STATIC DEFORMATIONS IN AN OBLIQUELY LAYERED MEDIUM. PART I. STRIKE-SLIP FAULT

Ryosuke SATO

*Geophysical Institute, Faculty of Science,
University of Tokyo, Tokyo, Japan*

(Received September 25, 1974)

Static surface deformations due to a strike-slip fault which takes place along the dipping boundary between two different media are investigated in a two-dimensional case by using the Mellin transform.

Numerical computations reveal that the smaller the rigidity of the upper layer is, the larger the maximum displacement becomes, and the larger the dip-angle is, the larger the difference becomes between maximum deformations in an obliquely layered medium and those in a uniform medium. When the rigidity of the upper medium is half of that of the lower medium, the maximum surface displacement is about 1.2 times as large as that in a uniform medium. When one intends to obtain focal parameters by comparing observed residual displacements with theoretical values for a uniform medium, one should bear in his mind the difference mentioned above.

1. Introduction

Since the dislocation theory was introduced to the field of seismology, studies on the determination of focal parameters have been numerous by analyzing not only seismic waves but also residual deformations.

Although some recent studies have discussed characteristics of seismic waves (SATO, 1973 a, 1973 b) and static deformations (BEN-MENACHEM and GILLON, 1970; SATO, 1971; SATO and MATSU'URA, 1973) due to a realistic source model in a horizontally layered medium, almost all the studies have been confined to problems in a semi-infinite or an infinite medium. As a more realistic case, it is considered that the dislocation takes place at the boundary between two different media, for which only SATO and MATSUZAKI (1974) showed that different radiation patterns from those in a uniform single medium are expected. It is important to study the effect of the free surface on the deformations in an obliquely layered medium when the displacement discontinuity occurs along the interface.

In the present paper, static deformations at the surface due to a two-dimensional strike-slip fault along the dipping boundary between two media are investigated by applying the Mellin transform. Deformations for a dip-slip

fault, which are practically important when considering earthquakes occurred in the island arc area, will be studied in the next paper.

2. Integral Expressions of Formal Solutions

Define the Mellin pair in the usual way as

$$\left. \begin{aligned} f(r) &= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \tilde{f}(s) r^{-s} ds, \\ \tilde{f}(s) &= \int_0^\infty f(r) r^{s-1} dr, \end{aligned} \right\} \quad (1)$$

and denote all the transformed quantities by superimposed bars.

We take the cylindrical coordinate system (r, θ, z) and assume that the behavior is uniform to the z -direction, that is, the two-dimensional problem is considered. The medium I with rigidity μ_1 occupies the region $\theta_0 \leq \theta \leq 0$ and the medium II with rigidity μ_2 does $\pi \geq \theta > \theta_0$, $\theta=0$ and $\theta=\pi$ being the free surface (Fig. 1).

Fig. 1. Geometry of the problem.

Denote the z -component of displacements by v , which satisfies the Laplace's equation

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} = 0. \quad (2)$$

The Mellin transform of v with respect to r is taken as

$$\left. \begin{aligned} \bar{v}_1(s) &= A_1(s) \cos s\theta, & \text{in I,} \\ \bar{v}_2(s) &= A_2(s) \cos s(\pi - \theta), & \text{in II,} \end{aligned} \right\} \quad (3)$$

which satisfy boundary conditions $p_{z\theta} = 0$ at $\theta=0$ and $\theta=\pi$.

Boundary conditions at $\theta=\theta_0$ are given by

$$\left. \begin{aligned} v_1 - v_2 &= f(r), \\ \mu_1 \frac{\partial v_1}{\partial \theta} - \mu_2 \frac{\partial v_2}{\partial \theta} &= 0. \end{aligned} \right\} \quad (4)$$

Putting (3) into (4) and denoting the Mellin transform of $f(r)$ by $\tilde{f}(s)$, it is found

$$\left. \begin{aligned} A_1 \cos s\theta_0 - A_2 \cos s(\pi - \theta_0) &= \tilde{f}(s), \\ A_1 \mu_1 \sin s\theta_0 + A_2 \mu_2 \sin s(\pi - \theta_0) &= 0, \end{aligned} \right\} \quad (5)$$

from which we have

$$\left. \begin{aligned} A_1 &= \mu_2 \tilde{f}(s) \sin s(\pi - \theta_0) / F(s), \\ A_2 &= -\mu_1 \tilde{f}(s) \sin s\theta_0 / F(s), \end{aligned} \right\} \quad (6)$$

with

$$F(s) = \frac{1}{2}(\mu_1 + \mu_2) \sin s\pi - \frac{1}{2}(\mu_1 - \mu_2) \sin s(\pi - 2\theta_0). \quad (7)$$

Thus the inverse Mellin transform gives integral expressions of displacements as

$$\left. \begin{aligned} v_1 &= \frac{\mu_2}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{1}{F(s)} \tilde{f}(s) r^{-s} \sin s(\pi - \theta_0) \cos s\theta \, ds, & \text{in I,} \\ v_2 &= -\frac{\mu_1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{1}{F(s)} \tilde{f}(s) r^{-s} \sin s\theta_0 \cos s(\pi - \theta) \, ds, & \text{in II.} \end{aligned} \right\} \quad (8)$$

3. Evaluation of Integrals

We can take any form of the dislocation if its Mellin transform can be found. From the practical point of view, the dislocation on the fault plane may be considered to be maximum at some depth. Hence we adopt

$$f(r) = D_0 \frac{a^2 \tan^2 \lambda}{(r-a)^2 + a^2 \tan^2 \lambda}, \quad (9)$$

which takes maximum value D_0 at $r=a$ and $D_0 \sin^2 \lambda$ at $r=0$ and is illustrated in Fig. 2 when $\sin \lambda = 0.1, 0.2$ and 0.4 . The Mellin transform of $f(r)$ is given by

$$\tilde{f}(s) = -\pi D_0 \sin \lambda \frac{\sin(s-1)(\pi-\lambda)}{\sin s\pi} \left(\frac{a}{\cos \lambda} \right)^s, \quad 0 < \operatorname{Re} s < \varepsilon, \quad (10)$$

where ε is a small real positive number but when calculating $f(r)$ from the inverse transform of (10), we can take $0 < \operatorname{Re} s < 2$, since $s=1$ is not a pole.

Fig. 2. The displacement discontinuity $f(r)$ at $\theta = \theta_0$.

For this displacement discontinuity at $\theta = \theta_0$, the surface displacements are

$$\left. \begin{aligned} [v_1]_{\theta=0} &= -\frac{1}{2\pi i} \pi \mu_2 D_0 \sin \lambda \\ &\quad \times \int_{\varepsilon-i\infty}^{\varepsilon+i\infty} \frac{\sin(s-1)(\pi-\lambda) \cdot \sin s(\pi-\theta_0)}{\sin s\pi \cdot F(s)} \left(\frac{a}{r} \frac{1}{\cos \lambda}\right)^s ds, \\ [v_2]_{\theta=\pi} &= \frac{1}{2\pi i} \pi \mu_1 D_0 \sin \lambda \int_{\varepsilon-i\infty}^{\varepsilon+i\infty} \frac{\sin(s-1)(\pi-\lambda) \cdot \sin s\theta_0}{\sin s\pi \cdot F(s)} \left(\frac{a}{r} \frac{1}{\cos \lambda}\right)^s ds. \end{aligned} \right\} \quad (11)$$

To obtain these surface displacements, it is only necessary to evaluate the integral of the type

$$I = \frac{1}{2\pi i} \int_C \frac{\sin(s-1)(\pi-\lambda) \cdot \sin s\alpha}{\sin s\pi \cdot F(s)} x^s ds, \quad (12)$$

where the contour C starts from $\varepsilon - i\infty$ to $\varepsilon + i\infty$ and is closed by a semi-circle with infinite radius in the left-half complex s -plane when $x > 1$ and in the right-half plane when $x < 1$.

3.1 The case when $x > 1$

There are two series of poles, one is $s = s_n = -n$ ($n=0, 1, \dots$) and the other is $s = \nu_n$ (< 0) which satisfies $F(\nu_n) = 0$.

When π/θ_0 or $\pi/(\pi-\theta_0)$ is an integer ($=m$), if $s = \nu_n$ is a root of $F(s) = 0$, then $s = \nu_n - m$ is also a root. There are $(m-1)$ roots, which are not integers, between $s=0$ and $s=-m$. If m is even, non-integral roots are $m-2$, because one of $(m-1)$ roots is an integer $-m/2$. For example, when $\theta_0 = \pi/6$ and $\mu_2/\mu_1 = 0.5$, roots of $F(s) = 0$ between $s=0$ and -6 are

$$-.8971, \quad -1.9177, \quad -3, \quad -4.0823, \quad -5.1029,$$

and hence non-integral roots are given by

$$-.8971 - 6n, \quad -1.9177 - 6n, \quad -4.0823 - 6n, \quad -5.1029 - 6n, \\ (n=0, 1, \dots).$$

Pole contributions can be obtained as follows (we do not consider here the case when π/λ is an integer):

a) $F(s_n) \neq 0$, $\sin s_n \alpha \neq 0$

For this case, $s = s_n$ is a single pole and the pole contribution is simply obtained as

$$I = \frac{\sin(s_n-1)(\pi-\lambda) \cdot \sin s_n \alpha}{\pi \cos s_n \pi \cdot F(s_n)} x^{s_n}. \quad (13)$$

b) $F(s_n) \neq 0$, $\sin s_n \alpha = 0$

s_n is not a pole for this case.

c) $F(s_n)=0$, $\sin s_n \alpha \neq 0$

s_n is a double pole whose contribution is found to be

$$I = [\{(\pi - \lambda) \cos(s_n - 1)(\pi - \lambda) + \sin(s_n - 1)(\pi - \lambda) \cdot \ln x\} \sin s_n \alpha + \alpha \sin(s_n - 1)(\pi - \lambda) \cdot \cos s_n \alpha] x^{s_n} / [\pi \cos s_n \pi \cdot F'(s_n)] \quad (14)$$

with

$$F'(s_n) = \frac{\pi}{2} (\mu_1 + \mu_2) \cos s_n \pi - \frac{\pi - 2\theta_0}{2} (\mu_1 - \mu_2) \cos s_n (\pi - 2\theta_0). \quad (15)$$

d) $F(s_n)=0$, $\sin s_n \alpha = 0$

s_n is a single pole and the contribution is

$$I = \frac{\alpha \sin(s_n - 1)(\pi - \lambda) \cdot \cos s_n \alpha}{\pi \cos s_n \pi \cdot F'(s_n)} x^{s_n}. \quad (16)$$

This can be regarded as a special case of (14) when $\sin s_n \alpha = 0$.

e) $F(\nu_n)=0$ (integral roots are excluded)

$$I = \frac{\sin(\nu_n - 1)(\pi - \lambda) \cdot \sin \nu_n \alpha}{\sin \nu_n \pi \cdot F'(\nu_n)} x^{\nu_n}. \quad (17)$$

Sum of these pole contributions gives I .

3.2 The case when $x < 1$

One series of poles is $s = -s_{n+1} = n + 1$ and the other is $s = -\nu_n (> 0)$.

Pole contributions for this case can be obtained by multiplying those in Section 3.1 by -1 , taking $-s_{n+1}$ and $-\nu_n$ in place of s_n and ν_n , respectively.

When the medium is uniform, that is, $\mu_1 = \mu_2 \equiv \mu$, then $F(s) = \mu \sin s\pi$. Hence there are no non-integral poles and only cases c) and d) are taken into account.

4. Numerical Results and Concluding Remarks

Series expressions obtained in the previous section are computed until the convergency is obtained. At $r=0$, surface displacements can be evaluated from a residue at a pole $s_n=0$ and are given by

$$\left. \begin{aligned} [v_1]_{\theta=0} &= \frac{\mu_2(\pi - \theta_0)}{\mu_1\theta_0 + \mu_2(\pi - \theta_0)} D_0 \sin^2 \lambda, \\ [v_2]_{\theta=\pi} &= -\frac{\mu_1\theta_0}{\mu_1\theta_0 + \mu_2(\pi - \theta_0)} D_0 \sin^2 \lambda. \end{aligned} \right\} \quad (18)$$

Taking $\theta_0 = 30^\circ$ and $\sin \lambda = 0.2$, Fig. 3 shows surface displacements when $\mu_2/\mu_1 = 3, 1$ and $1/3$. For a uniform medium, the surface displacements can

be obtained by the conventional method for the same dislocation as

$$v = \frac{D_0}{\pi} x \sin \theta_0 \cdot a^2 \tan^2 \lambda \int_0^\infty \frac{1}{(\xi - a)^2 + a^2 \tan^2 \lambda} \frac{1}{(x + \xi \cos \theta_0)^2 + \xi^2 \sin^2 \theta_0} d\xi, \quad (19)$$

taking x -axis along the surface and origin of the coordinates at $r=0$. Numerical evaluation of the present expression when $\mu_2/\mu_1=1$ coincides with

Fig. 3. Surface displacement/ D_0 when $\theta_0=30^\circ$ and $\sin \lambda=0.2$.

Fig. 4. Surface displacement/ D_0 for various dip-angles when $\mu_2/\mu_1=2$ and $\sin \lambda=0.2$.

that of (19). When the upper layer is soft or small rigidity, large deformations are seen in Fig. 3 as expected.

Fixing μ_2/μ_1 to 2 and $\sin \lambda$ to 0.2, surface displacements are shown in Fig. 4 for various dip-angles and fixing θ_0 to 30° and μ_2/μ_1 to 2, they are illustrated in Fig. 5 for different dislocations (cf. Fig. 2).

Fig. 5. Surface displacement/ D_0 for different values of $\sin \lambda$ when $\theta_0 = 30^\circ$ and $\mu_2/\mu_1 = 2$.

Fig. 6. Ratio of maximum displacement for an obliquely layered medium to that for a uniform medium as a function of μ_2/μ_1 .

Ratio of maximum surface displacement for a given μ_2/μ_1 to that for a uniform medium is plotted in Fig. 6, calculated by the interpolation and the inverse interpolation technique, taking the dip-angle as a parameter. The conclusion of the present paper is summarized in this figure as follows: The larger the dip-angle is, the larger the difference becomes between maximum deformations in an obliquely layered medium and those in a uniform medium. The smaller the rigidity of the upper layer is, the larger the maximum displacement becomes and conversely the larger the rigidity is, the smaller the displacement becomes. When μ_2/μ_1 is 2, the maximum displacement is about 1.2 times as large as that in a uniform medium.

When intending to obtain focal parameters by comparing observed residual deformations with theoretical values derived under the assumption of uniform medium, one should bear in his mind the difference mentioned above.

REFERENCES

- BEN-MENACHEM, A. and A. GILLON, Crustal deformation by earthquakes and explosions, *Bull. Seism. Soc. Amer.*, **60**, 193-215, 1970.
- SATO, R., Crustal deformation due to dislocation in a multi-layered medium, *J. Phys. Earth*, **19**, 31-46, 1971.
- SATO, R., Seismic waves due to a dislocation source model in a multi-layered medium, Part I. Theory, *J. Phys. Earth*, **21**, 155-172, 1973 a.
- SATO, R., Seismic waves due to a dislocation source model in a multi-layered medium, Part II. Numerical calculations for a point source, *J. Phys. Earth*, **21**, 401-414, 1973 b.
- SATO, R. and M. MATSU'URA, Static deformations due to the fault spreading over several layers in a multi-layered medium, Part I: Displacement, *J. Phys. Earth*, **21**, 227-249, 1973.
- SATO, R. and T. MATSUZAKI, Radiation patterns due to a slip at a boundary between two media (in Japanese), *Zisin*, **27**, 367-370, 1974,