, °K	Model GC-U2 T, °K	Model GC-M T, °K
100	2340	2390	2080
200	3000	2470	2710
400	3600	3380	3320
600	3870	3650	3670
800	4010	3850	3910
1000	4120	3990	4080
1500	4280	4220	4350
2000	4370	4340	4480
2900	4410	4410	4510
4000	4420	4420	4430
5000	4400	4400	4380
6370	4390	4390	4350

in Table 3. The present temperature within model GC-M is, at all depths, about 1000-1400

Fig. 4. Variation of surface heat flow with time for GC models.

degrees higher than that calculated by MACDONALD (1959, his model 14) disregarding the energy released by the formation of the core.

Figure 4 shows the history of surface heat flow for GC models. The almost constant surface heat flow was reached at the end of the uniform Earth and after the beginning of the core-mantle separation it began to increase rapidly. At $t > 3.3 \times 10^9$ years the heat flow decreases gradually with time until the present time. In all these models the maximum value of the heat flow took place at $t = 3.3 \times 10^9$ years, i.e., 1.2×10^9 years ago, which occurred some 1.8×10^9 years after the beginning of the core-mantle separation. For model GC-U6, the final surface heat flow is 69.2 ergs/cm²-sec; for model GC-U2, 70.9 ergs/cm²-sec; for model GC-M, 60.5 ergs/cm²-sec. In models GC-U6 and GC-UC the present heat flows are in excess of the observed heat flow (UYEDA, 1967b), and the present temperatures exceed the melting temperature of diopside at depths between 100 and 1500 km. This is the result of the gradual emission of gravitational energy.

(A-2) Initial hot Earth (GH);

In model GH we investigate the thermal history of an initially hot Earth with consideration of the energy emitted in a gradual growth of the core. It is assumed that the gravitational energy has been gradually emitted over the whole body of the Earth since 4500 My ago. For the values in the model, the mean rate of the gravitational energy release is 6.25 ergs/g-year.

In the present calculations we shall use three models about the concentration of radioactive elements in the mantle. In one model, as a result of the gradual growth of the core since 4500 My ago, we assume that the radioactivity has been gradually concentrated in the upper 600 km of the Earth since 4500 My ago. In a second model we assume that the radioactivity has been gradually concentrated in the upper 200 km since 4500 My ago. Finally, we assume that the radioactivity has been gradually concentrated in the whole mantle since 4500 My ago. These three models are referred to as model GH-U6, GH-U2, and GH-M respectively.

The development of temperature as a function of time within model GH-U6 is shown in Figure 5. It is seen that, after a rapid increase in temperature, at $t > 2.2 \times 10^9$ years the temperature at depths between 100 and 5500 km decreases with time until the present time, while near the Earth's center there is a continual increase in temperature. At $t = 2.2 \times 10^9$ years the temperature in the mantle attained to the maximum. The build-up of radioactive heat sources near the surface is assumed in this model. However, the temperature of the material at the 600 km depth has not been raised above the temperature at a greater depth within the mantle. This is the result of the high initial temperature. The present temperature is 6460°K at the Earth's center, 5900°K at the core-mantle boundary, which exceeds the melting temperature of diopside at depths between 100 and 2900 km.

In models GH-U6, GH-U2, and GH-M the present temperature distributions are listed in Table 4. These present temperatures lie

Fig. 5. The development of the temperature distribution within model GH-U6.

Table 4. Present temperature distributions for GH models.

Depth km	Model GH-U6 T, °K	Model GH-U2 T, °K	Model GH-M T, °K
100	2540	2500	2380
200	3250	3210	3070
400	3960	3760	3800
600	4340	4140	4250
800	4590	4430	4580
1000	4790	4670	4840
1500	5200	5120	5320
2000	5490	5440	5640
2900	5900	5870	6040
4000	6320	6320	6410
5000	6430	6430	6480
6370	6460	6460	6490

well above the estimated melting temperature of diopside in the mantle. Thus, GH models

fail to meet the requirement of a solid mantle. This results from the gradual emission of gravitational energy and high initial temperature. In model GH-U2 the present temperature is higher than that of GH-U6 in the uppermost layer of the Earth. In model GH-M the present temperatures is higher than that of model GH-U6 at all depths greater than 900 km.

Figure 6 shows the history of surface heat flow for GH models. At the very beginning stage in the Earth's history the heat flow decreased slightly with time. This is due to the high initial temperature. At the time between 0.3 to 1.8×10^9 years the heat flow increased rapidly with time, but at $t > 1.8 \times 10^9$ years it decreases with time until the present time. In GH models there was a maximum surface heat flow at $t = 1.8 \times 10^9$ years or 2.7×10^9 years ago. The maximum heat flow occurred 1.8×10^9 years after the

beginning of the core-mantle separation. For model GH-U6, the present surface heat flow is $76.0 \text{ ergs/cm}^2\text{-sec}$; for model GH-U2, $76.6 \text{ ergs/cm}^2\text{-sec}$; for model GH-M, $70.5 \text{ ergs/cm}^2\text{-sec}$. In all these models the present flows are in excess of the observed heat flow.

(B) *Rapid formation of the core*

(B-1) *Initial cold Earth (RC);*

In model RC we investigate the thermal history of an initially cold Earth with consideration of the energy emitted in a rapid growth of the core. It is assumed that the energy had been rapidly emitted over the whole body of the Earth at the time between 1.5 to 1.6×10^9 years.

In the present calculations we shall use three models about the concentration of radioactive elements in the mantle. In one model, as a result of the rapid growth of the core at $t = 1.5 \times 10^9$ years, we assume that the radioactivity had been rapidly concentrated in the upper 600 km of the Earth at the time be-

Fig. 6. Variation of surface heat flow with time for GH models.

Fig. 7. The development of the temperature distribution within model RC-U6.

tween 1.5 to 1.6×10^9 years. In a second model we assume that the radioactivity had been rapidly concentrated in the upper 200 km at the time between 1.5 to 1.6×10^9 years. Finally, for the sake of comparison, it is assumed that the radioactivity had been rapidly concentrated in the whole mantle at the time between 1.5 to 1.6×10^9 years. These three models are referred to as model RC-U6, RC-U2, and RC-M respectively.

The development of temperature as a function of time within model RC-U6 is shown in Figure 7. It is seen that at all depths there was a sharp increase in temperature at the time between 1.55 to 1.65×10^9 years. After the very rapid rise in temperature, the outer layers of the Earth cool rapidly, while at all depths greater than 1400 km temperature does not greatly change with time. It is clearly seen that the gravitational energy has dominated the thermal history of the Earth. The present temperature is 4260°K at the Earth's center, 4100°K at the core-mantle boundary, which exceeds the melting temperature of diopside at depths between 100 and 900 km.

In models RC-U6, RC-U2, and RC-M the present temperature distributions are listed in Table 5. In model RC-U2 the present temperature is higher than that of RC-U6 in the

Table 5. Present temperature distributions for RC models.

Depth km	Model RC-U6 T, $^\circ\text{K}$	Model RC-U2 T, $^\circ\text{K}$	Model RC-M T, $^\circ\text{K}$
100	2150	2160	1810
200	2820	2720	2440
400	3390	3030	3050
600	3640	3260	3410
800	3740	3420	3660
1000	3810	3540	3850
1500	3930	3760	4150
2000	4000	3910	4320
2900	4100	4070	4370
4000	4180	4180	4250
5000	4230	4230	4240
6370	4260	4260	4260

Fig. 8. Variation of surface heat flow with time for RC models.

uppermost layer, which exceeds the melting temperature of diopside at depths between 100 and 700 km.

Figure 8 shows the history of surface heat flow for RC models. In all these models, at $t = 1.6 \times 10^9$ years, there was a sharp increase in heat flow followed by a rapid decrease. The maximum value of the heat flow took place at $t = 1.63 \times 10^9$ years, i.e., 2.87×10^8 years ago, which occurred about 1.3×10^8 years after the differentiation of the core. For model RC-U6, the present surface heat flow is $62.8 \text{ ergs/cm}^2\text{-sec}$; for model RC-U2, $63.1 \text{ ergs/cm}^2\text{-sec}$; for model RC-M, $51.4 \text{ ergs/cm}^2\text{-sec}$, which are in agreement with the observed heat flow. In model RC-U2 the present heat flow is almost identical with that in model RC-U6. This results from the rapid core-mantle separation and the rapid near surface concentration of the radioactive heat sources at a time 3.0×10^9 years. Hence, in future, the heat flow in model RC-U2 will be lower than that in model RC-U6.

(B-2) Initial hot Earth (RH):

In model RH we investigate the thermal history of an initially hot Earth with con-

Fig. 9. The development of the temperature distribution within model RH-U6.

sideration of the energy emitted in a rapid growth of the core. It is assumed that the energy had been rapidly emitted over the whole body of the Earth at the time between 0.0 to 1.0×10^8 years.

In the present calculations we shall use three models about the concentration of radioactive elements in the mantle. In one model, as a result of the rapid growth of the core at $t=0.0$, we assume that the radioactivity had been rapidly concentrated in the upper 600 km of the Earth at the time between 0.0 to 1.0×10^8 years. In a second model we assume that the radioactivity had been rapidly concentrated in the upper 200 km at the time between 0.0 to 1.0×10^8 years. Finally, we assume that the radioactivity had been rapidly concentrated in the whole mantle at the

time between 0.0 to 1.0×10^8 years. These three models are referred to as model RH-U6, RH-U2, and RH-M respectively.

The development of temperature as a function of time within model RH-U6 is shown in Figure 9. It is seen that at all depths there was a sharp increase in temperature at the time between 0.2 to 1.1×10^8 years. This brief temperature rise at the very beginning soon ceased and cooling commenced. At $t > 1.1 \times 10^8$ years the outer layers of the Earth cool rapidly, while at all depths greater than 3000 km the temperature decreases slowly with time until the present time. It is clearly seen that the gravitational energy has dominated the thermal history of the Earth. The rate of cooling was greater in the past than it is now. This is due to the rapid formation

Table 6. Present temperature distribution for RH models.

Depth km	Model RH-U6 T, °K	Model RH-U2 T, °K	Model RH-M T, °K
100	2330	2290	2140
200	3160	2920	2860
400	3710	3370	3560
600	4050	3690	4000
800	4250	3930	4310
1000	4420	4130	4550
1500	4740	4510	5000
2000	4980	4810	5320
2900	5370	5250	5670
4000	5800	5760	5960
5000	6000	5990	6060
6370	6100	6100	6120

of the core and the upper mantle, and the high initial temperature. The present temperature is 6100°K at the Earth's center, 5370°K at the core-mantle boundary, which exceeds the melting temperature of diopside at depths between 100 and 1700 km.

In models RH-U6, RH-U2, and RH-M the present temperature distributions are listed in Table 6. In model RH-U2 the present temperature is lower than that in model RH-U6 at all depths, which exceeds the melting temperature of diopside at depths between 100 and 1200 km. In model RH-M the present temperature is higher than that in model RH-U6 at all depths greater than 700 km. This results from the uniform distribution of radioactive heat sources in the mantle.

Figure 10 shows the history of the surface heat flow for RH models. In all these models, at the very beginning, there was a sharp increase in surface heat flow followed by a rapid decrease. The maximum value of the heat flow took place at $t=1.1 \times 10^8$ years, i.e., 4.39×10^9 years ago, which occurred some 1.1×10^8 years after the differentiation of the core. For model RH-U6, the present surface heat flow is 68.7 ergs/cm²-sec; for model RH-U2, 67.4 ergs/cm²-sec; for model RH-M, 62.4 ergs/cm²-sec. In model RH-U2 the modern value is in rough agreement with the observ-

Fig. 10. Variation of surface heat flow with time for RH models.

ed heat flow. The heat flow in model RH-U2 has been lower than that in model RH-U6 since 2400 My ago. This results from the rapid core-mantle separation and the rapid near surface concentration of the radioactive heat sources at a time 4.5×10^9 years ago.

§ 5. Discussion and Conclusions

In the process of the core formation, gravitational energy is transformed to thermal energy, to the amount of about 2800 joules/g on an average value for the whole mass of the Earth. This is equivalent to an average rise of temperature of about 2200°. The energy is higher than that given by RUNCORN (1965) and BIRCH (1965) using different methods and is lower than that given by UREY (1952), STACEY (1963), and SHIMAZU (1963) using approximate methods. The energy due to the formation of the core during geological time makes it difficult to account for an appropriate heat flow and a solid mantle. The large amount of energy released favors the view that the core was substantially complete

at the time between 4500 My and 3000 My ago, and that its process reached completion in a geologically short time. A rapid growth of the core is also confirmed by STACEY (1963), TOZER (1965), and SHIMAZU (1965, 1967) by different thermodynamical calculations. For the moon, IRIYAMA and SHIMAZU (1967) have obtained a similar conclusion by a simplified calculation. For the Earth's mantle, a different view has been obtained by LEE (1968). He argues that the mantle has differentiated slowly throughout the Earth's history by selective fusion of the silicates. This will be the subject for a future study.

The main factor to control the internal temperature is gravitational energy due to the core-mantle separation. The molten state of the core is possible to attain even if the Earth is formed by accretion of cold materials and the core is lacking in radioactive heat sources.

The author has calculated total radiogenic energy (RE), total heat loss (LE), gravitational energy (GE), and strain energy (SE):

$$RE = \int_0^\tau dt \int_M \sum_i H_i \exp(-\lambda_i t) dM, \quad (18)$$

$$LE = 4\pi R^2 \int_0^\tau l(t) dt, \quad (19)$$

where τ is the age of the Earth, λ_i is the decay constant of the element i , H_i is its rate of heat generation, and $l(t)$ is the surface heat flow value at time t . We assume that the radioactivity of the Earth equals that of chondritic meteorites (see Section 2-A-1). After integrating we obtain:

$$RE = 1.10 \times 10^{38} \text{ ergs.}$$

For the surface heat flow value $l(t)$ we adopt that of model RC-U2 in Section 4-B-1. After integrating we obtain:

$$LE = 0.387 \times 10^{38} \text{ ergs.}$$

As described in Section 4-A-1, the value $GE-SE$ is 1.68×10^{38} ergs, which gives the total release of thermal energy on core formation process. The value $RE+GE-LE-SE=2.39 \times 10^{38}$ ergs gives the total heat accumulated during the Earth's evolution (model RC-U2).

Fig. 11. The diagram of energy balance during the Earth's evolution (model RC-U2).

The total heat loss LE is lower than that suggested by SHIMAZU (1963) and, however, is within the range given by LUBIMOVA (1969). The scheme of energy balance during the Earth's evolution is shown in Figure 11.

Calculations given in Section 3-1 yield values of energies released during the core formation process of Venus, Mars, and Mercury. Data which enter into the calculations are summarized in Tables 7, 8, 9, and 10. Table 7 shows parameters for density

Table 7. Parameters for density models of Venus, Mars, and Mercury.

Differentiated model	
Core	
Venus:	$\rho = 7.227 + 2.0676P - 0.1879P^2$
Mars:	$(\rho = 7.227 + 2.068P)^*$
Mercury:	$\rho = 7.000 + 2.068P$
$P = \text{pressure in } 10^{12} \text{ dynes/cm}^2$	
Mantle	
$(d\rho/dr = \rho^3 g / \alpha \rho^2)^*$	
Venus:	$\alpha = 35 \text{ (km/sec)}^2$
Mars:	$(\alpha = 51)^*$
Mercury:	$\alpha = 51$
Undifferentiated model	
$\alpha = 29 \text{ (km/sec)}^2$	

* MACDONALD (1962).

Table 8. Density and pressure distributions for the differentiated and the undifferentiated models of Venus.

Radius km	Undifferentiated		Differentiated	
	Density g/cm ³	Pressure Mb	Density g/cm ³	Pressure Mb
0	6.50	1.725	11.69	2.951
300	6.49	1.715	11.66	2.919
600	6.47	1.692	11.59	2.845
900	6.44	1.658	11.48	2.738
1200	6.41	1.615	11.33	2.600
1500	6.36	1.561	11.15	2.434
1800	6.31	1.499	10.92	2.242
2100	6.25	1.427	10.65	2.028
2400	6.17	1.347	10.33	1.794
2700	6.09	1.258	9.97	1.545
3000	5.99	1.163	9.57	1.286
3150			9.36	1.153
			Boundary of core	
3150			5.16	1.153
3300	5.88	1.061	5.11	1.082
3600	5.76	0.953	5.02	0.945
3900	5.63	0.840	4.93	0.813
4200	5.48	0.724	4.83	0.685
4500	5.31	0.605	4.73	0.560
4800	5.12	0.484	4.63	0.436
5100	4.91	0.364	4.52	0.314
5400	4.67	0.245	4.40	0.195
			Boundary	
5400			3.57	0.195
5700	4.39	0.130	3.38	0.102
6056	3.99	0.000	3.11	0.000

models of these planets. Tables 8, 9, and 10 show the calculated densities and pressures for the undifferentiated and the differentiated models of Venus, Mars, and Mercury respectively. The gravitational potential energies for the undifferentiated Venus, Mars, and Mercury are, respectively, -1.594×10^{39} ergs, -4.855×10^{37} ergs, and -1.777×10^{37} ergs. The gravitational energies for the differentiated Venus, Mars, and Mercury are, respectively, -1.696×10^{39} ergs, -4.965×10^{37} ergs, and -1.911×10^{37} ergs. The values of energy change in reconstruction of these planets are presented in Table 11 for comparison with

Table 9. Density and pressure distributions for the differentiated and the undifferentiated models of Mars.

Radius km	Undifferentiated		Differentiated	
	Density g/cm ³	Pressure Mb	Density g/cm ³	Pressure Mb
0	4.29	0.272	8.09	0.412
300	4.28	0.268	8.05	0.400
600	4.26	0.260	7.99	0.370
900	4.24	0.247	7.90	0.325
1200	4.20	0.230	7.78	0.265
1324			7.72	0.236
			Boundary of core	
1324			3.84	0.236
1500	4.15	0.209	3.81	0.217
1800	4.10	0.183	3.77	0.184
2100	4.03	0.154	3.72	0.152
2400	3.95	0.122	3.66	0.119
2700	3.85	0.087	3.60	0.084
3000	3.74	0.050	3.54	0.048
3380	3.58	0.000	3.45	0.000

Table 10. Density and pressure distributions for the differentiated and the undifferentiated models of Mercury.

Radius km	Undifferentiated		Differentiated	
	Density g/cm ³	Pressure Mb	Density g/cm ³	Pressure Mb
0	5.70	0.263	7.80	0.386
300	5.69	0.259	7.77	0.374
600	5.65	0.242	7.72	0.347
900	5.60	0.219	7.63	0.304
1200	5.52	0.189	7.51	0.249
1500	5.43	0.152	7.37	0.180
1800	5.31	0.109	7.21	0.101
1943			7.12	0.059
			Boundary of core	
1943			3.14	0.059
2100	5.17	0.060	3.10	0.039
2440	4.97	0.000	3.03	0.000

the energies with radioactive decay. If the lost gravitational energy is completely converted to heat, the average temperature rise is about 1600°, 130°, and 310° for Venus,

Table 11. Constants used in the calculation and computed gravitational, and radioactive energies of Venus, Mars, and Mercury.

	Venus	Mars	Mercury
Radius* (km)	6056	3380	2440
Mass* (10^{27} g)	4.8655	0.6396	0.3296
Mass of core (10^{27} g)	1.404	0.08055	0.2350
Loss of gravitational energy (10^{38} ergs)	1.02	0.0110	0.0134
Radioactive energy** (10^{38} ergs)	0.902	0.119	0.0611

* REYNOLDS and SUMMERS (1969).

** It is assumed that the radioactivity of these planets equals that of chondritic meteorites.

Mars, and Mercury respectively. In the Earth and Venus the gravitational energy is larger than the energy of radioactive decay. For Mars and Mercury the lost gravitational energy is much lower than that given by LUBIMOVA (1967) by a simplified calculation. The difference will be attributed to different sizes of core used.

The effect of thermal convection is not taken into account in the present calculation. As being pointed out by LUBIMOVA (1969), there is not yet a satisfactory theory of the thermal history of the Earth based on convective transfer of heat. Moreover, as ELSASSER (1963) and BRANCAZIO (1964) pointed out, it seems probable that the viscosity of the mantle materials increases by a factor of 10^2 to 10^3 from the top of the mantle to the bottom. On this basis, the convective flow will be concentrated near the top of the mantle. Such a convective heat transfer will not dominate the thermal history of the whole Earth.

The effect of fusion (absorption of latent heat) is not taken into account in the present calculation. IRIYAMA (1966) has calculated the effect of fusion in a discussion of the thermal history of the Earth, and confirmed that the effect of latent heat is rather minor for the subsequent history.

The initial temperatures within the terrestrial planets fix the date for the differentiation of the mantle (IRIYAMA 1968). However, a lack of data on the thermal state at 4500 My ago precludes detailed conclusions.

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