THEORETICAL AND EXPERIMENTAL STUDIES OF A SHEAR WAVE GENERATOR

I. WAVE FIELDS RADIATED FROM SEVERAL TYPES OF FORCE SYSTEMS

Isao Onda,* Shauzow Komaki,** and Minami Ichikawa***

*Faculty of Technology, Gunma University, Kiryu, Japan
**Department of Foundation Engineering, Saitama University, Urawa, Japan
***Kajima Institute of Construction Technology, Chofu, Tokyo, Japan

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An efficient force system for an appropriate design of a shear wave generator is found to be a horizontal single force or a horizontal single couple in the vertical plane. The transverse displacement of waves due to this type of force system is of SH wave, involving small amplitudes of P and SV waves.

1. Introduction

Instrumentation of elastic shear waves has furnished valuable information on the exploration of soil foundations. In almost all experiments, the shear wave source was situated on the surface of the soil, and was not powerful enough (e.g., Shima and Ohta, 1967). Waves, therefore, could not be observed from a long distance, and it led to a hard survey of the velocity structure in deeper subsoil. To overcome that difficulty, an arrangement in which the source is situated in one borehole and the geophones in other boreholes should be considered as a method of prospecting deep structures.

Before the practical design of the shear wave generator is discussed and presented, it will be benefit to clarify the characteristics of waves radiated by several types of force systems. For simplicity, it is assumed that the medium (or the subsoil) is homogeneous and isotropic, bounded by a horizontal free surface, and that the boreholes are drilled vertically in the medium. It is convenient for the interpretation of experimental results that the displacements are given in the cylindrical coordinates, where the z-axis agrees with the borehole of the source which is taken as the coordinate origin. The waves radiated by several types of force systems were discussed by Love (1904), Nakano (1923), Matuzawa (1926), and Kawasumi (1933), in connection with study of the earthquake mechanisms. Their solutions, however, were not expressed in the cylindrical coordinates.
The purpose of this paper is to classify the wave characteristics due to several types of simple force systems and to suggest an appropriate design for a shear wave generator. In section 2 the wave fields due to these force systems are calculated and in the next section they are summarized. Section 4 is concerned with a wave field due to a special force system, a horizontal single force. The last section deals with the effect of the source length.

In a subsequent paper (ICHIKAWA et al., 1975), a shear wave generator will be developed following the conclusion of this paper.

2. Waves Radiated by Force Systems

The z-axis is taken as coinciding with the borehole of the source which is the coordinate origin, as shown in Fig. 1. The position of geophone is given by the cylindrical coordinates \((r, \theta, z)\). The Cartesian coordinates are also employed, in specifying the direction of force.

![Fig. 1. Positions of the source and a geophone, and the coordinates.](image)

The general solution of an elastic wave equation is expressed by the form

\[
U = \text{grad} \Phi + \text{curl} (0, 0, \Psi_1) + \text{curl} \text{curl} (0, 0, \Psi_2),
\]

where \(U\) is the displacement vector, the quantities in the parentheses imply \((r, \theta, z)\) components, and potentials \(\Phi, \Psi_1, \Psi_2\) satisfy the equations

\[
\frac{\partial^2 \Phi}{\partial t^2} = V_p^2 \Phi, \quad \frac{\partial^2 \Psi_j}{\partial t^2} = V_s^2 \Psi_j, \quad j = 1, 2.
\]

In Eq. (1), the first, the second and the third terms refer to the longitudinal (P), the horizontally polarized shear (SH) and the other shear (SV) waves, respectively. Even if waves are written in another form, the expression can always be transformed into an Eq. (1) type. Thus, the characteristics of each wave are easily split in terms of the three potentials \(\Phi, \Psi_1, \Psi_2\). The wave
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important to the determination of shear-wave velocity structure is the SH wave, because it is coupled with P wave.

2.1 Single force along the x-axis (see Fig. 2)

![Fig. 2. Single force along the x-axis.](image)

After SATO (1969), waves due to a single force of magnitude \( \chi(t) \) acting at the origin in the direction of the x-axis are expressed by

\[
U = \nabla \phi + \nabla \times \nabla \times (\psi, 0, 0)_{\text{Cartesian}},
\]

where the curl-operators are applied to a vector in the Cartesian coordinates \((x, y, z)\). In Eq. (2),

\[
\phi = \frac{1}{4\pi \rho} \frac{\partial \phi}{\partial x} = \cos \theta \frac{\partial \phi}{\partial x}, \quad \psi = \frac{1}{4\pi \rho} \phi = \frac{1}{R} F \left( t - \frac{R}{V_p} \right),
\]

\[
F(t) = \int_0^t \int_0^t \chi(t') \, dt' = \sqrt{r^2 + z^2}.
\]

With transformation from the Cartesian into the cylindrical coordinates, we have (see Appendix)

\[
U = \nabla \phi + \nabla \times \nabla \times (\psi_1, 0, 0, \psi_2)_{\text{Cartesian}},
\]

\[
\psi_1 = \frac{\sin \theta}{8\pi^2 \rho V_s^2} \int_{-\infty}^{\infty} \tilde{\chi}(\omega)e^{\text{iat}} \, d\omega \int_0^{\infty} J_1(\xi r)e^{-\nu' |z|} \frac{d\xi}{\nu'},
\]

\[
\psi_2 = -\frac{\cos \theta}{8\pi^2 \rho} \int_{-\infty}^{\infty} \tilde{\chi}(\omega)e^{\text{iat}} \frac{d\omega}{\omega^2} \int_0^{\infty} J_1(\xi r) \frac{d\xi}{dz} \frac{d\xi}{\nu'}
\]

where

\[
\tilde{\chi}(\omega) = \int_{-\infty}^{\infty} \chi(t)e^{-\text{iat}} \, dt, \quad \nu' = \sqrt{\xi^2 - \omega^2/V_s^2}.
\]

Consequently it follows that there exist three types of waves and each wave has only one nodal plane.
2.2 *Single force directed upwards* (see Fig. 3)
Similarly as in the preceding case, the potentials are obtained as:

\[
U = \text{grad } \phi + \text{curl curl } (0, 0, \psi_{\text{Cartesian}}) \\
= \text{grad } \phi + \text{curl curl } (0, 0, \psi_z),
\]

(6)

Fig. 3. Single force directed upwards.
Fig. 4. Horizontal single couple along the x-axis.
Fig. 5. Double force along the x-axis without moment.
Fig. 6. Horizontal single couple in the x-z plane.
Fig. 7. Vertical single couple in the x-z plane.
Fig. 8. Vertical double force without moment.
Fig. 9. Center of expansion.
Fig. 10. Center of rotation about the vertical axis.
Fig. 11. Center of rotation about the x-axis.
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where

\[ \Phi = \frac{1}{4 \pi \rho} \frac{\partial \phi}{\partial z}, \quad \Psi_z = -\frac{1}{4 \pi \rho} \phi \]

and potentials \( \phi \) and \( \psi \) are given by Eqs. (3). In this case, \( \Psi_1 \) does not appear. Consequently the SH wave is not radiated.

2.3 Horizontal single couple along the x-axis (see Fig. 4)

Waves radiated by a force system of double forces with moment, shown in Fig. 4, are expressed by:

\[
U = \text{grad} \Phi + \text{curl} \text{curl} (\Psi, 0, 0)_{\text{Cartesian}} = \text{grad} \Phi + \text{curl} \text{curl} (\Psi \cos \theta, -\Psi \sin \theta, 0),
\]

where

\[
\Phi = -\frac{1}{4 \pi \rho} \frac{\partial \phi}{\partial z} = \frac{\sin 2\theta}{8 \pi \rho} r \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \phi}{\partial r} \right),
\]

\[
\Psi = \frac{1}{4 \pi \rho} \frac{\partial \phi}{\partial y} = \frac{\sin \theta}{4 \pi \rho} \frac{\partial \phi}{\partial r},
\]

whence

\[
\Psi_{1} = -\frac{1}{16 \pi^2 \rho V_a^2} \int_{-\infty}^{\infty} \tilde{M}(\omega) e^{i \omega t} d\omega \int_{0}^{\infty} \left\{ J_0(\xi t) \cos 2\theta - J_0(\xi r) e^{-\nu |z|} \xi \frac{d\xi}{\nu'} \right\} d\xi,
\]

\[
\Psi_{2} = -\frac{\sin 2\theta}{16 \pi^2 \rho} \int_{-\infty}^{\infty} \tilde{M}(\omega) e^{i \omega t} d\omega \int_{0}^{\infty} J_0(\xi t) \frac{\partial}{\partial x} \frac{d\xi}{\nu'},
\]

and, in the definition of \( \phi \) and \( \psi \) in Eqs. (3), the moment \( M(t) \) should be taken in place of \( \psi(t) \).

It is found that three potentials \( \Phi, \Psi_1, \Psi_2 \) possess two orthogonalized nodal planes in the azimuth.

2.4 Double force along the x-axis without moment (see Fig. 5)

\[
U = \text{grad} \Phi + \text{curl} \text{curl} (\Psi, 0, 0)_{\text{Cartesian}},
\]

where

\[
\Phi = \frac{1}{4 \pi \rho} \frac{\partial \phi}{\partial x} = \frac{1}{8 \pi \rho} \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \phi}{\partial r} \right) + \cos 2\theta \cdot r \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \phi}{\partial r} \right) \right\},
\]

\[
\Psi = \frac{1}{4 \pi \rho} \frac{\partial \phi}{\partial x} = -\frac{\cos \theta}{4 \pi \rho} \frac{\partial \phi}{\partial r},
\]

whence

\[
\Psi_{1} = -\frac{\sin 2\theta}{16 \pi^2 \rho V_a^2} \int_{-\infty}^{\infty} \tilde{N}(\omega) e^{i \omega t} d\omega \int_{0}^{\infty} J_0(\xi t) e^{-\nu |z|} \xi \frac{d\xi}{\nu'},
\]

\[
\Psi_{2} = \frac{1}{16 \pi^2 \rho} \int_{-\infty}^{\infty} \tilde{N}(\omega) e^{i \omega t} d\omega \int_{0}^{\infty} \left\{ J_0(\xi r) - J_0(\xi t) \cos 2\theta \right\} \frac{\partial}{\partial x} \frac{d\xi}{\nu'},
\]

and also the intensity \( N(t) \) should be taken in place of \( \chi(t) \) in Eqs. (3).
Therefore, this type of force system radiates waves with two nodal planes in the azimuth.

2.5 **Horizontal single couple in the x-z plane (see Fig. 6)**

\[
U = \nabla \Phi + \nabla \times \nabla \times (\Psi, 0, 0)_{\text{Cartesian}},
\]

where
\[
\Phi = \frac{1}{4\pi \rho} \frac{\partial^2 \phi}{\partial x \partial z} = \frac{\cos \theta}{4\pi \rho} \frac{\partial^2 \phi}{\partial r \partial z},
\]
\[
\Psi = -\frac{1}{4\pi \rho} \frac{\partial \phi}{\partial z}.
\]

These potentials are obtained by a formal differentiation with respect to \( z \), of the potentials (5) which describe the disturbance due to a horizontal single force. Thus, the potentials for each wave are given by

\[
\Phi = \frac{\cos \theta}{4\pi \rho} \frac{\partial^2 \phi}{\partial r \partial z},
\]
\[
\Psi_1 = \frac{\sin \theta}{8\pi^2 \rho V_s^2} \left\{ \int_{-\infty}^{\infty} M(\omega) e^{i\omega t} d\omega \int_{0}^{\infty} \frac{J_1(\xi r) e^{-\nu|z|}}{\nu} \frac{d\xi}{d\nu} \right\},
\]
\[
\Psi_2 = -\frac{\cos \theta}{8\pi^2 \rho} \left\{ \int_{-\infty}^{\infty} M(\omega) e^{i\omega t} d\omega \int_{0}^{\infty} \frac{J_1(\xi r) e^{-\nu|z|}}{\nu} \frac{d\xi}{d\nu} \right\}.
\]

2.6 **Vertical single couple in the x-z plane (see Fig. 7)**

\[
U = \nabla \Phi + \nabla \times \nabla \times (0, 0, \Psi)_{\text{Cartesian}} = \nabla \Phi + \nabla \times \nabla \times (0, 0, \Psi_2),
\]

where
\[
\Phi = -\frac{1}{4\pi \rho} \frac{\partial^2 \phi}{\partial x \partial z} = \frac{\cos \theta}{4\pi \rho} \frac{\partial^2 \phi}{\partial r \partial z},
\]
\[
\Psi_1 = 0,
\]
\[
\Psi_2 = \frac{1}{4\pi \rho} \frac{\partial \phi}{\partial x} = \frac{\cos \theta}{4\pi \rho} \frac{\partial \phi}{\partial r}.
\]

The SH wave is not radiated.

2.7 **Vertical double force without moment (see Fig. 8)**

\[
U = \nabla \Phi + \nabla \times \nabla \times (0, 0, \Psi)_{\text{Cartesian}}
\]

where
\[
\Phi = \frac{1}{4\pi \rho} \frac{\partial^2 \phi}{\partial z^2},
\]
\[
\Psi_1 = 0, \quad \Psi_2 = \Psi = -\frac{1}{4\pi \rho} \frac{\partial \phi}{\partial z}.
\]

These potentials are also given by a formal differentiation of those due to a vertical single force with respect to \( z \). Thus, this type of force system does
not radiate SH wave, and the generated waves have no nodal plane in the azimuth.

2.8 Center of expansion (see Fig. 9)

\[
U = \text{grad} \Phi + \text{curl curl}(\psi_x, \psi_y, \psi_z)_{\text{Cartesian}}
\]

where

\[
\Phi = \frac{1}{4\pi \rho}\nabla^2 \phi ,
\]

\[
\psi_x = -\frac{1}{4\pi \rho} \frac{\partial \phi}{\partial x} , \quad \psi_y = -\frac{1}{4\pi \rho} \frac{\partial \phi}{\partial y} , \quad \psi_z = -\frac{1}{4\pi \rho} \frac{\partial \phi}{\partial z} .
\]

Since the rotation of the vector \((\psi_x, \psi_y, \psi_z)_{\text{Cartesian}}\) vanishes, the displacement becomes

\[
U = \text{grad} \Phi , \quad \psi_1 = \psi_2 = 0 .
\]

Only the P wave with no nodal plane is radiated.

2.9 Center of rotation about the vertical axis (see Fig. 10)

\[
U = \text{curl curl}(\psi_x, \psi_y, 0)_{\text{Cartesian}} = \text{curl curl}(\psi_x \cos \theta + \psi_y \sin \theta, -\psi_x \sin \theta + \psi_y \cos \theta, 0) ,
\]

where

\[
\psi_x = -\frac{1}{4\pi \rho} \frac{\partial \phi}{\partial y} = -\frac{\sin \theta}{4\pi \rho} \frac{\partial \phi}{\partial r} ,
\]

\[
\psi_y = \frac{1}{4\pi \rho} \frac{\partial \phi}{\partial x} = \frac{\cos \theta}{4\pi \rho} \frac{\partial \phi}{\partial r} .
\]

Substitution of these potentials into the above equation yields

\[
U = \text{curl curl} \left( 0, -\frac{1}{4\pi \rho} \frac{\partial \phi}{\partial r} , 0 \right) ,
\]

whence

\[
\psi_1 = \frac{1}{4\pi \rho} \nabla^2 \phi ,
\]

\[
\Phi = \psi_2 = 0 .
\]

This type of force system radiates neither the P nor SV waves. The SH wave has no nodal plane.

2.10 Center of rotation about the x-axis (see Fig. 11)

\[
U = \text{curl curl}(\psi_x, 0, \psi_z)_{\text{Cartesian}} = \text{curl curl}(\psi_x \cos \theta, -\psi_x \sin \theta, \psi_z) ,
\]
where
\[ y_z = \frac{1}{4\pi\rho} \frac{\partial \phi}{\partial z}, \]
\[ y_z = -\frac{1}{4\pi\rho} \frac{\partial \phi}{\partial \xi} = -\frac{\cos \theta}{4\pi\rho} \frac{\partial \phi}{\partial \eta}, \]
and thus
\[
\begin{align*}
\psi_1 &= -\frac{\sin \theta}{8\pi^3 \rho V_z^2} \left\{ \int_{-\infty}^{\infty} \mathcal{M}(\omega)e^{i\omega t} d\omega \int_{0}^{\infty} J_1(\xi \eta) \frac{d\xi}{\nu} \right\}, \\
\psi_2 &= -\frac{\cos \theta}{8\pi^3 \rho V_z^2} \left\{ \int_{-\infty}^{\infty} \mathcal{M}(\omega)e^{i\omega t} d\omega \int_{0}^{\infty} J_1(\xi \eta) e^{-\nu |\xi|} \frac{d\xi}{\nu} \right\}.
\end{align*}
\]
(16)

The P wave is not radiated, and there is a nodal plane for S waves.

2.11 Other types of force system

The medium under consideration is linear, so that waves due to other types of force system can be derived by means of suitable superposition of the above-mentioned types of force system. For example, the wave field due to a single force directed to the azimuth \( \alpha \) and the zenithal angle \( \delta \) is evaluated as follows. Since the direction cosines of the force vector to the \( x, y, z \) axes are \( \sin \delta \cdot \cos \alpha, \sin \delta \cdot \sin \alpha, \cos \delta \), respectively, we have

![Fig. 12. Single force in directed with an Euler's angle \((\alpha, \delta, 0)\).](image)

\[ U = \text{grad} \Phi + \text{curl} \text{curl} (\psi \sin \delta \cdot \cos \alpha, \psi \sin \delta \cdot \sin \alpha, \psi \cos \delta), \]

where
\[ \Phi = \frac{1}{4\pi\rho} \left( \sin \delta \cdot \cos \alpha + \frac{\partial \phi}{\partial \xi} \sin \delta \cdot \sin \alpha + \frac{\partial \phi}{\partial \eta} \cos \delta \right), \]
\[ \psi = -\frac{1}{4\pi\rho} \phi. \]
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Thence we have

\[
\psi_1 = \frac{\sin \delta \cdot \sin (\theta - \alpha)}{8\pi^2 \rho V_s^2} \int_{-\infty}^{\infty} \tilde{V}(\omega) e^{i\omega t} d\omega \int_{0}^{\infty} J_1(\xi r) e^{-\nu l z} \frac{d\xi}{\nu} ,
\]

\[
\psi_2 = -\frac{\sin \delta \cdot \cos (\theta - \alpha)}{8\pi^2 \rho} \int_{-\infty}^{\infty} \tilde{V}(\omega) e^{i\omega t} d\omega \int_{0}^{\infty} J_1(\xi r) \frac{d\xi}{\nu^3} \frac{d\omega}{\omega^2} \frac{d\xi}{\nu} + \Psi \cdot \cos \delta .
\]

Other examples of the force system which are important to the study of earthquake mechanism are concerned with the strike slip and the dip slip faults. The disturbances due to such force systems are deduced in terms of the suitable superposition of two single couples (Sato, 1969).

3. Feature of Waves Radiated by the Sources

In the preceding section, solutions for waves generated by several simple types of force system were deduced; the characteristics of these waves are summarized in the following table.

<table>
<thead>
<tr>
<th>Type</th>
<th>Waves generated</th>
<th>Numbers of nodal plane</th>
<th>Force system</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>o o o</td>
<td>1</td>
<td>Horizontal single force</td>
</tr>
<tr>
<td>II</td>
<td>o x o</td>
<td>0</td>
<td>Vertical single force</td>
</tr>
<tr>
<td>III</td>
<td>o o o</td>
<td>2</td>
<td>Horizontal single couple</td>
</tr>
<tr>
<td>IV</td>
<td>o o o</td>
<td>2</td>
<td>Horizontal double force without moment</td>
</tr>
<tr>
<td>V</td>
<td>o o o</td>
<td>1</td>
<td>Horizontal single couple in the vertical plane</td>
</tr>
<tr>
<td>VI</td>
<td>o x o</td>
<td>1</td>
<td>Vertical single couple</td>
</tr>
<tr>
<td>VII</td>
<td>o x o</td>
<td>0</td>
<td>Vertical double force without moment</td>
</tr>
<tr>
<td>VIII</td>
<td>o x o</td>
<td>0</td>
<td>Center of expansion</td>
</tr>
<tr>
<td>IX</td>
<td>x o x</td>
<td>0</td>
<td>Center of rotation about vertical axis</td>
</tr>
<tr>
<td>X</td>
<td>x o o</td>
<td>1</td>
<td>Center of rotation about horizontal axis</td>
</tr>
</tbody>
</table>

From an experimental point of view, it is desirable that the force system acting at the source is simple because of ease of design of the source apparatus and that the number of nodal planes of generated waves is few by reason of diminution of errors in the measurements.

Table 1 shows that the most efficient force system for the generation of SH wave is of the rotation center about the vertical axis (type IX). Although this fact has been well-known, the design of the generator seems to be very difficult.

The next most efficient are those of the horizontal single force (type I), the horizontal single couple in the vertical plane (type V) and the rotation
center about the horizontal axis (type X), in which there is only one nodal plane. Though P wave is not radiated, however, the design of the generator based on the rotation center about the horizontal axis (type X) is definitely more difficult than that based on the rotation center about the vertical (type IX).

The azimuthal characteristics of waves due to the horizontal single couple in the vertical plane (type V) is equivalent to those due to the horizontal single force (type I), because the differentiation of waves generated by the latter system with respect to \( z \) agrees with waves generated by the former system.

It is concluded that the force system of the type I or type V is the most relevant in the design of the shear wave generator.

4. Wave Field from a Horizontal Single Force

The most relevant force system for the determination of shear-wave velocity structure of subsoil is of a horizontal single force or horizontal single couple in the vertical plane. In this section, the wave field due to a horizontal single force is discussed.

For simplicity, a harmonic time function is assumed for the source, that is, \( \chi(t) \propto \tilde{\chi}(\omega) \exp(-i\omega t) \), whence \( F(t) \propto \tilde{\chi}(\omega) \exp(-i\omega t)/\omega^2 \). From Eq. (4), the wave field is split into three parts:

\[
U = U^p + U^{SH} + U^{SV},
\]

and the displacement vector \((u, v, w)\) for the longitudinal wave is expressed, from Eq. (4), as

\[
\begin{align*}
    u^p &= -\frac{\tilde{\chi}(\omega) \cos \theta}{4\pi \rho \omega^2} \int_0^\infty \frac{d}{dr} J_1(\xi r) e^{-i\nu |\xi|} \frac{\xi^2 d\xi}{\nu}, \\
    v^p &= \frac{\tilde{\chi}(\omega) \sin \theta}{4\pi \rho \omega^2} \int_0^\infty \frac{J_1(\xi r) e^{-i\nu |\xi|} \xi^2 d\xi}{\nu}, \\
    w^p &= \frac{\tilde{\chi}(\omega) \cos \theta}{4\pi \rho \omega^2} \text{sgn}(z) \int_0^\infty J_1(\xi r) e^{-i\nu |\xi|} \xi^3 d\xi;
\end{align*}
\]

for the SH wave,

\[
\begin{align*}
    u^{SH} &= \frac{\tilde{\chi}(\omega) \cos \theta}{4\pi \rho V_s^2} \int_0^\infty \frac{J_1(\xi r) e^{-i\nu' |\xi|} \xi d\xi}{\nu'}, \\
    v^{SH} &= -\frac{\tilde{\chi}(\omega) \sin \theta}{4\pi \rho V_s^2} \int_0^\infty \frac{d}{dr} J_1(\xi r) e^{-i\nu' |\xi|} \frac{d\xi}{\nu'}, \\
    w^{SH} &= 0;
\end{align*}
\]

and for the SV wave,
It is found that the vertical component of SH wave, \( u^{SH} \), is null and the transverse ones of P and SV, \( u^P \) and \( u^{SV} \), and the radial one of SH, \( u^{SH} \), have the factor \( r^{-1} \). In a far field, the other components \( u^P \), \( u^{SV} \), \( w^P \), \( w^{SV} \), \( f_{\omega}^P \), and \( f_{\omega}^{SV} \) are predominant, because of the relation \( J_1'(z) = J_0(z) - z^{-1}J_1(z) \). And the radial and vertical components are proportional to \( \cos \theta \), while the transverse one is proportional to \( \sin \theta \), where \( \theta \) is measured from the direction of force. Namely, in the direction of \( \theta = \pi/2 \), the former vanishes and the latter predominates.

Consequently, the observation of the SH wave should be carried out in the direction perpendicular to that of the force applied, by means of the transversely sensitive geophones. However, it should be noted that the transverse record in the near field is disturbed by small amplitude P and SV waves.

5. Effect of the Source Length

The effect of the source length on the wave field can be determined by the superposition of point source over that length. The force is assumed to act uniformly at \(-a \leq z \leq +a\), and the geophone is set at \( z_0 > a \). Thus, the transverse displacement of the SH wave is given by

\[
u^{SH} = \frac{\bar{\xi}(\omega) \sin \theta}{4\pi \rho V_0^2} \frac{1}{2a} \int_{-a}^{a} dz' \int_{z_0-a}^{z_0+a} \frac{dJ_1(\xi r)}{dr} e^{-\nu'z_0} \frac{dz}{\nu'} \frac{d\xi}{\nu'}
\]

Since \( a \) is not large, for practical purposes, it is assumed that \(|\nu'a| < 1\), and thus

\[
u^{SH} = \frac{\bar{\xi}(\omega) \sin \theta}{4\pi \rho V_0^2} \int_{0}^{\infty} \frac{dJ_1(\xi r)}{dr} e^{-\nu'z_0} \left[ 1 + O(\nu'a)^2 \right] \frac{dz}{\nu'}
\]

The factor \( \nu' \) is inversely proportional to the wavelength at far stations. The effect of the source length on the SH wave, therefore, is of the order of the square of the ratio of the source length to the wavelength, similar to the effect of the source length of the gun-type SH-wave generator on the free
surface (ONDA and KOMAKI, 1968). In the other components, also, the same effect of the source length is derived.

Appendix. Separation of SH and SV Components in Eq. (2), in Case of a Horizontal Single Force

The shear wave field due to a single force along the x-axis is expressed by

\[ U^S = \text{curl curl} \ A, \quad A = (\Psi, 0, 0)_{\text{Cartesian}}. \]  \hspace{1cm} (2')

Transformation from the Cartesian to the cylindrical coordinates \((r, \theta, z)\) yields the expression of vector \(A\)

\[ A = (\Psi \cos \theta, -\Psi \sin \theta, 0), \]

whence each component of the shear wave is computed by

\[ U^S = \text{curl curl} \ A = (u^S, v^S, w^S), \]

where

\[
\begin{align*}
  u^S &= -\cos \theta \cdot (\partial^2 \Psi / \partial z^2 + r^{-1} \partial \Psi / \partial r), \\
  v^S &= \sin \theta \cdot (\partial^2 \Psi / \partial z^2 + \partial^2 \Psi / \partial r^2), \\
  w^S &= \cos \theta \cdot \partial^2 \Psi / \partial r \partial z.
\end{align*}
\] \hspace{1cm} (A-1)

The function \(\Psi\) is expressed by means of Fourier and Hankel transforms with respect to the \(t\)- and \(r\)-coordinates, respectively, as

\[ \Psi = \frac{1}{8\pi^2 \rho} \int_{-\infty}^{\infty} \chi(\omega) e^{\omega t} \frac{d\omega}{\omega^2} \int_0^{\infty} e^{-\nu' \xi} J_0(\xi \tau) \frac{\xi}{\nu'} d\xi, \]

where \(\chi(\omega)\) and \(\nu'\) are given by Eq. (5'). Thence, except for the common factor \((8\pi^2 \rho)^{-1} \int_{-\infty}^{\infty} \chi(\omega) \exp (i\omega \tau) \omega^{-2} d\omega \int_0^{\infty} e^{-\nu' \xi} d\xi\), Eqs. (A-1) yields

\[
\begin{align*}
  u^S &= \cos \theta \cdot e^{-\nu' \xi} \left\{ \frac{\xi}{r} J_1(\xi \tau) - \left( \frac{\omega^2}{V_s^2} \right) J_0(\xi \tau) \right\}, \\
  v^S &= -\sin \theta \cdot e^{-\nu' \xi} \left\{ \frac{\omega^2}{V_s^2} J_0(\xi \tau) - \frac{\xi}{r} J_1(\xi \tau) \right\}, \\
  w^S &= -\cos \theta \cdot \frac{d}{dz} e^{-\nu' \xi} J_1(\xi \tau).
\end{align*}
\]

On the other hand, a shear wave field can be split into SH and SV wave fields:

\[ U^{SH} = \text{curl} (0, 0, \Psi') = \left( \frac{1}{r} \frac{\partial \Psi'_1}{\partial \theta}, -\frac{\partial \Psi'_1}{\partial r}, 0 \right), \]
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\[ U^{SV} = \text{curl} \text{ curl} (0, 0, \Psi_z) \]
\[ = \left( \frac{\partial \Psi_z}{\partial r}, \frac{1}{r} \frac{\partial \Psi_z}{\partial \theta}, - \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \Psi_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \Psi_z \right). \]

Since the \( z \)-component of \( U^\text{SH} \) vanishes, the SV potential \( \Psi_z \) is determined from equating the \( z \)-component of \( U^\text{S} \) to that of \( U^\text{SV} \), that is, \( w^\text{S} = w^\text{SV} \), and thus
\[ \Psi_z = -\cos \theta \cdot \frac{e^{-\nu^\text{SH} |r|}}{\xi^z} \frac{J_1(\xi r)}{\xi}, \]
from which the other components of the SV wave are evaluated as
\[ u^\text{SV} = -\cos \theta \cdot e^{-\nu^\text{SH} |r|} \frac{\xi^z}{\xi^r} \frac{d J_1(\xi r)}{dr}, \]
\[ v^\text{SV} = \sin \theta \cdot e^{-\nu^\text{SH} |r|} \frac{\xi^z}{\xi^r} J_1(\xi r). \]

As the SH wave is reduced to \( U^\text{S} - U^\text{SV} \), the SH-radial displacement is evaluated as
\[ u^\text{SH} = \cos \theta \cdot e^{-\nu^\text{SH} |r|} \cdot \frac{\omega^2}{V_u^2} \frac{J_1(\xi r)}{r}, \]
whence
\[ \Psi_1 = \sin \theta \cdot e^{-\nu^\text{SH} |r|} \cdot \frac{\omega^2}{V_u^2} \frac{J_1(\xi r)}{\xi}. \]

The SH-potential \( \Psi_1 \) is also derived from the transverse displacement, and the same result is obtained.

When the omitted common factor is restored, the representation of Eqs. (5) is obtained.

REFERENCES


