Steady Solution for Thermal Convection within the Earth's Mantle

By

Jun IRIYAMA, Kazuo FUKUTA

*Department of Fundamental Science, Faculty of Engineering,
Chubu Institute of Technology, Kasugai, Aichi, Japan*

and

Yasuo SHIMAZU

*Department of Earth Sciences, Faculty of Science,
Nagoya University, Nagoya, Japan*

Abstract

A linear stability problem for the mantle of the earth is considered. Upper and lower mantle convection currents are assumed to reflect a rapid change in the viscosity at transition zone. The theory of mantle convection proposed by TOZER is adopted as a basis for the present investigation. For appropriate values of physical parameters it is found that the mantle should be in a convective state. Using the minimum temperature assumption, the internal temperature distribution in the presence of mantle convection current is considered.

The temperature at the bottom of the upper mantle convection cell is about 1450–1650°K regardless of conduction transfer and the thickness of the convecting zone. The temperature gradient within the upper mantle convecting zone differs very little from the adiabatic gradient. The temperature at the core-mantle boundary is about 2100–3800°K.

§1. Theoretical Background

The onset of convection at some zone within a chemically homogeneous planet is governed by the steepness of the temperature gradient dT/dr relative to the adiabatic gradient $(dT/dr)_{ad}$ at that zone. If dT/dr were less than or equal to $(dT/dr)_{ad}$ under these conditions, then there would be no energy transfer by convection. Convection current carries large amounts of energy from the interior of the earth, and therefore a steady situation will be realized eventually, in which both conduction and convection compete as heat transport mechanism (COX and GIULI, 1968). The actual $T(r)$ curve within the convective zone has a slope intermediate between that of the conductive $T(r)$ curve and that of the adiabatic $T(r)$ curve.

We assume that the thermal convection will occur if the Rayleigh number (the dimensionless parameter)

$$Ra = g\alpha d^3 \delta T / \kappa \nu \quad (1)$$

exceeds a critical value of 2600, where g is the acceleration of gravity, α the thermal coefficient of expansion ($=2 \times 10^{-5}/\text{deg}$), d the thickness of the layer, δT the temperature difference between the upper and lower boundary of the considering layer calculated from conduction theory, κ the thermal diffusivity, and ν the kinematic viscosity.

The theory of mantle convection proposed by TOZER (1967) is adopted here as a basis for the present calculation. We assume that convection is to reduce the temperature at the bottom of the convecting layer a minimum. If heat is fully transported to the upper part of the layer by convection current, then the temperature at the lower boundary of the convecting layer will become a minimum. The ratio of the heat brought to the upper surface of a convecting system to the heat that is brought by conduction is given in terms of the Nusselt number, a dimensionless quantity defined as (e.g., KNOPOFF, 1964)

Fig. 1. An outline of the calculation.

$$Nu = H / (\kappa \rho C_p \delta T / d) \quad (2)$$

where H is the total heat escaping at the upper surface and ρ the density. Thus, the temperature at the bottom of convection cell will decrease in proportion to the Nusselt number in the system and is (TOZER, 1967)

$$T_m = T_{UB} + \delta T / Nu \quad (3)$$

where T_{UB} is the temperature at the upper boundary of the fluid layer calculated from conduction theory. The Nusselt number is

calculated from the Rayleigh number in the convecting system by using the empirical results by SILVESTON (1958).

An outline of the present calculation is shown in Figure 1.

§ 2. Model

We consider the mantle convection model under the fixed boundary conditions and internal heat generation. The upper and lower mantle convection currents are assumed

to reflect a rapid change in the viscosity at transition zone, i.e., Bullen's region C. Three models are considered with respect to the conductive temperature distribution within the mantle. These are classified into the following types: (1) high temperature types (models H1 and H2; IRIYAMA, 1970, his models RC-U2 and RH-U2), (2) low temperature types (models L1 and L2), and (3) oceanic and shield types (models OCEAN and SHIELD; CLARK and RINGWOOD, 1964). These are illustrated in Figures 2, 3, 4, 5, and 6.

The thermal conductivity of the medium is given by

$$K_T = C + 16n^2ST^3/3\epsilon \quad (4)$$

where C is the ordinary lattice conductivity ($=2.5 \times 10^5$ ergs/cm/sec/deg), n the refractive index ($=1.7$), S the Stefan-Boltzmann constant, and ϵ the opacity (10/cm).

We assume that the shear viscosity of the medium is (TOZER, 1967)

$$\eta = 700T \exp(E/kT) \quad (5)$$

where E is the activation energy, and k the Boltzmann's constant. It is supposed that a rapid change of the viscosity occurs in the

transition zone of the mantle. Thus, in the calculations the activation energy within upper mantle convection zone is set equal to 4.0 eV. At the upper boundary of lower mantle convection zone it is set equal to 5.6 eV. The lower (upper) boundary of the upper (lower) mantle convection cell is assumed to be at depths between 500 and 700 km. The lower boundary of the lower mantle convection is assumed to be at the core-mantle boundary.

§ 3. Results and Conclusions

The parameters (UB , d , T_{UB} , and T_{LB}) adopted in the calculations and the physical values (η , Ra , Nu , and T_m) obtained from the calculations are illustrated in Tables 1, 2, 3, 4, 5, and 6 and Figures 2, 3, 4, 5, and 6. The blank space in the table shows that convection current is absent from the model. In the calculation of Rayleigh number the viscosity is set equal to its value at the upper boundary of the convecting region.

Main results are:

1. The temperature at the bottom of upper mantle convection cell is about 1450–1650°K regardless of the conduction equation

Table 1. Data for oceanic mantle (model OCEAN).

Model	UB , km	d , km	T_{UB} , °K	T_{LB} ,** °K	η , poise	Ra	Nu	T_m , °K
0-500'	50	450	1200	2055	5.36×10^{22}	7.06×10^3	2.83	1502
	500	2400	2055	3280	6.58×10^{24}	1.68×10^4	4.07	2356
0-500	80	420	1420	2055	1.58×10^{20}	1.17×10^6	9.97	1484
	500	2400	2055	3280	1.09×10^{25}	7.83×10^3	3.02	2463
0-500''	110	390	1520	2055	1.92×10^{19}	6.05×10^6	14.39	1557
	500	2400	2055	3280	1.48×10^{24}	5.24×10^4	4.95	2302
0-600'	50	550	1200	2143	5.36×10^{22}	1.42×10^4	3.81	1447
	600	2300	2148	3280	3.28×10^{25}	2.55×10^3		
0-600	80	520	1420	2143	1.58×10^{20}	2.43×10^6	11.75	1482
	600	2300	2143	3280	1.16×10^{25}	6.86×10^3	2.92	2529
0-600''	110	490	1520	2143	1.92×10^{19}	1.39×10^7	17.40	1556
	600	2300	2143	3280	1.51×10^{24}	4.71×10^4	4.83	2378
0-700'	50	650	1200	2221	5.36×10^{22}	2.54×10^4	4.67	1418
	700	2200	2221	3280	8.07×10^{25}	9.71×10^2		
0-700	80	620	1420	2221	1.58×10^{20}	4.74×10^6	13.64	1477
	700	2200	2221	3280	1.34×10^{25}	5.31×10^3	2.51	2642
0-700''	110	590	1520	2221	1.92×10^{19}	2.81×10^7	20.38	1555
	700	2200	2221	3280	1.55×10^{24}	4.11×10^4	4.69	2447

* UB ; the depth of the upper boundary of fluid layer.

** T_{LB} ; the temperature at the lower boundary of the fluid layer calculated from conduction theory.

Table 2. Data for shield mantle model (model SHIELD).

Model	UB, km	d, km	T_{UB} , °K	T_{LB} , °K	η , poise	Ra	Nu	T_m , °K
S-500'	120	380	1285	1990	4.44×10^{21}	4.53×10^4	4.89	1429
	500	2400	1990	3270	5.71×10^{25}	2.03×10^3		
S-500	155	345	1396	1990	2.73×10^{20}	3.99×10^5	7.82	1472
	500	2400	1990	3270	1.56×10^{25}	5.89×10^3	2.58	2486
S-500''	180	320	1475	1990	4.85×10^{19}	1.41×10^6	10.39	1525
	500	2400	1990	3270	3.47×10^{24}	2.44×10^4	4.61	2267
S-600'	120	480	1285	2085	4.44×10^{21}	1.03×10^5	5.77	1423
	600	2300	2085	3270	6.89×10^{25}	1.31×10^3		
S-600	155	445	1396	2085	2.73×10^{20}	1.00×10^6	9.26	1471
	600	2300	2085	3270	1.60×10^{25}	5.23×10^3	2.42	2576
S-600''	180	420	1475	2085	4.85×10^{19}	2.06×10^6	11.32	1529
	600	2300	2085	3270	3.12×10^{24}	2.48×10^4	4.65	2340
S-700'	120	580	1285	2173	4.44×10^{21}	2.03×10^5	6.71	1417
	700	2200	2173	3270	8.33×10^{25}	9.76×10^2		
S-700	155	545	1396	2173	2.73×10^{20}	1.26×10^5	6.04	1525
	700	2200	2173	3270	3.48×10^{24}	2.04×10^4	4.01	2455
S-700''	180	520	1475	2173	4.85×10^{19}	8.22×10^6	15.45	1520
	700	2200	2173	3270	3.99×10^{24}	1.75×10^4	4.11	2440

Table 3. Data for hot model (model H1).

Model	UB, km	d, km	T_{UB} , °K	T_{LB} , °K	η , poise	Ra	Nu	T_m , °K
H1-500'	40	460	1050	3150	1.18×10^{25}	1.00×10^2		
	500	2400	3150	4070				
H1-500	60	440	1400	3150	2.48×10^{20}	2.23×10^6	11.52	1552
	500	2400	3150	4070	1.68×10^{24}	3.48×10^4	4.51	3354
H1-500''	70	430	1610	3150	8.12×10^{18}	4.33×10^7	22.45	1679
	500	2400	3150	4070	7.67×10^{22}	6.37×10^5	8.69	3256
H1-600'	40	560	1050	3260	1.18×10^{25}	1.90×10^4	3.94	1610
	600	2300	3260	4070	3.87×10^{23}	1.21×10^5	5.99	3395
H1-600	60	540	1390	3260	3.13×10^{20}	3.55×10^6	12.79	1536
	600	2300	3260	4070	2.58×10^{24}	1.88×10^4	3.93	3466
H1-600''	80	520	1790	3260	2.31×10^{17}	1.98×10^9	53.07	1818
	600	2300	3260	4070	4.31×10^{21}	8.18×10^6	15.43	3312
H1-700'	40	660	1050	3345	1.18×10^{25}	3.23×10^2		
	700	2200	3345	4070				
H1-700	60	640	1400	3345	2.48×10^{20}	7.65×10^6	15.20	1528
	700	2200	3345	4070	3.20×10^{24}	1.42×10^4	3.67	3542
H1-700''	80	620	1790	3345	2.31×10^{17}	3.71×10^9	61.11	1815
	700	2200	3345	4070	4.56×10^{21}	6.67×10^6	14.74	3394

solution and the thickness of the convecting layer.

2. If the temperature at the bottom of upper mantle convection cell is smaller than about 1450°K, then the lower mantle convection does not occur.

3. Variation of depth of the lower boundary of the upper mantle convection between

500 and 700 km affects little on the temperature within the convecting region of the upper mantle. In this case, the temperature at the lower boundary of lower mantle convection changes by some 300 degrees.

4. If the convection is present, then the temperature gradient within the upper mantle differs very little from the adiabatic gradient.

Table 4. Data for cold model (model L1).

Model	UB , km	d , km	T_{UB} , °K	T_{LB} , °K	η , poise	Ra	Nu	T_m , °K
L1-500'	250 500	250 2400	1228 1505	1505 2808	2.27×10^{22}	1.15×10^3		
L1-500	290 500	210 2400	1280 1505	1505 2808	5.09×10^{21} 3.18×10^{25}	2.33×10^3 3.03×10^3	1.33 1.79	1448 2232
L1-500''	330 500	170 2400	1330 1505	1505 2808	1.35×10^{21} 6.89×10^{25}	3.42×10^3 1.45×10^3	1.87	1423
L1-600'	220 600	380 2300	1183 1593	1593 2808	9.21×10^{22}	1.55×10^3		
L1-600	230 600	370 2300	1200 1593	1593 2808	5.36×10^{22} 1.27×10^{25}	2.31×10^3 6.70×10^3	1.40 2.84	1480 2021
L1-600''	240 600	360 2300	1213 1593	1593 2808	3.58×10^{22} 6.47×10^{25}	3.04×10^3 1.43×10^3	1.79	1425
L1-700'	200 700	500 2200	1150 1673	1673 2808	2.76×10^{23}	1.56×10^3		
L1-700	210 700	490 2200	1170 1673	1673 2808	1.41×10^{23} 1.51×10^{25}	2.70×10^3 5.12×10^3	1.66 2.43	1473 2139
L1-700''	220 700	480 2200	1183 1673	1673 2808	9.21×10^{22}	3.74×10^3	2.09	1417

Table 5. Data for hot model (model H2).

Model	UB , km	d , km	T_{UB} , °K	T_{LB} , °K	η , poise	Ra	Nu	T_m , °K
H2-500'	50 500	450 2400	1280 3545	3545 5250	5.09×10^{21} 2.44×10^{23}	1.74×10^5 3.98×10^5	6.49 7.81	1629 3763
H2-500	60 500	440 2400	1500 3545	3545 5250	2.91×10^{19} 3.96×10^{23}	1.92×10^7 2.52×10^5	18.70 7.05	1609 3787
H2-500''	65 500	435 2400	1600 3545	3545 5250	4.49×10^{18} 5.03×10^{22}	1.59×10^8 1.75×10^6	30.10 10.91	1698 3701
H2-600'	50 600	550 2300	1280 3690	3690 5250	5.09×10^{21} 5.07×10^{23}	3.40×10^5 1.81×10^5	7.54 6.55	1599 3928
H2-600	60 600	540 2300	1500 3690	3690 5250	2.91×10^{19} 4.95×10^{23}	3.80×10^7 1.86×10^5	21.81 6.58	1600 3927
H2-600''	65 600	535 2300	1600 3690	3690 5250	4.49×10^{18} 1.05×10^{23}	2.10×10^8 7.96×10^5	32.05 9.13	1665 3861
H2-700'	50 700	650 2200	1280 3815	3815 5250	5.09×10^{21} 4.37×10^{23}	5.91×10^5 1.85×10^5	7.78 6.57	1605 4033
H2-700	60 700	640 2200	1500 3815	3815 5250	2.91×10^{19} 5.74×10^{23}	6.70×10^7 1.43×10^5	24.66 6.20	1594 4046
H2-700''	65 700	635 2200	1600 3815	3815 5250	4.49×10^{18} 1.15×10^{23}	3.73×10^8 6.46×10^5	36.45 8.71	1661 3980

The temperature at the core-mantle boundary is about 3800°K for model H2 and 2100°K for model L1. The result is close to that given by LYUSTIKH (1969) by a quantitative study.

5. For model H1-600 the Rayleigh number within the convecting region of the upper mantle is 3.55×10^6 and that of the lower

mantle is 1.88×10^4 ; for model H2-600, 3.80×10^7 and 1.86×10^5 ; for model S-600, 1.00×10^6 and 5.23×10^3 ; for model O-600, 2.43×10^6 and 6.86×10^3 ; for model L2-600, 4.47×10^3 and 3.92×10^3 ; and for model L1-600, 2.31×10^3 and 6.70×10^3 , respectively. These show that the upper mantle of the earth is considerably less thermally stable than the lower mantle.

Table 6. Data for cold model (model L2).

Model	UB , km	d , km	T_{UB} , °K	T_{LB} , °K	η , poise	Ra	Nu	T_m , °K
L2-500'	150	350	1190	1680	7.36×10^{22}	1.73×10^3		
	500	2400	1680	3060				
L2-500	160	340	1210	1680	3.92×10^{22}	2.79×10^3	1.64	1496
	500	2400	1680	3060				
L2-500''	170	330	1230	1680	2.14×10^{22}	4.38×10^3	2.22	1433
	500	2400	1680	3060				
L2-600'	120	480	1110	1785	1.14×10^{24}	4.38×10^2		
	600	2300	1785	3060				
L2-600	150	450	1190	1785	7.36×10^{22}	4.47×10^3	2.22	1458
	600	2300	1785	3060				
L2-600''	170	430	1230	1785	2.14×10^{22}	1.20×10^4	3.41	1393
	600	2300	1785	3060				
L2-700'	130	570	1140	1880	3.90×10^{23}	2.27×10^3	1.40	1668
	700	2200	1880	3060				
L2-700	135	565	1158	1880	2.10×10^{23}	3.90×10^3	2.15	1493
	700	2200	1880	3060				
L2-700''	140	560	1170	1880	1.41×10^{23}	5.50×10^3	2.55	1448
	700	2200	1880	3060

Fig. 2. Steady convection solutions $T(r)$ for hot (H1, IRIYAMA, 1970, his model RC-U2) and cold (L1) models.

H1: 4, 5 and 6; Upper boundary of the upper mantle convection at depth 60 km. Upper boundary of the lower mantle convection at depth 700, 600 and 500 km, respectively.

L1: 1, 2 and 3; Upper boundary of the upper mantle convection at depth 210, 230 and 290 km, respectively. Upper boundary of the lower mantle convection at depth 700, 600 and 500 km, respectively.

Ad. Temp.; Adiabatic temperature distribution by VERHOOGEN (1951).

Fig. 3. Steady convection solutions $T(r)$ for oceanic and shield mantle models (from 0-400 km depth, CLARK and RINGWOOD, 1964).

OCEAN: 1, 4 and 6; Upper boundary of the upper mantle convection at depth 80 km. Upper boundary of the lower mantle convection at depth 700, 600 and 500 km, respectively.

SHIELD: 2, 3 and 5; Upper boundary of the upper mantle convection at depth 155 km. Upper boundary of the lower mantle convection at depth 700, 600 and 500 km, respectively.

Fig. 4. Steady convection solutions $T(r)$ for model H1.

1, 2, 3, 4, 5, 6, 7 and 8; Upper boundary of the upper mantle convection at depth 50, 50, 60, 60, 60, 70, 70 and 70 km, respectively. Upper boundary of the lower mantle convection at depth 700, 600, 700, 600, 500, 700, 600 and 500 km, respectively.

Fig. 5. Steady convection solutions $T(r)$ for cold models (L1 and L2).

L1: 1, 2 and 5; Upper boundary of the upper mantle convection at depth 210, 230 and 290 km, respectively. Upper boundary of the lower mantle convection at depth 700, 600 and 500 km, respectively.

L2: 3, 4 and 6; Upper boundary of the upper mantle convection at depth 135, 150 and 160 km, respectively. Upper boundary of the lower mantle convection at depth 700, 600 and 500 km, respectively.

Fig. 6. Steady convection solutions $T(r)$ for hot (H2, IRIYAMA, 1970, his model RH-U2) model.

1, 2, 3, 4, 5, 6, 7, 8 and 9; Upper boundary of the upper mantle convection at depth 50, 50, 50, 60, 60, 60, 65, 65 and 65 km, respectively. Upper boundary of the lower mantle convection at depth 700, 600, 500, 700, 600, 500, 700, 600 and 500 km, respectively.

6. The thermal structure for the convecting zone under the oceans differs very little from that under the continents.

7. If the temperature at the bottom of the upper mantle convection cell is low, then the temperature of the lower mantle convection system is high. This results from the fact that the viscosity becomes larger or the Rayleigh number smaller as the temperature becomes lower.

References

- Clark, S. P., and A. E. Ringwood, Density and constitution of the mantle, *Rev. Geophys.*, **2**, 35-88, 1964.
- Cox, J. P., and R. T. Giuli, *Principles of Stellar Structure, Vol. 1; Physical Principles*, Gordon & Breach, New York, 1968.
- Iriyama, J., Thermal history of the earth with consideration of the formation of its core, *Jour. Phys. Earth*, **18**, 295-311, 1970.
- Knopoff, L., The convection currents, *Rev. Geophys.*, **2**, 89-122, 1964.
- Lyustikh, E. N., Problem of convection in the earth's mantle, in *The Earth's Crust and Upper Mantle Geophys. Monograph 13*, pp. 689-692, American Geophysical Union, Washington, D. C., 1969.
- Silveston, P. L., Wärmedurchgang in waagerechten Flüssigkeitsschichten, *Forsch. Ing. Wes.* **24**, 29-32, 59-69, 1958.
- Tozer, D. C., Towards a theory of thermal convection in the mantle, in *The Earth's Mantle*, pp. 325-353, Academic Press, London, 1967.
- Verhoogen, J., The adiabatic gradient in the mantle, *Trans. Am. Geophys. Un.*, **32**, 41-43, 1951.

(Received April 23, 1971)