STABILITY OF TRANSFORM FAULT AS INFERRED FROM VISCOUS FLOW IN THE UPPER MANTLE

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The most stable flow pattern responsible for a ridge transform fault system is studied based on a mantle model of a layered viscous fluid. If the flow beneath the ridge is caused by a passive convection, the stable flow on the earth's surface is most likely perpendicular to the ridge axis only for components of small wavelength, less than about 5.5 times of the thickness of the lithosphere, and oblique to the ridge axis for long wave components. This difference may suggest the reason why transform faults are formed on every oceanic ridge. It is concluded that a very small viscosity in the asthenosphere and negligibly small resistances against sliding motion are required to maintain the system. It is also suggested that the amount of ridge offset depends not only on the lithosphere thickness but also on the total direction of the system with respect to the plate motion. From the characteristic feature of the equatorial mid-Atlantic ridge, the thickness of the viscous lithosphere is estimated to be about 30 km or less under the rigid plate and the viscosity in the asthenosphere may be as small as one-hundredth of that in the lithosphere.

1. Introduction

The origin, the evolution and the stability of transform faults of oceanic ridges have been fundamental problems of plate tectonics because of their important bearing on convection currents beneath the ridges. On every oceanic ridge, transverse fracture zones are so numerous that the apparent sinuosity of the east Pacific rise, for example, is merely the consequence of successive offsets of a single ridge crest, the transform faults, (Menard, 1966).

Sizes of these faults are remarkably various, the largest of the seismically active ones is as large as 850 km (Table 1). Similar characteristics of spreading centers are clearly shown in physiographic diagrams of ocean floors (Heezen et al., 1967, 1968).

The orthogonal character between ridge and fault is almost universal among oceanic ridges except those in the Indian ocean and the equatorial
Atlantic. In view of the evidence that the movements on the earth's surface are expressible in terms of several rigid plates in relative motions with each other (Le Pichon, 1968), the apparent sinuosity of the ridge or the relative location of each transform fault, must be stable at least as long as the ocean floor spreading is in a steady state. Even in the equatorial Atlantic, where the ridge crest is offset westward over 3000 km by a series of left lateral faults, the general trend is consistently parallel to the both continental margins of America and of Africa. And the fact that the most of fracture zones are very long straight lines consistently parallel to each other, (Le Pichon, 1968), may show other evidence of the stability of the ridge transform fault systems. Therefore, in the study of convection currents under ridges, flow patterns and the physical properties of the mantle must show such a feature in which a nearly orthogonal ridge transform fault system is easily formed and the pattern is maintained for a long time.

Recently, Brune (1972) obtained a stable ridge transform fault pattern by freezing wax from the surface and exerting a tensile force on the surface. When the temperature of material and the ratio of the spreading velocity to surface cooling were appropriately chosen, the resultant pattern was an orthogonal ridge transform fault system having precisely those properties outlined by Wilson (1965). He suggested that a symmetric spreading was produced in the condition in which no tensile strength across ridges and negligibly small resistances against sliding motion were necessary for maintaining the faults.

The convection currents are therefore presumed as passive beneath oceanic ridges; that is, upwelling of material at a ridge crest is a result only of hydrostatic forces in the fluid. It would be worthy to note in this respect that Matsuzawa (1968) stated a similar conclusion in his geological study of the African rift system, revealing that the rift system was nothing but a field of maximum tensile forces during the Miocene-Pliocene period and was not necessarily the upwelling center of a large scale mantle convection. If the ridge is the field of driving forces of global plate movements, these faults are presumably caused by the pressures produced by a thermal convection under the ridge. According to Talwani et al. (1965), however, free-air gravity anomalies over the north mid-Atlantic ridge are close to zero (within ±50 mgal) and zero or slightly positive over the crest of the east Pacific rise. In the Indian ocean, the mid-oceanic ridge, including the Carlsberg ridge, is generally characterized by a relative gravity high, but the anomaly is as small as 20 ~ 30 mgal [Le Pichon and Talwani, 1969]. These gravity anomalies, markedly contrasting with those over trenches, are also in favor of the passive convection beneath the ridge. Although the contribution of a thermal convection of global size to the plate motions is not neglected, the driving forces
are in most parts due to the sinking lithospheric plates under island arcs as discussed by Isacks and Molnar (1969).

2. Flow within a Viscous Fluid

Peculiar stress distribution within the upper mantle is produced by the spreading of an ocean floor as well as by a large scale mantle convection caused by horizontal density variation in the mantle. The latter is however unlikely to be the direct cause of such a small scale flow pattern as a transform fault system but can move the rigid plate on the earth's surface by which the transform fault system is produced. Assuming that all the driving forces, such as dragging by the sinking plate under island arcs, forces caused by the thermal convection and gravity sliding near ridge crests (Jacoby, 1970), are working through the motion of a rigid plate, the flow near the ridge crest may be represented by the flow in which the velocity is given on the surface of a viscous fluid.

The viscosity distribution is also an important parameter to the investigation of the ridge transform fault system. Accordingly, a three-layered structure is assumed to approximate the viscosity distribution in the upper mantle. Because the temperature dependence of viscosity is extremely large, the top layer is a rigid plate except in very close regions around the ridge axis, the second layer is the lithosphere assumed as a viscous fluid but not as a rigid plate, and the third is the asthenosphere with a low viscosity relative to that in the lithosphere. The hypothetical interface between the rigid plate and the viscous lithosphere is defined as the surface of the upper mantle in the following discussions because the boundary conditions are applied in this or in lower interfaces.

For a very slow motion of an incompressible fluid, the steady state velocity $\mathbf{v}$ must satisfy both the equation of motion and the equation of continuity as follows;

$$\eta P^2 \mathbf{v} + \rho \text{grad} U - \text{grad} P = 0 \quad \text{(1)}$$

and

$$\text{div} \mathbf{v} = 0 \quad \text{(2)}$$

where $U$ is the gravitational potential, $P$, the pressure, $\eta$, the viscosity and $\rho$, the density.

In the first place, we investigate the flow in a homogeneous viscous fluid. In the case of no fault, a cylindrical co-ordinate system $(r, \xi, z)$ with the $z$ axis parallel to the ridge axis is convenient. Figure 1 shows a vertical section of the homogeneous mantle beneath the rigid plate, where $O$ is the ridge axis and $AC$, the surface. The curl of (1) gives

$$\mathcal{F}^2 (\text{curl} \mathbf{v}) = 0 \quad \text{(3)}$$
Fig. 1. Cylindrical co-ordinate system used in a homogeneous viscous fluid. 

If all the quantities are independent of $z$, the first type of solution to (2) and (3) is represented as:

$$v_r = \frac{1}{r} \frac{\partial \Psi}{\partial \xi}, \quad v_\xi = -\frac{\partial \Psi}{\partial r} \quad \text{and} \quad v_z = 0,$$

where $\Psi$ is the stream function which satisfies

$$\nabla^2 \Psi = 0. \quad (5)$$

The second type of solution represents a flow parallel to the $z$ axis and perpendicular to the gravity force. Therefore the equation of motion is reduced to

$$\eta \nabla^2 v_z - \frac{\partial P}{\partial z} = 0. \quad (6)$$

From the assumption that all the quantities are independent of $z$, Eqs. (2) and (6) are reduced to

$$\nabla^2 v_z = v_x = v_\xi = 0. \quad (7)$$

Solutions to (5) and (7) are required to give the surface flow that are spreading with a constant velocity of $V$. Boundary conditions are therefore given in the form

$$v_x = V_P, \quad v_\xi = 0, \quad v_z = V_H \quad \text{at} \quad \xi = 0,$$

$$v_x = V_P, \quad v_\xi = 0, \quad v_z = -V_H \quad \text{at} \quad \xi = \pi,$$

where $V_P^2 + V_H^2 = V^2$ (for $V_P$ and $V_H$, see Fig. 2).

The first and the second type of flow that satisfy (8) are given as follows; First type:

$$v_r = V_P \left( 1 - \frac{2\xi}{\pi} \right) \cos \xi - \frac{2}{\pi} \sin \xi,$$

$$v_\xi = -V_P \left( 1 - \frac{2\xi}{\pi} \right) \sin \xi$$

and

$$v_z = 0.$$
Second type: \[ u_r = u_z = 0, \]
\[ u_\xi = V_H \left( 1 - \frac{2\xi}{\pi} \right). \] (10)

The resultant flow pattern satisfying above boundary conditions represents an oblique ocean floor spreading with a constant velocity of \( V \) towards the direction of \( \tan^{-1}(V_r/V_H) \) from the ridge axis. The stresses caused by the flow are represented by the following stress tensor;
\[
S = \begin{pmatrix}
\sigma_{rr} & \sigma_{r\xi} & \sigma_{r\zeta} \\
\sigma_{\xi r} & \sigma_{\xi\xi} & \sigma_{\xi\zeta} \\
\sigma_{\zeta r} & \sigma_{\zeta\xi} & \sigma_{\zeta\zeta}
\end{pmatrix} - P \cdot E
\] (11)

where \( P \) is the pressure, \( E \), a 3 \times 3 unit matrix, and
\[
\begin{align*}
\sigma_{rr} &= 2\eta \cdot \partial v_r/\partial r, \\
\sigma_{\xi\xi} &= 2\eta \cdot \partial v_\xi/\partial \xi, \\
\sigma_{\zeta\zeta} &= 2\eta \cdot \partial v_\zeta/\partial \zeta, \\
\sigma_{r\xi} &= \sigma_{\xi r} = \gamma (\partial v_\xi/\partial r - v_\xi/r + 1/r \cdot \partial v_r/\partial \xi), \\
\sigma_{r\zeta} &= \sigma_{\zeta r} = \gamma (1/r \cdot \partial v_\zeta/\partial \xi + \partial v_\xi/\partial \xi), \\
\sigma_{\xi\zeta} &= \sigma_{\zeta\xi} = \gamma (\partial v_\xi/\partial \zeta + \partial v_\zeta/\partial \xi).
\end{align*}
\] (12)

Substituting (9) and (10) into (12) we can write the deviatoric stress tensor \( S' \) in the form
\[
S' = S + P \cdot E = \begin{pmatrix}
0 & a & 0 \\
a & 0 & b \\
0 & b & 0
\end{pmatrix}
\] (13)

where \( a = -4\eta V_r \cos \xi/(\pi r) \) and \( b = -2\eta V_\zeta/(\pi r) \).

The equation of the principal stresses, \( R_1, R_2 \) and \( R_3 \), is given by
\[
\begin{vmatrix}
-R & a & 0 \\
a & -R & b \\
0 & b & -R
\end{vmatrix} = 0
\] (14)

and the equation of principal axes, by
\[
\begin{pmatrix}
0 & a & 0 \\
a & 0 & b \\
0 & b & 0
\end{pmatrix} \begin{pmatrix}
\alpha_r \\
\alpha_\xi \\
\alpha_\zeta
\end{pmatrix} = \begin{pmatrix}
R_r \alpha_r \\
R_\xi \alpha_\xi \\
R_\zeta \alpha_\zeta
\end{pmatrix}, \quad i = 1, 2, 3
\] (15)

where \( \alpha_r, \alpha_\xi \) and \( \alpha_\zeta \) are the direction cosines of the principal axes. From (13), (14) and (15) we get
\[
\begin{align*}
R_1 &= 0; & \alpha_r : \alpha_\xi : \alpha_\zeta &= b : 0 : -a, \\
R_2 &= \sqrt{a^2 + b^2}; & \alpha_r : \alpha_\xi : \alpha_\zeta &= -a : \sqrt{a^2 + b^2} : b, \\
R_3 &= -\sqrt{a^2 + b^2}; & \alpha_r : \alpha_\xi : \alpha_\zeta &= a : -\sqrt{a^2 + b^2} : b.
\end{align*}
\] (16)
Thus, the maximum stress is $R_2$, the minimum, $R_3$ and the intermediate, $R_1$. The maximum shear stress on the surface is exerted parallel either to the plane $\xi = 0$ or to the plane determined by the null axis $(b : 0 : -a)$ and the $\xi$ axis.

If the lithosphere has a finite shear strength, a number of vertical slip faults parallel to the null axis may be produced near the ridge axis as shown in Fig. 2, where the null axis is not perpendicular to the flow direction except for the case of $V_H = 0$ but makes a higher angle with the flow direction than that between the ridge axis and the flow direction. These characteristics of a viscous flow suggest that the following processes take place during the development of a transform fault system: (1) An ocean floor spreading is supposed

Fig. 2. Showing a graphical determination of null axis from the angle between the flow direction $OV$ and the ridge axis $OZ$. The null axis $OB$ must satisfy the following relation on the surface; $OA : AB = V_H : 2V_P$.

Fig. 3. Proposed scheme for the development of a ridge transform fault system. (1) Initial stage of an oblique flow with surface velocity indicated by arrows. (2) Generation of dip-slip faults parallel to the null axis. (3) Growth of new ridge axis along some of the faults produced in the step (2). (4) Formation of an oblique transform fault pattern. Each ridge segment is still subjected to a shear stress similar to that in (1). If the oblique pattern is not yet stable, small dip-slip faults around the ridge axis can grow into new ridge axes as shown in (3). (5) Completion of an orthogonal pattern.
to have begun with an oblique flow as shown in Fig. 3. (2) Immediately after that, shear stresses near the ridge axis become sufficiently high to produce a number of vertical faults on the surface. (3) Because of their dip-slip character these faults are not expected to grow into strike-slip faults but are supposed to stimulate the generation of new ridge axes in an echelon arrangement with an equal probability along the old ridge axis. (4) And the following movement of the lithosphere will produce strike-slip faults connecting new ridges at both ends. This process may be a development of an oblique ridge transform fault system, in which the fault strike is not perpendicular to the ridge axis. It is noted that the stress field around the new ridge axis is still similar to that in Step (1), although the new null axis makes a higher angle with the flow direction than in the case of Step (2). Therefore the subsequent plate motion may produce a number of vertical faults around the new ridge axis as shown in (4) of Fig. 3. Some of these faults may grow into new ridge axes. (5) Repeating these procedures the flow pattern tends to be an orthogonal system ((5) in Fig. 3) until an energy balance is accomplished between the ridge and the transform fault.

3. Energy Balance between Ridges and Transform Faults

In a layered fluid, an oblique flow is also resolved into the first and the second type similar to (9) and (10), respectively. These two types of flow termed as the V-type and H-type flows, respectively, are schematically shown in Fig. 4 together with the method how to resolve a flow into two. The energy dissipation in the oblique flow is given by

\[
H = \frac{1}{2 \eta} \int_0^\infty \int_0^r \sigma_{ij} \, r \, d\xi \, dr
\]

per unit width, where \( \sigma_{ij} \) is the deviatoric stress given by (12) and the summation over repeated indices implied (McKENZIE, 1969).

The energy dissipation \( H \), equivalent to the work necessary to maintain

![Fig. 4. Separation of a flow into V-type and H-type flows.](image-url)
the flow, can be separated into the two parts, one is associated with the V-type, and the other, with the H-type flow shown in Fig. 4. The length of a ridge axis is given by 1/sin θ per unit width. If a constant velocity is assumed on the surface, (17) is possibly written in the form

\[ H = (H_v V^2 \sin^2 \theta + H_H V^2 \cos^2 \theta) / \sin \theta \]  

(18)

where \( H_v \) is the energy dissipation per unit width of the V-type flow having a unit surface velocity, \( H_H \), the similar energy dissipation of the H-type flow, and \( \theta \), the direction of the ridge axis measured from the direction of flow and defined here as the ridge angle (Fig. 4). Equation (18) is rewritten as;

\[ H = H_H V^2 (k - 1) \sin \theta + \frac{1}{\sin \theta} \]

(19)

where \( k = H_v / H_H \).

The ratio \( k \) is a constant not dependent on \( \theta \) but on the viscosity distribution in the fluid. The most stable flow having a constant surface velocity \( V \) is therefore determined by

\[ \frac{\partial H}{\partial \theta} = 0 = \cos \theta (k - 1) + \frac{1}{\sin \theta}, \quad 0 < \theta \leq 90^\circ \]  

(20)

where the restriction \( 0 < \theta \leq 90^\circ \) comes from the symmetrical character of the flow. Therefore, the minimum in (19) is obtained under the condition that

\[ \theta = 90^\circ \quad \text{with } k \leq 2, \]

\[ \theta < 90^\circ \quad \text{with } k > 2. \]  

(21)

From (13), (17) and (20), the ratio \( k \) for a homogeneous fluid must be 2, although both of \( H_v \) and \( H_H \) become infinite if the integration in (17) covers the entire range of \( r \) from 0 to \( \infty \). Therefore only V-type flows are stable in a homogeneous viscous fluid, but it should be noted that \( k = 2 \) implies a weak minimum because the first three derivatives, \( \partial^3 H / \partial \theta^3 \), \( \partial^2 H / \partial \theta^2 \) and \( \partial H / \partial \theta \), are all zero if \( \theta = 90^\circ \). It is suggested that a considerable variation in flow pattern is expected in layered structures.

Before entering into a detailed discussion of \( H_v \) and \( H_H \) for layered structures, the stability condition similar to (20) is presented for a ridge transform fault system. In Fig. 5, showing a part of the system, \( AB \) and \( CD \) are assumed as ridge axes in the flow with a constant surface velocity \( V \) parallel to transform faults \( BC \) and \( DE \) but not necessarily perpendicular to the ridge axes. Thus, the flow is assumed as making an angle \( \theta \) with ridge axes.

Since the transform faults are systematically shifted in a left lateral or in a right lateral sense, the apparent direction of the system is defined as \( \phi \) representing the angle between the line \( AE \) and the flow direction. Then, the
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Fig. 5. Showing a part of a ridge transform fault system with a ridge angle $\theta$. If the interval between faults and the length of fault are constant, respectively, the total direction of the system is defined by the angle $\phi$ between $AE$ and the flow direction.

The energy dissipation $H_R$ associated with the ridge axis $CD$ is given by

$$H_R = V^3 L H_\mu \left( (k - 1) \sin \theta + \frac{1}{\sin \theta} \right)$$  

where $L$ is the interval between the two faults $BC$ and $DE$. The energy dissipation $H_T$ caused by the transform fault $DE$ is represented by

$$H_T = \alpha V^3 L H_\mu (\cot \phi - \cot \theta)$$  

where $\alpha$ is the ratio of the energy dissipation caused by a transform fault to that caused by the H-type flow with a surface velocity equal to the slip velocity of the fault. If coupling between the two flows each associated with ridges $AB$ and $CD$, respectively, and separated each other by the transform fault $BC$ is moderately weak, so that the Eqs. (22) and (23) are still valid but an energy exchange between the two flows is possible, the total energy of the system is approximated by the sum of $H_R$ and $H_T$. Hence the stability condition follows

$$\frac{\partial (H_R + H_T)}{\partial \theta} = 0.$$

From (22) and (23), (24) is reduced to

$$\cos \theta (1 - (k - 1) \sin^2 \theta) = \alpha.$$  

The structure of the transform fault beneath the ridge is not yet known,
but is supposed to change gradually into a viscous flow at a deep. Approximating the upper mantle by a two-layered model, the transform fault is assumed to be a vertical fault in the upper layer and a H-type in the lower layer. Let $\eta_0$ and $\eta_1$ be the viscosities in the upper and in the lower layer, respectively, $d$, the layer thickness, and $\sigma_f$, the frictional force against sliding motion of the fault. Then the ratio $\alpha$ is represented in the form

$$\alpha = \frac{H_v}{H_H} = \frac{\eta_1}{\eta_0} + \frac{2V\sigma_f d}{V^2 H_H}$$

where $H_H$ is approximated by the energy dissipation in a fluid of a uniform viscosity of $\eta_0$. From (26) order of magnitude estimation of $\alpha$ may be possible with the use of typical parameters as follows; $d=50$ km (McKenzie, 1967), $V=10^{-7}$ cm/sec, $\sigma_f=100$ bars, $\eta_0=10^{22}$ poise and $\eta_1=10^{21}$ $\sim$ $10^{22}$ poise (McConnell, 1968). $H_H$ is the most doubtful quantity because nonlinear effects may predominate near the ridge axis. The expression of $H_H$ is obtained from (13) and (17) as follows;

$$H_H = \frac{4\eta_0}{\pi} \log \frac{R}{r_0}$$

where $R$ is the approximate dimension of the flow supposed to be less than the earth's radius but larger than the size of a typical transform fault, and $r_0$, being half the width of the nonlinear region, may be comparable to or larger than half the width of a typical rift valley on oceanic ridges. If we put $R=1000$ km and $r_0=50$ km, tentatively, $H_H$ amounts to about $4 \times 10^{23}$ erg/cm/sec.

Therefore, the first and the second term in (26) is about $0.1 \sim 0.01$ and about $0.02$, respectively. Although these values are not reliable, it is suggested that the transform faults of oceanic ridge contribute very little to the energy of flow in the upper mantle.

4. Estimation of the Ratio $H_v/H_H$ for a Layered Model

As shown in the last section, the ratio of $H_v$ to $H_H$ is the most important

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Fig. 6. A two-layered upper mantle model. The flow within the mantle is schematically shown by arrows. $d$, thickness of upper layer; $\eta_0$ and $\eta_1$, viscosity coefficients in the upper and lower layers, respectively.
parameter to determine the stable flow pattern. In a layered structure, rectangular co-ordinates with the \( xy \) plane parallel to interfaces and with the \( y \) axis parallel to the ridge axis are convenient. Figure 6 shows a vertical section of the two-layered model used in this study, where the upper layer is the viscous lithosphere with a viscosity \( \eta_0 \) and with a thickness \( d \), and the lower, a half space with a low viscosity \( \eta_1 \). The \( y \) axis is normal to the section and the distance between the ridge and the neighboring trench is chosen as \( \pi \) in order to express the solutions in terms of Fourier components. Expressions of velocities and stresses for a V-type flow in rectangular co-ordinates are given in terms of the stream function \( \Psi \) that satisfies (5).

\[
\begin{align*}
\psi_x &= \partial \Psi / \partial x, \quad \psi_z = -\partial \Psi / \partial z, \\
\sigma_{xz} &= -P + 2\eta \cdot \partial \psi_y / \partial x, \\
\sigma_{zz} &= -P + 2\eta \cdot \partial \psi_y / \partial z,
\end{align*}
\]

and

\[
\sigma_{zz} = \eta (\partial \psi_x / \partial x + \partial \psi_y / \partial z)
\]

where \( P \) is the pressure to be determined from

\[
\eta \partial \psi_x / \partial x = 0.
\]

Stresses for a H-type flow are given by the velocity \( \psi_y \) as;

\[
\sigma_{xz} = \eta \cdot \partial \psi_y / \partial x \quad \text{and} \quad \sigma_{zz} = \eta \cdot \partial \psi_y / \partial z
\]

in the case of \( P \psi_y = 0 \).

For a symmetrical flow pattern, the velocities \( \psi_x \) and \( \psi_y \) are given in the form

\[
\psi_x \text{ or } \psi_y = \sum_{m=1}^{\infty} V_m F_m(z) \sin mx
\]

where \( V_m \) is an arbitrary constant, \( F_m(z) \), the function of \( z \) represented by a linear combination of \( \exp(\pm mz) \) and \( z \cdot \exp(\pm mz) \). A similar expression for \( \Psi \) is also possible. Equations (28) and (29) give the following expressions for velocities and stresses in a V-type flow;

\[
\begin{pmatrix}
\psi_{xj}/\sin mx \\
\psi_{zj}/\cos mx \\
\sigma_{xzj}/2\eta_j m \sin mx \\
\sigma_{zzj}/2\eta_j m \cos mx
\end{pmatrix} =
\begin{pmatrix}
1 & -1 & z-1/m & -z-1/m \\
1 & 1 & z & z \\
-1 & -1 & -z+1/m & -z-1/m \\
-1 & 1 & -z & z
\end{pmatrix}
\begin{pmatrix}
A_j \exp(-mz) \\
B_j \exp(mz) \\
C_j \exp(-mz) \\
D_j \exp(mz)
\end{pmatrix}
\]

where \( j = 0 \) and \( 1 \) imply those quantities in the upper and in the lower layers, respectively, and \( A_j, B_j, C_j \) and \( D_j \) are the arbitrary constants to be determined by boundary conditions. As has been stated in the preceding sections, the flow on the surface must be the same with the plate movements. Then
we assume the surface velocities as:

\[ v_{x_0} = \sin mx \quad \text{and} \quad v_{z_0} = 0 \quad \text{at } z = -d , \tag{33} \]

where co-ordinates used here are such that the \( z \) axis is taken downward positive, the surface is given by \( z = -d \) and the interface between the two layers, by the \( xy \) plane. The following are four of the boundary conditions to satisfy the continuities of stresses and velocities across the interface:

\[ \begin{align*}
  v_{s1} &= v_{x_0} , & v_{s1} &= v_{z_0} , & \text{at } z = 0 . \\
  \sigma_{s1} &= \sigma_{x_0} , & \sigma_{s1} &= \sigma_{z_0} , & \text{at } z = 0 .
\end{align*} \tag{34} \]

The last two are \( B_1 = C_1 = 0 \) in order to make \( v_{s1} , \, v_{z1} , \, \sigma_{s1} \) and \( \sigma_{z1} \to 0 \) at \( z = \infty \).

The use of these boundary conditions together with (32) gives the following equation to determine the unknown constants:

\[
\begin{pmatrix}
  e & -1/e & -de - e/m & d/e - 1/m e \\
  e & 1/e & -de & -d/e \\
 1 & -1 & -1/m & -1/m \\
1 & 1 & 0 & 0 \\
-1 & -1 & 1/m & -1/m \\
-1 & 1 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
  A_0 \\
  B_0 \\
  C_0 \\
  D_0 \\
  A_1 \\
  C_1
\end{pmatrix}
=
\begin{pmatrix}
  1 \\
  0 \\
  0 \\
  0 \\
\eta_3/\eta_0 \\
\eta_3/\eta_0
\end{pmatrix}, \tag{35}
\]

where \( e = \exp(md) \).

Since the energy dissipation in the fluid must be equal to the work done on the surface, it is given by the following equation:

\[
H_{\nu m} = -\int_{-\pi}^{\pi} \sigma_{s} v_{s} \, dx = 2\pi \eta_0 m \left[ (A_0 - C_0 d - C_0/m) \exp(md) + (B_0 - D_0 d + D_0/m) \exp(-md) \right]. \tag{36}
\]

If the surface flow is given by

\[
v_x = v(x) \quad \text{for } 0 < x < \pi
\]

and

\[
v_x = -v(x) \quad \text{for } -\pi < x < 0
\]

the energy dissipation per unit width becomes

\[
H_{\nu} = \sum_{m=1}^{\infty} V_m^2 H_{\nu m} \tag{38}
\]

where

\[
V_m = \frac{1}{\pi} \int_0^{\pi} v(x) \sin mx \, dx .
\]

These are formulas concerned with the V-type flow. For the H-type flow,
however, all the results are represented by the following simple formulas:

\[ \psi = \sin mx \quad \text{at } z = -d, \]
\[ \psi = C(\cosh mz - (\eta_1/\eta_0) \sinh mz) \sin mx \quad \text{at } -d \leq z \leq 0, \]
\[ = C \exp (-mx) \sin mx \quad \text{at } z > 0, \]
\[ \sigma_{ys} = mC(\eta_0 \sinh mz - \eta_1 \cosh mz) \sin mx \quad \text{at } -d \leq z \leq 0, \]
\[ = -m\eta_1 C \exp (-mx) \sin mx \quad \text{at } z > 0 \]

and

\[ H_{IIm} = \pi mC(\eta_0 \sinh md + \eta_1 \cosh md) \]

where

\[ C = (\cosh md + (\eta_1/\eta_0) \sinh md)^{-1}. \]

Equation (38) is also adopted in this case.

Accordingly the energy ratio \( H_{Vr}/H_{IIm} \) depends not only on the viscosity ratio but also on the wavelength \( L(=2\pi/m) \) as shown in Fig. 7, where the energy ratio is plotted against \( L/d \) for several values of viscosity ratio. If \( \eta_1/\eta_0 < 1 \), the following are general features for a wide range of viscosity:

\[ H_{Vr}/H_{IIm} < 2 \quad \text{for } L/d \leq 5.5, \]
\[ H_{Vr}/H_{IIm} > 2 \quad \text{for } L/d \geq 5.5. \]

The asymptote of the ratio at short wavelengths is 2, the same value expected in a homogeneous fluid. The approximation of a homogeneous fluid is practically attained with a wavelength that is less than the layer thickness.

![Fig. 7. Energy ratio \( H_{Vr}/H_{IIm} \) plotted against the wavelength relative to layer thickness. Since the original data consist of line spectra, they are connected by smooth curves in this figure.](image)
For longer wave components, however, a small viscosity ratio gives rise to a high value in energy ratio as shown in Fig. 7. It is noted, however, that the ratio tends to decrease with wavelength and the maximum of the ratio is about 4 for possible ranges of layer thickness relative to the ridge-trench distance and of viscosity ratio in the upper mantle.

5. A Stable Ridge Transform Fault System

Whether transform faults are developed or not, the stable ridge angle $\theta$ between the flow direction and the ridge axis is to be determined by (25), but the apparent direction of the system with respect to the flow direction might have been fixed since the outbreak of ocean floor spreading. The present direction of the system might differ from the most stable one. However, the change into the most stable one is not easy because a large amount of asymmetric lithosphere production is required near the ridge. Instead, a number of transform faults are developed so as to form a relatively stable flow pattern near the ridge. Thus, the direction of a ridge axis is generally not the same with that of the system, but depends on the energy ratio of a V-type flow to that of a H-type flow. In order to discuss the problem in more detail, the stability condition (25) is graphically presented in Fig. 8 for several values of $\alpha$ defined

![Fig. 8. Relation between the stable ridge angle $\theta$ and the energy ratio $H_V/H_H$. $\alpha$, energy ratio of transform fault to the corresponding H-type flow as defined by (26). $\theta \geq \phi$ must be satisfied, where the equality stands for the case of no transform fault.](image-url)
in (26), where the abscissa is the energy ratio $H_v/H_H$, the ordinate, the stable ridge angle $\theta$, and $\phi$, the apparent direction of the system. Transform faults are stable only when $\theta > \phi$. The limiting case of no fault is given by $\theta = \phi$, because the flow pattern represented by $\theta < \phi$ gives no minimum in the energy dissipation, and existing flow patterns on the present earth's surface never meet such a condition. If the energy to maintain a transform fault is negligibly small, the ridge transform fault system becomes orthogonal or oblique depending on the energy ratio whether it is less than or larger than 2. The larger the ratio is, the lower the stable ridge angle $\theta$ becomes. As mentioned previously, $\phi$ is probably not determined by the stability condition but by the initial condition at the outbreak of an ocean floor spreading as typically shown in the equatorial mid-Atlantic ridge.

The origin of sinuosity of the east Pacific rise or of ridges in the Indian ocean is not clear because of doubtful topographic features parallel to these ridges. Nevertheless, a very long and straight ridge axis perpendicular to flow may not be the case in view of the following discussion:

In the case of no transform fault, the surface velocity on both side of a ridge axis may be approximated by the following formula;

$$v(x) = \begin{cases} V & \text{at } 0 \leq x \leq w \\ \frac{V}{w} (\pi - x) & \text{at } \pi - w \leq x \leq \pi \end{cases}$$

(41)

where $w$ is half the width of the nonlinear region around the ridge axis, within which the assumption of a constant velocity is not adopted in order to avoid extremely high stresses near the axis. From (38) and (41) we can write the Fourier coefficients as;

$$V_m = \frac{V}{m} \left(1 - (-1)^n\right) \frac{\sin \frac{mw}{m}}{mw}.$$  

(42)

Although there is yet little data concerning the nonlinear behavior of upper mantle material, some prominent rift valleys on ridge axes can afford such information that can presume it. Considering that the dimensions of valleys are 20~30 km in the north Atlantic and in the Indian ocean (Ewing and Heezen, 1960; Heezen et al., 1963), we assume $w = 0.5d \sim d$, where $d$ is the lithosphere thickness near the ridge. From (36), (38) and (39) we get

$$H_v/H_H = 2.4 \sim 2.8,$$

if $\frac{\eta_1}{\eta_0} < 0.05$, suggesting the instability of a V-type flow in the upper mantle. This might have an effect on the outbreak of ocean floor spreading if there is no appaeably weak belt on the earth's surface. The existing flow pattern on the present earth is the ridge transform fault system as schematically shown in Fig. 9 where a series of ridges is offset by a number of transform faults, $AB, \ldots, A'B'$ to the direction of $RR'$. If all the transform
Fig. 9. A transform fault system and the corresponding average velocity distribution obtained from the mean on a line parallel to RR'. The belt bounded by AA' and BB' is defined as a fault zone in the text.

Faults are confined within the belt bounded by AA' and BB', the belt is defined as the fault zone of the system.

The fault length is therefore given by

\[ L_r = \frac{W_f}{\sin \phi} \]  

where \( W_f \) is the width of the fault zone and \( \phi \), the direction of the fault zone defined by the angle between RR' and the direction of flow.

In order to approximate the system by a simple two-dimensional flow with the ridge axis RR', the velocities on a line parallel to RR' are averaged. Hence, the average velocity becomes a continuous function of the distance from RR' as shown in Fig. 9. It is apparent that the average velocity is identical in form with that given by (41) but the width 2w is replaced by \( W_f \), the width of the fault zone. As this is nothing but the exclusion of short-wave components, we make use of a more simple method that all the Fourier components given by (42) are separated into the two groups L and S, where L-group consists of components of longer wavelengths than the fault zone width, and S-group, of the remaining short-wave components. The advantages of the method are (1) the total energy is given by the sum of each energy and (2) the velocity reconstituted from the L-group components is a good approximation to the average velocity shown in Fig. 9. On account of these features, it is possible to assume that the L-group represents the flow of long-wave components without any fault pattern. If we assume again, although it might be too rough, that the short-wave components of the three-dimensional flow bounded by a transform fault at one side are approximated by the S-group, a further discussion of the transform fault pattern may be possible. Figure 10 shows the dependence of the two energy ratios of L and S groups on the wavelength by which the grouping has been made.
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Fig. 10. Energy ratios of L and S groups versus fault zone width \( W_f \). \( (H_v/H_H)_L \) and \( (H_v/H_H)_S \); energy ratios associated with longer and shorter wavelengths than \( W_f \), respectively. The Fourier coefficients used in these ratios are obtained by (42) with \( w = d = 0.01 \).

We note that this wavelength gives a measure of a fault zone. Each energy ratio is represented by

\[
(H_v/H_H)_L = \frac{\sum_{m=1}^{M} H_{r_m} V_m^2}{\sum_{m=1}^{M} H_{H_m} V_m^2}
\]

and

\[
(H_v/H_H)_S = \frac{\sum_{m=\lambda+1}^{M} H_{r_m} V_m^2}{\sum_{m=\lambda+1}^{M} H_{H_m} V_m^2}
\]

where \( M \) is the integer nearest to \( 2\pi/W_f \) provided that \( W_f \) is the width of the fault zone and \( V_m \) is given by (42). As the direction of fault zone is given a priori, we can find the ratio \( H_v/H_H \) from Fig. 8 by putting \( \theta = \phi \) for which the critical condition of no fault is satisfied. The energy ratio thus obtained is adopted to the curve \( (H_v/H_H)_L \) in Fig. 10 to find \( W_f/d \). For short-wave components, the same \( L/d \) determines the energy ratio \( (H_v/H_H)_S \) shown in the same figure. Using Fig. 8 again, we can find the stable ridge angle \( \theta \) from the energy ratio \( (H_v/H_H)_S \), which may be applicable to the small scale flow involved in a ridge transform fault system if the previously mentioned hypotheses are accepted. The length of fault is given by (43).

6. The Equatorial Mid-Atlantic Ridge

A fairly large number of transform faults have been detected on oceanic ridges and vary in size from the smallest, only 50 km, to the largest, about
Table 1. List of some pronounced transform faults.

<table>
<thead>
<tr>
<th>Location</th>
<th>Length (Approx.) km</th>
<th>Azimuth measured from ridge axis (Approx.) deg.</th>
<th>Sense of displacement left or right lateral</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mid-Atlantic Ridge</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gibbs F.Z. (53°N)</td>
<td>350</td>
<td>80--90</td>
<td>L</td>
<td>a, b, c</td>
</tr>
<tr>
<td>Oceanographer F.Z. (35°N)</td>
<td>130</td>
<td>80</td>
<td>R</td>
<td>d</td>
</tr>
<tr>
<td>Atlantis F.Z. (30°N)</td>
<td>50</td>
<td>70</td>
<td>R</td>
<td>e</td>
</tr>
<tr>
<td>Vema F.Z. (11°N)</td>
<td>300</td>
<td>80</td>
<td>L</td>
<td>f, g, h</td>
</tr>
<tr>
<td>7°N</td>
<td>450</td>
<td>60--80</td>
<td>L</td>
<td>f, i</td>
</tr>
<tr>
<td>St. Paul’s Rock (1°N)</td>
<td>550</td>
<td>70</td>
<td>L</td>
<td>f</td>
</tr>
<tr>
<td>Romanche F.Z. (0°)</td>
<td>800</td>
<td>80--90</td>
<td>L</td>
<td>f, i</td>
</tr>
<tr>
<td>Chain F.Z. (1°S)</td>
<td>290</td>
<td>90</td>
<td>L</td>
<td>f, i</td>
</tr>
<tr>
<td>7°S</td>
<td>250</td>
<td>90</td>
<td>R</td>
<td>j</td>
</tr>
<tr>
<td>23°S</td>
<td>330</td>
<td>90</td>
<td>R</td>
<td>a, j</td>
</tr>
<tr>
<td>East Pacific Rise</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blanco F.Z. (43°N)</td>
<td>360</td>
<td>75</td>
<td>L</td>
<td>k</td>
</tr>
<tr>
<td>Rivera F.Z. (19°N)</td>
<td>450</td>
<td>90</td>
<td>L</td>
<td>l</td>
</tr>
<tr>
<td>Siquellos F.Z. (8°N)</td>
<td>130</td>
<td>90</td>
<td>L</td>
<td>l</td>
</tr>
<tr>
<td>5°S</td>
<td>250</td>
<td>90</td>
<td>R</td>
<td>m</td>
</tr>
<tr>
<td>55°S</td>
<td>850</td>
<td>55--65</td>
<td>R</td>
<td>a, n</td>
</tr>
<tr>
<td>Indian Ocean</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Owen F.Z. (59°E, 13°N)</td>
<td>300</td>
<td>70</td>
<td>R</td>
<td>a, o, p</td>
</tr>
<tr>
<td>Vema Trench (68°E, 9°S)</td>
<td>450</td>
<td>40--45</td>
<td>R</td>
<td>a, q</td>
</tr>
<tr>
<td>Amsterdam F.Z. (80°E, 36°S)</td>
<td>450?</td>
<td>90?</td>
<td>R</td>
<td>a</td>
</tr>
<tr>
<td>Mozambique F. Z. (30°E, 50°S)</td>
<td>400</td>
<td>40--50</td>
<td>L</td>
<td>a</td>
</tr>
</tbody>
</table>

References:
- a: Barazangi and Dorman (1969), Plate 3.
- b: Johnson (1967), Fig. 1.
- d: Fox et al. (1969), Fig. 4.
- e: Hefzen and Tharp (1965), Fig. 6.
- f: Sykes (1967), Fig. 4.
- g: Van Andel et al. (1969), Fig. 1.
- h: Hefzen and Nafe (1964a), Fig. 2.
- i: Hefzen et al. (1964b), Figs. 1 and 2.
- j: Vacquier and Hefzen (1964), Fig. 1.
- k: Vine (1968), Fig. 4.
- l: Scull et al. (1971), Fig. 7.
- m: Menard (1966), Fig. 1.
- n: Sykes (1963), Fig. 2.
- p: Isacks et al. (1968), Fig. 5.
- q: Hefzen and Nafe (1964).

850 km as shown in Table 1 in which only pronounced transform faults are listed in order to indicate typical fault sizes and associated ridge angles.

According to Menard (1965), fracture zones on oceanic ridges are consistently roughly perpendicular to the ridge axes, but there are not a few exceptions as typically shown by some of those on the equatorial mid-Atlantic ridge (Fig. 11) and by many on the southwestern branch of mid-Indian ocean ridge. However, such slanting transform faults relative to ridge axes are of special interest because of the possibility that the lithosphere thickness and the viscosity distribution within the upper mantle are estimated from such a peculiar flow pattern on the earth’s surface. As shown in Fig. 10, the ratio \((H_v/H_L)\) increases with wavelength except for \(W_f/d \leq 4\). If the fault zone is as large as 7 times or more of the lithosphere thickness \(d\), \((H_v/H_L)\) is greater.
An increase in \( \alpha \) is also one of the causes of an oblique flow pattern (Fig. 8). It is noted throughout this study that all the transform faults are presumed to grow into sizes expected from the stability condition (25). But, in reality, the temperature in the lithosphere may be not so high to reject the existence of small faults far from stable, so that the size of fault implied in this study must be taken as the average of only prominent ones.

The most typical example is the transform fault pattern in the equatorial mid-Atlantic ridge, where the ridge crest between 15°N and 5°S is displaced by transform faults to a direction of nearly 35° in a left lateral sense as shown in Fig. 11 (Sykes, 1967). The general trend of the ridge axes is not likely perpendicular to the transform faults. A ridge angle of about 70° may be representative as shown in Table 1 and Fig. 11. The amount of each offset is considerably dispersive, ranging from hardly detected very small ones to ones as large as 800 km. But there are five prominent faults in this region as shown in Fig. 11. The average of these faults is estimated as \( L_f = 480 \) km. Although those parameters such as the ridge-trench distance \( D_{rt} \) and the width of nonlinear region around a ridge axis, \( 2w \), are also involved in the energy ratio curves in Fig. 10, the former has little effect on these curves if \( \eta_1/\eta_0 \ll 1 \) and the latter does not give much effect on determining \( W_f/d \). Therefore the following estimations are based only on the calculation with \( d = 0.01 D_{rt} \), but the ratio of \( w \) to \( d \) is eventually conditioned to satisfy \( w = 20 \sim 30 \) km approximately.
The energy ratio obtained from Fig. 8 with $\phi = 35^\circ$ is however beyond the upper limit of energy ratio in Fig. 10. Considering that this is resulted from the error in measuring $\phi$ on Fig. 11, we can assume an error of a few degrees. If we presume a little bit high angle, say $\phi = 38^\circ$, $(H_v/H_u)_L$ becomes about 3.5 with $\alpha < 0.01$. Then it follows that $W_f/d = 10 \sim 13$ with $\eta_1/\eta_0 < 0.01$ and, hence, $\theta = 70^\circ \sim 60^\circ$. As the width of fault zone is approximately 300 km if $L_f = 480$ km and $\phi = 38^\circ$ are adopted, the viscous part of the lithosphere amounts to about 30 km or less and the viscosity ratio $\eta_1/\eta_0$ may be less than 0.01. A higher value in $\phi$ gives rise to a thicker lithosphere and a higher ridge angle but if the limit is assumed as $\phi = 40^\circ$, the lithosphere not including the rigid plate may be less than 40 km thick and the ridge angle may be more than $80^\circ$. In conclusion, the most probable thickness is about 30 km which gives a ridge angle of about $70^\circ$.

7. Discussions and Conclusion

As shown in Figs. 8 and 10, a high viscosity in the lower layer and a high transform fault energy relative to the corresponding H-type flow are not favored on oceanic ridges in the Atlantic or in the Pacific ocean because a high viscosity does not give a low angle of $\phi$ and a high transform fault energy is against the stability of an orthogonal or nearly orthogonal pattern. The length of a developed transform fault relative to the thickness of the lithosphere not including rigid part is dependent not only on $\phi$ but also on the difference between $\phi$ and $\theta$. When $\phi$ is relatively high, the corresponding energy ratio in Fig. 8 is small, hence, a relatively short fault length results as shown in Fig. 10. If the width of fault zone is less than about 7 times of the layer thickness, an orthogonal system becomes stable. However, such data as the transform fault length, its distribution and ridge angles are not yet sufficient to make comparisons with theory except for those of the equatorial mid-Atlantic ridge. Single transform faults such as the one on the east Pacific rise at $55^\circ$S (Sykes, 1963), the Rivera fracture zone near Mexico (Sclater et al., 1971), the Vema trench of the mid-Indian ocean ridge (Heezen and Nafe, 1964) are also interesting but the method described here may not be applied because the two-dimensional approximation may not hold in these cases. On the contrary, the transform faults of the southwestern branch of mid-Indian ocean ridge are of special interest because of a possibly small spreading velocity (Ewing, 1969) and of their similarity to those in the equatorial Atlantic, but the distribution of faults and ridge axes are complex (Le Pichon and Heirtzler, 1968); the details are not yet known to make a quantitative comparison between them.

However, if the Mozambique fracture zone is representative, the fault
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length is less than 400 km and the ridge angle is 40° to 50°, apparently lower than that in the equatorial Atlantic. Hence, the viscosity in the asthenosphere and/or the energy to maintain transform faults might be higher than those in the mid-Atlantic.

The same calculations were carried out with a three-layered model representing an increase of viscosity with depth below 400 km from the earth's surface (McCONNELL, 1968), but the effects of a high viscosity in the bottom layer were very small and had little effect on the conclusions obtained with a two-layered model.

The numerical calculations in this study were carried out by FACOM 230-60 at the computer center of Nagoya University (Problem No. 4001BP1006).

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