CHARACTERISTIC PERIODS AND Q FOR OSCILLATIONS OF THE EARTH FOLLOWING AN INTERMEDIATE-FOCUS EARTHQUAKE

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A set of fundamental and overtone oscillations of the earth are observed from spectral analyses of digital data recorded worldwide by the new high gain, long-period seismographs, following the Celebes Sea earthquake of 11 June 1972. The characteristic theoretical periods and the $Q=Q(\omega)$, obtained from the spectral ratio of two different time windows, are used as an aid for modal identification. The fundamental spheroidal and torsional oscillations, $l=13-100$ and many overtones, $n=1, 2, 3, 4$, and 5 are identified. The overtone oscillations with high Q of about 600 are well marked on the spectrum of Q values. The observed periods are compared to those for existing earth models and the mean of previous observations, the agreement is good. The average absolute deviations between the three theoretical and observed suites of spheroidal mode periods are between 0.710 sec to 0.807 sec and that of the mean period is 0.714 sec. Over 50 new overtone mode of spheroidal oscillations and over 10 new overtone mode of torsional oscillations are reported.

1. Introduction

Novel developments in seismic instrumentation and techniques of data analysis offer a new opportunity for studying the free vibrations of the earth. Previously, eigen-vibrations of the earth were observed in the spectra of seismograms obtained from the following great earthquakes: Kamchatka, August 9, 1952 (BENIOFF *et al.*, 1961); Chile, May 22, 1960 (BENIOFF *et al.*, 1961); BOGERT, 1961; NESS *et al.*, 1961; ALSOP *et al.*, 1961; ALSOP, 1964b. BOLT, 1963; CONNES *et al.*, 1962; NOWROOZI and ALSOP, 1968; and DZIEWONSKI and LANDISMAN, 1970); Kurile Islands, October 13, 1963 (ALSOP, 1964a, ABE *et al.*, 1970; DZIEWONSKI and LANDISMAN, 1969); Alaska, March 1964 (NOWROOZI, 1965; SMITH, 1966; SLICHTER, 1967; DZIEWONSKI and GILBERT, 1972);

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and Rat Islands, February 4, 1965 (NOWROOZI, 1966). These earthquakes had magnitudes near 8 on the Richter scale and had relatively shallow depths of focus.

In addition, two deep-focus earthquakes also excited eigen-vibrations of the earth. These were (1) the Peru-Bolivia earthquake of August 1963, which had a focal depth of 543 km and a magnitude of $7\frac{3}{4}$, and (2) the Colombian earthquake of 31 July 1970, which had a focal depth of 651 km and a magnitude of 7.1. ALSOP and BRUNE (1965) reported on spheroidal oscillations for the Peru-Bolivia earthquake, and DARTLER *et al.* (1971), NOWROOZI (1972), and MENDIGUREN (1972) reported on eigen-vibrations for the Colombian earthquake.

The characteristic periods of the earth to be discussed in this paper were measured from the spectra of seismograms following the Celebes Sea intermediate-focus earthquake of 11 June 1972. The location, focal depth, and magnitude given by NOAA are 3.9°N , 124.3°E , 325 km, and $m_b=5.8$. Pasadena reported a surface-wave magnitude of 7.5 for this earthquake.

2. Instrumentation and Data Used

The high-gain digital seismograms recorded at Kipapa (KIP), Hawaii, and Kongsberg (KON), Norway, were used in this study. The high-gain system is composed of a set of three orthogonal Geotech seismometers with a natural period of 30 sec and three galvanometers with a natural period of 100 sec. The output of each galvanometer is amplified by a photo-tube-amplifier and recorded both digitally and photographically. The dynamic range of the digital recording used in this work is in order of 70 db, and it is limited by a photo tube amplifier. Sealed seismometer tanks, hermetically sealed bulkhead doors, and other special techniques developed in a two-year pilot program at the LDGO Ogdensburg Observatory are used to reduce long-period noise

Table 1. Distances, azimuths, components and magnifications.

	Δ	Azimuth epicenter to station	Azimuth station to epicenter	Components	Magnification at 30 sec period
KIP	0	0		Z	50400
	76.01	69.01	268.72	N , longitudinal and transverse	35000
				E	33700
KON		0		Z	100000
	99.39	332.42	65.46	N , longitudinal and transverse	76900
				E	80700

generated by atmospheric pressure changes. Using these techniques, useful magnifications of about 50000 and 100000 at periods of 30 sec were obtained for the vertical component of ground motion at Kipapa and Kongsberg, respectively. These magnifications are 10–100 times higher than those of conventional long-period stations. The details of a typical high-gain system were given by POMEROY *et al.* (1969).

The epicentral distances, azimuth and back azimuth to the recording stations, components used in time series analyzes, and magnification for each component are given in Table 1.

The minimum detectable signals at periods of 40 sec for the vertical, N-S and E-W components of the high-gain digital system at Kongsberg are 0.65×10^{-3} , 0.47×10^{-3} , 0.53×10^{-3} micron, respectively, and those for Kipapa are comparable.

3. Method of Analysis and Reduction of Digital Data

Ground motions were originally recorded digitally on magnetic tape at one-second intervals; the magnetic tape outputs were examined for drop-outs. A moving window with a length of 4000 sec was used for this purpose. The drop-outs were found by comparing each data point in the window with three times the standard deviation of all the data in the window. The number of drop-outs were less than ten in an interval of 8 hours and often drop-outs were approximately 1–10 sec long; these were replaced by values of the preceding points. Mean and trend were removed from the data within each window, and the results were plotted for visual inspection. Subsequently,

Fig. 1. The frequency response of a Butterworth filter and subsequent decimation.

the edited data were passed through a 6-pole Butterworth, low-pass, anti-alias filter, and the sampling rate was reduced by a factor of 20. The corner frequency of the filter was chosen such that signal-to-alias noise ratio in the reduced data was 60–80 db. The relative amplitude response of the filter is given in Fig. 1. For each component, 32 hours of digital data were analyzed starting approximately about the onset of the P phase. The total length of data was then divided into four equal sections of 8-hour intervals for estimation of period and Q values using the usual method of spectral ratio as described by NESS *et al.* (1961), NOWROOZI (1968), and DRATLER *et al.* (1971).

4. Procedure for Interpretation of Fundamental and Overtone Modes

Modes whose periods have sufficient separation can be interpreted with confidence by comparing the observed period of a spectral peak with the theoretical period for a realistic earth model. In this method, however, the difference between the period of neighboring modes must be larger than the uncertainty of the estimated period subject to identification. Modes whose periods do not satisfy this condition or whose periods are very close cannot be identified with any degree of certainty by this technique alone. So far, few useful methods have been suggested and applied for interpretation of overtone modes that have periods close to each other and to those of fundamental modes.

BRUNE (1964) suggested that the phase-velocity correspondence criterion which is best applicable to the interpretation of seismograms that contain a successive train of reflected and transmitted body phases (see ALSOP and BRUNE, 1965). Seismograms written after a deep-focus earthquake, in general, meet this criterion.

NOWROOZI (1972) applied this technique for analysis of the Colombia earthquake of 31 July 1970, which had a focal depth of 651 km. DRATLER *et al.* (1971), using seismograms of the same earthquake, identified the overtone modes of spheroidal oscillations from their very slow decay, high Q , and observed period.

MENDIGUREN (1972) developed a novel method for identification of various modes. This method consists of the summation of spectra observed at many stations, after correction for the phase determined from the known earthquake source mechanism. He also applied this technique to the same deepfocus Colombian earthquake of 31 July 1970.

The high- Q technique of DRATLER *et al.* (1971) is subject to possible error if applied for identification of overtone modes excited by shallow-focus earthquakes. Shallow-focus earthquake often have numerous large after-shocks, which may in turn excite additional eigen-vibrations of the earth and

thus confuse the decay rate. For example, the great Alaskan earthquake of March 1964 was followed by 6 aftershocks that had magnitudes greater than 6 within the first 10 hours after occurrence of the main shocks. For the Rat Islands earthquake of 1965, NOWROOZI (1968) estimated an error of 3.5% in estimated Q values because of aftershock activities. However, deep and intermediate-focus earthquakes are ideal for the application of the high- Q technique, as no substantial aftershock activities are associated with them. So far this technique has not been applied successfully and convincingly for identification of overtone modes excited by shallow focus earthquake.

In general, the observable surface amplitudes of overtone modes excited by shallow-focus earthquakes are so small that their spectral amplitudes are close to the noise level and are often lost in side-bands of the more energetic spectral peaks of the neighboring fundamental modes. When a source is suitably located at depth, certain overtone modes will be excited with sufficient amplitudes to be observable on the surface of the earth.

In the present work the interpretation of higher modes is based mainly on their higher Q values in comparison to those of other modes; the criterion of smoothness in phase velocity is used when information from high- Q values is ambiguous.

The spectrum of Q values is estimated from the ratio of spectral amplitudes from two time windows with a known time-lag. When Q values are plotted against all permissible periods in the time interval used for spectral analysis, the high- Q overtone modes stand out from the fundamental modes, which have smaller Q values. Thus the Q spectrum is helpful in interpretation of overtone modes.

5. *Results of Analysis*

Although the results presented in this work are based on interpretation of more than 40 spectra of various sections of seismograms and their combined analysis, in this section, I discuss only a few typical spectra in order to demonstrate both the quality of spectral peaks and the technique used. Figure 2 shows the superposition of two amplitude spectra obtained from the analysis of two portions of a seismogram written at Kongsberg. The spectral peaks show the vertical component of motion and are not corrected for instrumental response. Upper and lower spectra are obtained from two portions of a seismogram, one starting about 8 hours and the other, 16 hours after onset of the P phase. From two amplitude spectra such as these, the Q spectrum is obtained.

Figure 3 shows the superposition of three Q -spectra. The reason for superposition of Q -spectra are: a) to show the reproducibility of the Q -spectra

22 2 1640005 1

Fig. 2. Amplitude spectra for two portions of the vertical component of motion recorded at Kongsberg, Norway. The upper and lower spectra are based on eight hours of record and start approximately 8 hours and 16 hours after the onset of P wave. The straight line across the spectrum is the 95% confidence level for the upper spectrum. The various identified spheroidal modes are marked, the symbol $5S_{32}$ means fifth overtone mode of spheroidal oscillation of order 32.

from various time windows, and b) to give a visual measure of the variation in the estimated Q values. The Q -spectra given in Fig. 3 present three estimates of Q -spectra from three ratios of amplitude spectra for vertical ground

Fig. 3. Superposition of three Q -spectra. The high- Q oscillations such as $5S_{32}$ and $5S_{10}$ stand out. Spectra are obtained from the vertical component of motion at Kongsberg. Other high Q modes which are given in Table 6 are obtained from a similar spectra of the vertical component of motion at Kipapa.

motion at Kongsberg. The spectral peaks with high- Q are preliminarily interpreted as overtone modes (see DRATLER *et al.*, 1971). The estimated periods for many overtone modes estimated from Q -spectra are identical to those estimated from amplitude spectra. When slight variations in the period of a spectral peak were observed, presumably due to lateral inhomogeneties in Earth structure (REITER, 1973), a mean period is tabulated. In terms of frequency, the variation was never more than 1-2 divided by window length, 8 hours. When attempts were made to estimate Q values from the horizontal component of motion, it was found that Q -spectra, in general, were not reproducible. This probably is due to lack of complete separation of the torsional field from the spheroidal field; thus Q values estimated only from the vertical component of motion are tabulated.

The high- Q criterion does sometimes yield ambiguous interpretation; Figure 4 gives Q spectra obtained from the ratio of two amplitude spectra of the longitudinal component of motion at Kipapa. The position of a number of modes are indicated on the abscissa. In the band of high- Q values, several overtone modes are present so that it is not possible to interpret this high- Q mode. If actual theoretical Q values for these modes were known and they had sufficient variations, it could have helped the interpretation.

Fig. 4. A spectrum obtained from the longitudinal component of motion at Kipapa. The many closely spaced overtone modes that are within the high- Q band make the interpretation ambiguous.

Various techniques were used to increase the signal-to-noise ratio and to enhance the spectral peaks. The effect of nodal lines for spheroidal oscillation was reduced by summing the spectra of the longitudinal and the vertical components of motion (see Fig. 5; Nowroozi, 1965). The product of two

Fig. 5. The sum of spectra from vertical and longitudinal components of motion recorded at Kongsberg. The sum reduces the effect of nodal lines.

spectra from two portions of the same records were made to enhance signal-to-noise ratio (see Figs. 6, 7).

Fig. 6. Product of two sum spectra for reducing signal-to-noise ratio.

Fig. 7. Product of two spectra from two portions of the transverse component of motion at Kipapa.

6. Observed Periods and Q Values of Fundamental and Overtone Modes

Summaries of the observed periods and Q values for fundamental and overtone modes are given in Tables 2–12. As the observed periods for most modes obtained from seismograms recorded at KON and KIP were the same,

Table 2a. Observed period and Q of fundamental spheroidal oscillations ($n=0$).

Mode number	Period second	Half-Width second	Q	Mode number	Period second	Half-Width second	Q
10				56			
11				57	160.00	0.44	
12				58			
13	472.13	3.87		59	155.67	0.42	296
14				60	153.19	0.41	
15	426.69	3.11		61	151.58	0.40	
16	405.63	2.86		62	149.22	0.38	259
17	389.190	2.63	110	63			
18	374.03	2.43	127	64	145.45	0.37	278
19	360.00	2.25	126	65			
20	346.99	2.09		66			
21	334.88	1.95	145	67			
22	323.59	1.82	207	68	138.46	0.33	
23				69	135.21	0.32	
24	306.38	1.63	201	70	134.58	0.31	
25	296.91	1.53	242	71			
26	288.00	1.44	191	72	130.91	0.30	
27	282.35	1.38	198	73	129.15	0.29	
28	274.28	1.31	188	74			
29				75			
30	261.82	1.19	217	76	125.76	0.27	
31				77			
32	250.43	1.01		78	121.01	0.25	
33				79			
34	240.00	1.00		80	119.01	0.24	341
35	234.15	0.95		81	118.03		
36				82	116.60	0.24	
37	225.00	0.88	258	83	114.74	0.23	
38	221.54	0.85		84			
39				85	112.06	0.22	
40				86			
41	208.695	0.76		87	110.77	0.21	
43	200.00	0.69		88	109.51	0.21	337
44	197.26	0.67	261	89			
45				90			
46	190.73	0.63		91	106.27	0.20	
47	187.01	0.61		92			
48				93	103.97	0.18	
49	181.13	0.57		94			
50				95	101.77	0.18	
51	175.61	0.53		96	100.70	0.18	
52				97	99.65	0.17	
53	170.41	0.50	295	98	98.29	0.16	
54	167.44	0.47		99	97.60	0.16	
55	165.52	0.47	233	100	96.97	0.16	

Table 2b. Observed period and Q value for first overtone mode of spheroidal oscillations ($n=1$).

Mode number	Period second	Half-Width second	Q
8	553.84	5.31	214
11	429.85	3.21	
15	316.48	1.74	
17	285.15	1.41	
20	252.63	1.11	
24	219.85	0.84	259
25	214.93	0.80	
26	206.45	0.74	
28	195.92	0.67	
29	189.47	0.62	
35	163.64	0.46	325
36	160.89	0.45	
38	154.01	0.41	
40	147.69	0.37	

Table 3. Observed period and Q value for second overtone mode of spheroidal oscillations ($n=2$).

Order	Period second	Half-Width second	Q
12	364.56	2.31	244
15	309.68	1.66	
17	274.28	1.31	
26	180.00	0.56	
27	173.49	0.52	
28	169.41	0.49	312
35	142.57	0.35	
36	138.46	0.33	
38	133.33	0.31	
39	130.32	0.29	
40	128.00	0.28	234

only one average period is tabulated. For the few occasions in which slight variations in period were observed, the period deviation was within the half-width period.

Figures 8 and 9 show plots of phase velocity as a function of period, using Jean's equation (see NOWROOZI, 1972), for spheroidal and torsional oscillation, respectively. The phase velocity for fundamental modes is very smooth up to periods of about 130 sec which have wavelength of about 550 km; modes with shorter periods probably are affected by lateral heterogeneities of the earth. The phase-velocity plot for the first overtone mode is also smooth, with a possible exception around 200 sec. The interpretation of the first overtone mode of spheroidal oscillations is difficult because of its rela-

Table 4. Observed period and Q for third overtone mode of spheroidal oscillations ($n=3$).

Order	Period second	Half-Width second	Q
7	374.03	2.43	
17	242.02	1.02	
21	210.22	0.77	
26	178.88	0.56	
27	172.46	0.52	
28	166.47	0.48	
30	155.67	0.42	306
32	145.45	0.37	
33	141.18	0.35	
36	129.15	0.29	368
40	116.13	0.23	
41	112.72	0.22	398
42	112.94	0.21	
43	109.71	0.21	
44	107.32	0.21	408
45	105.49	0.19	342
47	102.49	0.18	
48	100.70	0.17	460
49	98.74	0.16	474

Table 5. Observed period and Q for fourth overtone mode of spheroidal oscillations ($n=4$).

Order	Period second	Half-Width second	Q
5	369.23	2.37	
6	334.88	1.95	
9	269.16	1.25	286
10	257.14	1.15	
14	225.00	0.89	288
16	211.76	0.78	
17	205.61	0.73	
19	192.00	0.64	291
20	187.01	0.61	
23	171.43	0.51	
25	161.80	0.45	260
29	146.19	0.37	
31	139.81	0.34	299
32	136.49	0.32	
33	132.11	0.30	
34	129.73	0.29	
35	127.43	0.29	
36	124.14	0.27	
39	117.55	0.24	
40	114.74	0.22	391

Table 6. Observed period and Q for fifth overtone mode of spheroidal oscillations ($n=5$).

Order	Period second	Half-Width second	Q
9	252.63	1.11	
10	238.02	0.98	406
13	202.82	0.71	514
14	194.59	0.66	
15	188.23	0.62	
16	181.71	0.60	
17	176.69	0.54	
20	163.64	0.46	
22	154.84	0.42	415
23	150.78	0.39	477
24	147.69	0.38	
25	144.00	0.36	433
32	123.61	0.26	626
34	118.03	0.24	448

Table 7. Observed period of fundamental mode torsional oscillations ($n=0$).

Order	Period second	Half-Width second
18	389.19	2.63
19	374.03	2.43
36	218.18	0.83
38	208.69	0.77
39	204.26	0.72
43	187.01	0.61
52	158.24	0.43
54	153.19	0.41
57	145.45	0.37
59	142.57	0.35
60	137.80	0.33
62	135.21	0.32
63	134.58	0.31
64	132.72	0.30
66	128.57	0.29
68	125.76	0.27
69	123.61	0.26
71	120.50	0.25
72	119.01	0.24
74	115.20	0.23
76	113.38	0.22
79	108.27	0.20
82	105.49	0.19
84	102.49	0.18

Table 8. Observed period of first overtone mode of torsional oscillations ($n=1$).

Order	Period second	Half-Width second
12	338.82	1.99
14	306.38	1.63
15	293.88	1.50
20	240.00	1.00
21	232.26	0.94
22	225.00	0.89
24	211.76	0.78
31	176.69	0.54
35	162.71	0.46

Table 9. Observed period of second overtone torsional oscillations ($n=2$).

Order	Period second	Half-Width second
7	360.00	2.25
9	323.59	1.82
11	290.91	1.47
12	274.28	1.31
15	236.06	0.97
20	197.26	0.67
26	167.44	0.49

Table 10. Observed period of third overtone mode of torsional oscillations ($n=3$).

Order	Period second	Half-Width second
1	309.63	1.66
6	285.15	1.41
8	266.66	1.23
10	250.43	1.09
14	214.92	0.80
16	198.62	0.68
19	178.88	0.56
22	163.64	0.46
24	154.84	0.42

Table 11. Observed period of fourth overtone mode of torsional oscillations ($n=4$).

Order	Period second	Half-Width second
4	228.57	± 0.91
13	190.73	0.61
16	175.61	0.53
17	170.41	0.50

Fig. 8. Phase velocity of fundamental and overtone modes of spheroidal oscillations.

Fig. 9. Phase velocity of fundamental and overtone modes of torsional oscillations.

Table 12. Observed overtone of fifth overtone mode of torsional oscillations ($n=5$).

Order	Period second	Half-Width second
18	147.69	0.38
20	140.49	0.34

tively small amplitude on the surface. The other overtone modes of spheroidal oscillations have smooth phase-velocity plots.

The phase velocity plot of fundamental torsional oscillations is smooth up to about a 150 sec period which has a wavelength of about 730 km; for periods shorter than 150 sec, torsional modes are probably affected by lateral heterogeneities. This wavelength is longer than the wavelength of 550 km for the spheroidal mode, and is due to the different depths of penetration for each oscillation. The overtone modes of torsional oscillations show a smooth phase-velocity plot. The phase velocities for ${}_1T_3$ and ${}_4T_4$ are outside the range of Fig. 9 and are not plotted.

Because of possible uncertainty in complete separation of spheroidal oscillations from torsional oscillations, only those Q values that were obtained from the vertical component of motion are tabulated. The tabulated values are an average of 2 to 3 estimates. Figure 3 shows the extent in variation of

Fig. 10. Q values of overtone modes of spheroidal oscillations.

Q values. Figure 10 shows a plot of Q values against period for overtone modes. There is an indication that Q values increase with increasing order of overtone number n ; however, this is only well observed for $n=5$, the fifth overtone mode.

7. Comparison of the Observed Period with the Mean and Theoretical Period

The mean period of previously observed modes reported by DZIEWONSKI

and GILBERT (1972), and theoretical periods obtained for four earth models, are used for comparison. The models are:

- (1) DZIEWONSKI and GILBERT (1973), model UTD 124A,
- (2) HADDON and BULLEN (1969), model HB1, and
- (3) DERR (1968, 1969), models D1 and D11.

The period difference in seconds between observed, mean, and theoretical periods for spheroidal and torsional modes against order number, l , are shown in Figs. 11 and 12. There is a total of only 5 fundamental modes of spheroidal oscillations that have differences in period of more than two seconds. These are modes with periods of less than 130 sec. For all the observed overtone modes of spheroidal oscillations, period differences as high as 3 sec exist for

Fig. 11. Observed period for spheroidal oscillations T_0 minus, mean period T_m , and theoretical periods for three earth models.

Fig. 12. Observed period of torsional oscillation minus mean and theoretical period for four earth models.

only ${}_1S_{11}$, ${}_9S_7$, and ${}_4S_6$ modes. The majority of the remaining period differences are within ± 1 sec, thus, the observations reported here are in close agreement with the previous mean and theoretical periods.

For torsional oscillations, the period differences are somewhat greater and show greater scatter than the spheroidal oscillations. This probably is due to a lack of complete separation of the torsional from the spheroidal modes. Nevertheless, the population of 54 period differences show a maximum absolute deviation of only 0.920 sec for fundamental modes. For overtone modes of the torsional oscillation the maximum deviation is only 1.370 sec which is for model UTD 124A of DZIEWONSKI and GILBERT (1972).

The average absolute deviation, D , is defined as:

$$D = \sum_n |T_0 - T_e| / N \text{ where } T_0 = \text{observed period, } T_e = \text{a reference period,}$$

either mean or theoretical period, and N is the total number of modes for which the mean of previous observation and theoretical periods of the earth model were available. This is used as a measure of agreement between observed period the mean and theoretical period. The absolute deviation between periods of the new fundamental spheroidal and torsional oscillations reported here and the mean of the previous observations as reported by DZIEWONSKI and GILBERT (1972) are 0.538 and 0.920 sec, respectively. For

Table 13. Average absolute deviation, D , in sec between observed, mean, and theoretical periods.

Mode	No.	Spheroidal oscillations			
		$T_0 - T_M$	$T_0 - T_{DG}$	$T_0 - T_{HB}$	$T_0 - T_D$
fundamental	26	0.538	0.599	0.516	0.458
1st overtone	3	1.076	2.237	2.160	2.217
2nd overtone	2	0.520	0.595	0.905	0.450
3rd overtone	1	1.980	1.30	3.030	
4th overtone	4	1.370	1.420	1.192	
5th overtone	2		0.1	0.475	0.445
overall	38	0.714	0.807	0.801	0.710

the overtone modes of spheroidal oscillations, the deviations vary from 0.52 sec to 1.98 sec. Thus, the results of this study are in excellent agreement with the mean of the previous observations where such data are available. Considering the total population of fundamental and overtone modes of spheroidal oscillations, where comparison can be made, the absolute deviations between periods reported here, the previous mean period, and periods for three theoretical models, (see Table 13), vary from 0.710 sec for model D1 of DERR (1969) to 0.807 sec for UTD 124A of DIEWONSKI and GILBERT (1972). Deviations for the mean period and for HB1 model of HADDON and BULLEN (1969) are 0.714 sec and 0.801 sec respectively. Thus, the theoretical periods of spheroidal modes for all three models are in good agreement with the present observations, and no decisive preference can be made as to which model best satisfies the new observations.

8. Conclusion

Fundamental and overtone modes of both spheroidal and torsional oscillations were observed from an intermediate focus earthquake, with a rather small body-wave magnitude of 5.8. Because of its suitable depth, this earthquake excited a set of overtone modes some of which had never been identified previously, over 50 new overtone mode of spheroidal oscillations and over 10 new overtone of torsional oscillations are reported. Q -spectra were used as an aid in identification of high- Q overtone modes. For periods of 100 to about 400 sec the Q of higher overtone modes showed a tendency to increase with increasing order of overtone number, n ; for $n=5$, this increase is well distinguished. For example, mode 5^{897} has a Q value of over 600. When possible, the observed periods were compared with the mean period of previous observations, and the period of existing theoretical models. Although the theoretical models are substantially different from each other, in the observed

period range reported here, no decisive conclusion can be made as to what model the data demand, thus indicating the difficult problem of none uniqueness of the Earth's structure. Although the observed periods reported here cannot be individually important for inferences of the internal structure of the earth as a whole, in a statistical sense, they can improve the accuracy of observed periods for each mode and thus are useful for inversion purposes. The success in measurement of free oscillations from relatively small earthquakes as reported here depended to a large extent on development of the new high-gain system with digital recording.

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