SPECTRAL ANALYSIS OF THE AFTERSHOCKS OF THE EARTHQUAKE OFF IZU PENINSULA, 1974

Kazuo SHIBUYA and Kiyoshi SUYEHIRO
Geophysical Institute, Faculty of Science, University of Tokyo, Tokyo, Japan
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Eleven aftershocks of the earthquake off Izu Peninsula, 1974 which occurred during May 16-18, 1974 at a location at almost the same distance from the temporary stations NHT and IRT, were chosen for a spectral analysis by the maximum entropy method to detect the effect of the aftershock region on the nature of seismic waves passing through it. Predominant frequencies were obtained and they showed always lower values for the spectra obtained at IRT than those at NHT. Station IRT is located at the southeastern end of the fault and receives the waves which have passed through the aftershock region, whereas the wave paths to NHT are roughly normal to the fault strike. Therefore the results are consistent with the hypothesis that the aftershock region has a comparatively low Q value.

1. Introduction

This paper reports a possible effect of the aftershock region on the nature of seismic waves passing through it for the aftershocks of the earthquake off Izu Peninsula of May 9, 1974 (M=6.9).

Immediately after the main shock that caused great damages around Minami Izu Town, aftershock observations were started at fifteen stations by the aftershock research groups (RESEARCH GROUP for AFTERSHOCKS, 1975). The observation by the Geophysical Institute of Tokyo University was carried out at three stations from May 16, a week after the main shock, until Nov. 29 using 4-ch. DAR 20-day data recorders (Fig. 1, Table 1) (ISHIBASHI et al., 1975).

By the precise determination of the space-time distribution of aftershocks, its correlation to the fault and the mechanism of the main shock have been studied in many cases (e.g. WATANABE and KUROISO, 1970). However, there are few studies on the physical properties of the fault after the main shock or the aftershock region (SUYEHIRO, 1968).

In checking the effect of the aftershock region on the propagation of seismic waves using spectral analysis, we must distinguish the pass effect from
Fig. 1. Map showing the epicentral distribution of aftershocks of the earthquake off Izu Peninsula, 1974 for the time interval 12h May 16–12h May 18 (JST), 1974 and the locations of three temporary stations, IRT, NHT, and OAT. Solid lines indicate Quaternary active faults in the south-western part of the peninsula. The large circle shows the epicenter of the main shock. This figure is redrawn from Fig. 4 in Ultra Sensitive Observation of Aftershocks of the Earthquake off Izu Peninsula, May 9, 1974, Ishibashi et al.

The structural parameters in the figure are those used in the hypocenter determination program.

Fig. 2. Map showing the hypocentral distribution at nearly equal distances (14 km) from NHT and IRT. Solid circles of 05, 06, ..., 31 indicate 11 shocks spectrally analyzed. Also shocks A and B in the first quadrant are analyzed to check the station effect.
other effects by a source spectrum, geometrical spreading, receiver effect on the spectrum hereafter referred to as the station effect, and the instrumental response. Therefore, two stations, IRT and NHT and the aftershocks of May 16–18 are so chosen that the two stations are at nearly equal distances (14 km).

Table 1. List of the ultra-sensitive temporary stations for aftershock observation of the Off Izu Peninsula Earthquake of May 9, 1974.

<table>
<thead>
<tr>
<th>Station</th>
<th>Abbr.</th>
<th>Coordinates</th>
<th>Component</th>
<th>Magnification*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Irozaki</td>
<td>IRT</td>
<td>36°36'46.8&quot;N</td>
<td>Vertical (high gain)</td>
<td>890 k</td>
</tr>
<tr>
<td></td>
<td></td>
<td>138°50'38.0&quot;E</td>
<td>Horizontal high gain (NS)</td>
<td>940 k</td>
</tr>
<tr>
<td></td>
<td></td>
<td>height 20 m</td>
<td>Horizontal low gain (EW)</td>
<td>63 k</td>
</tr>
<tr>
<td>Neginohata</td>
<td>NHT</td>
<td>34°48'28.7&quot;N</td>
<td>Vertical (high gain)</td>
<td>1200 k</td>
</tr>
<tr>
<td></td>
<td></td>
<td>138°49'34.0&quot;E</td>
<td>Horizontal high gain (NW-SE)</td>
<td>1080 k</td>
</tr>
<tr>
<td></td>
<td></td>
<td>height 130 m</td>
<td>Horizontal low gain (NW-SE)</td>
<td>66 k</td>
</tr>
<tr>
<td>Ochiai</td>
<td>OAT</td>
<td>34°43'06.3&quot;N</td>
<td>Vertical (high gain)</td>
<td>840 k</td>
</tr>
<tr>
<td></td>
<td></td>
<td>138°57'48.6&quot;E</td>
<td>Horizontal high gain (NS)</td>
<td>240 k</td>
</tr>
<tr>
<td></td>
<td></td>
<td>height 110 m</td>
<td>Horizontal low gain (NS)</td>
<td>39 k</td>
</tr>
</tbody>
</table>

* Measured at 10 Hz on monitoring play-back, characteristic frequencies are 3.5 Hz for vertical component and 1 Hz for horizontal component, recorded by DAR 4-ch. data recorder with 1.0 mm/sec tape speed during May 12–Nov. 29, 1974. (after ISHIBASHI et al.)

Table 2. List of the 11 aftershocks chosen for spectral analysis.

<table>
<thead>
<tr>
<th>No.</th>
<th>Origin time</th>
<th>Hypocenter*</th>
<th>Epicentral distance</th>
<th>Magnitude by F-P***</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>y m d h m s</td>
<td>x km y km z km</td>
<td>NHT km IRT km</td>
<td>2.30 log (F-P)–1.23</td>
</tr>
<tr>
<td>05</td>
<td>74 5 16 21 33 18.4</td>
<td>-7.3 3.0 7.3</td>
<td>14.3 12.5</td>
<td>0.9</td>
</tr>
<tr>
<td>06</td>
<td>74 5 16 21 45 22.5</td>
<td>-8.0 4.7 8.0</td>
<td>13.2 14.3</td>
<td>0.9</td>
</tr>
<tr>
<td>07</td>
<td>74 5 16 21 49 12.9</td>
<td>-8.6 4.6 7.2</td>
<td>13.6 14.6</td>
<td>0.9</td>
</tr>
<tr>
<td>11</td>
<td>74 5 17 04 43 33.8</td>
<td>-8.3 3.8 8.8</td>
<td>14.2 13.8</td>
<td>1.5</td>
</tr>
<tr>
<td>13</td>
<td>74 5 17 06 05 59.7</td>
<td>-7.7 3.4 7.9</td>
<td>14.1 13.1</td>
<td>1.7</td>
</tr>
<tr>
<td>15</td>
<td>74 5 17 08 37 40.1</td>
<td>-8.3 3.1 5.5</td>
<td>14.7 13.3</td>
<td>1.4</td>
</tr>
<tr>
<td>16</td>
<td>74 5 17 08 58 14.5</td>
<td>-7.4 3.0 8.3</td>
<td>14.4 12.6</td>
<td>1.5</td>
</tr>
<tr>
<td>28</td>
<td>74 5 18 02 17 36.7</td>
<td>-9.2 3.6 6.3</td>
<td>14.8 14.4</td>
<td>1.5</td>
</tr>
<tr>
<td>29</td>
<td>74 5 18 03 43 17.3</td>
<td>-7.8 4.3 7.9</td>
<td>13.4 13.8</td>
<td>1.2</td>
</tr>
<tr>
<td>30</td>
<td>74 5 18 04 12 44.8</td>
<td>-8.7 4.6 9.9</td>
<td>13.7 14.7</td>
<td>1.4</td>
</tr>
<tr>
<td>31</td>
<td>74 5 18 04 20 45.5</td>
<td>-8.1 4.2 10.4</td>
<td>13.7 13.9</td>
<td>0.9</td>
</tr>
<tr>
<td>A</td>
<td>74 5 17 08 18 50.0</td>
<td>5.5 4.3 3.8</td>
<td>12.5 10.8</td>
<td>1.2</td>
</tr>
<tr>
<td>B</td>
<td>74 5 18 03 57 40.1</td>
<td>2.7 3.1 4.8</td>
<td>12.3 9.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>

* Origin of the coordinate is shown in Fig. 2

(\(\phi_0, \lambda_0\))=(34°40', 138°50').

** Hypocenter parameters are determined by the structural model of Fig. 1 using first order iterative approximation method. Taking into account the reading error of 0.02–0.05 sec and other effects, the accuracy of these values is ±0.5 km.
from these aftershocks but the seismic path to one station IRT is through the aftershock region and the other station NHT, is not (Fig. 2). The shocks, which give sufficient SN ratios, have almost the same magnitudes determined from the F-P versus M relation (Table 2). Assuming that the shape of the source spectrum is independent on the azimuth, and ascertaining that the station effect is not significant, energy spectra of the first one second of vertical velocity seismograms of the same shock at two stations are compared.

One of the features presented in this paper is the application of the MEM spectral estimation instead of the conventional method. In order to clarify the difference between the predominant frequencies of the first direct waves that passed through the aftershock region and those that did not, contaminations by S-phase, reflected waves, and converted waves, etc. must be avoided. In this respect only the data from the first 1.0–1.5 sec are used, for the S-P time at the two stations lies between 1.5–2.0 sec. Therefore the maximum lag time $\tau_0$ of the autocorrelation function (ACF) reaches only up to 0.7–1.0 sec. For example, assuming the ACF $f(\tau)$ (Fig. 3),

$$f(\tau) = f(-\tau)$$

$$f(\tau) = e^{-\gamma |\tau|} \cos \omega_0 \tau \quad 0 \leq \tau \leq \tau_0$$

$$f(\tau) = 0 \quad \tau > \tau_0,$$

Fourier transforming Eq. (1) (power spectrum),

\[ F(\omega) = \int_{-\tau_0}^{\tau_0} f(\tau) e^{-i\omega \tau} d\tau \]

where $\omega_0 = 2\pi f_0$ becomes sharper as $\tau_0$ gets larger.
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\[ F(\omega) = \frac{1}{\sqrt{2\pi}} \cdot 2 \cdot \int_0^\infty e^{-\tau} \cos \omega_0 \tau e^{-j\omega \tau} d\tau \]

\[ = \sqrt{\frac{2}{\pi}} e^{-\tau_0} \sin \left( (\omega_0 - \omega) \tau_0 - \phi \right) + \frac{e^{-\tau_0} \sin \left( (\omega_0 + \omega) \tau_0 + \phi' \right)}{\sqrt{\tau^2 + (\omega_0 - \omega)^2}} \]

\[ + \frac{\tau}{\tau^2 + (\omega_0 - \omega)^2} + \frac{\tau}{\tau^2 + (\omega_0 + \omega)^2} \]

where

\[ \tan \phi = \frac{\omega_0 - \omega}{\gamma}, \quad \tan \phi' = \frac{\omega_0 + \omega}{\gamma}, \]

\( F(\omega) \) has a vibrating wave form that has the maximum peak at \( \omega_0 \). As shown schematically in Fig. 4, the peak at \( \omega_0 \) becomes sharper as \( \tau_0 \) becomes larger. Therefore, in estimating the predominant frequency from very short transient phenomena of the first P wave arrival, the conventional Blackman-Tukey method is insufficient as shown in section 4, meanwhile the MEM spectral estimation can produce rather reasonable spectra with a small amount of data. A further discussion will be given later.

2. Data Analysis

The direct recording magnetic tape made at a 1 mm/sec tape speed was played back at 10 mm/sec and compiled in an FM tape. Figures 5 and 6

Fig. 5. Monitor record of shocks in the lower part of Amagi Area to check the station effect. Fig. 5-1: shock A, Fig. 5-2: shock B. They show FM compiled record and filter output traces through 5.1, 6.4, ..., 40.3 Hz analog resonator.
Fig. 6. Vertical velocity seismograms of the 11 shocks. Monitored by pen recorder from compiled FM tape. On FM tape compilation the gain of IRT is 6 dB higher than that of NHT.
show the trace output to a pen recorder from the compiled FM tape which gives the data, the time scale of which, is ten times that of the original record. The frequency response of the observation system including recording and playback is given in Fig. 7. A variation of the frequency response through the FM tape compilation is found to be negligible by checking the responses in three ways; the first without the FM compilation, the second with the FM compilation and the time scale magnification by five times, and the third by ten times. As can be seen from Fig. 7, responses at NHT and IRT are almost the same. Therefore after the correction for the instrumental response, a comparison of energy spectra is meaningful in the range of 5–50 Hz. The compiled data were A-D converted at a sampling rate of 3.74 ms for IRT and 3.79 ms for NHT in real time, and the spectra were estimated by a computer. Figure 8 indicates the procedure given above.

3. Spectral Estimation and Results

As mentioned above, the MEM spectral estimation was carried out, which assumed the digitized seismic trace as a random process and fitted an

Assuming the digitized data of time series \( x(t) \) to be an AR process of order \( p \), it can be expressed as;

\[
x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \cdots + \alpha_p x_{t-p} + \epsilon_t
\]

where the DC component and the ultra low frequency component are eliminated in advance.

Rewriting Eq. (3),

\[
\epsilon_t = x_t - \sum_{s=1}^{p} \alpha_s x_{t-s}
\]

\( z \)-transforming Eq. (4),
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\[ E(z) = X(z) - X(z)A(z) \]  
(5)

where

\[ A(z) = \sum_{m=1}^{p} \alpha_m z^{-m} . \]  
(5')

From Eq. (5),

\[ |X(z)|^2 = \frac{|E(z)|^2}{|1 - A(z)|^2} = \frac{|E(z)|^2}{|\Gamma(z)|^2} \]  
(6)

where

\[ \Gamma'(z) = \sum_{m=1}^{n+1} \gamma_m z^{-m} \]  
(6')

\((1, -\alpha_1, -\alpha_2, \ldots, -\alpha_p) \equiv (\gamma_1, \ldots, \gamma_{p+1}).\)  
(7)

The energy spectral density function is obtained by putting \(z = e^{-j2\pi f}\)

\[ S(f) = \frac{\sigma^2}{|\Gamma'(e^{-j2\pi f})|^2} \]  
(8)

where

\[ |E(e^{-j2\pi f})|^2 = \sigma^2 (= \text{const.}) \]  
(8')

Fig. 9. Relation of the order \(p\) of an autoregressive process and the final prediction error (FPE). Broken line expresses an ideal case where the order \(p\) at minimum FPE can be easily determined. Solid line shows schematically the actual case of a seismic trace. In this paper, taking into account SN ratio and FPE, \(p=2\) for NHT and \(p=7-12\) for IRT are chosen in most cases.
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Fig. 10-1—Fig. 12-1 (NHT), Fig. 10-2—Fig. 12-2 (IRT). Energy spectra of the 11 shocks obtained at NHT and IRT. Frequency response correction is made to the spectra (Fig. 7). Values are relative to the peak.

Fig. 13-1. MEM spectra of shocks A and B at NHT. Predominant frequency of A is 11.5 Hz and that of B is 19 Hz.

Fig. 13-2. MEM spectra of shocks A and B at IRT. Predominant frequency of A is 13 Hz and that of B is 17 Hz. Comparison of these figures confirms that the difference in station effects of NHT and IRT is insignificant and we can compare energy spectra of NHT and IRT after system response correction.
Eq. (8') implies that ACF of a random noise is of a δ-function type.

The final prediction error (FPE) and the order $p$ of $\sigma^2$ were determined by \textsc{Akaike}'s method (\textsc{Akaike}, 1969). The correlation of $p$ and $\sigma^2$ for the real data is schematically shown in Fig. 9 by the solid line. One should be careful not to overestimate $p$ by taking into account the SN ratio of the seismic trace. Thus the estimated results are indicated in Figs. 10-12.

To check the station effects, seismic traces of two shocks which had occurred at nearly equal distances to NHT and IRT, but whose wave paths to the stations were not in the aftershock region (Fig. 2) are spectral analyzed. The filtered traces are shown in Fig. 5, which are also confirmed by the MEM spectra (Fig. 13) indicating an insignificant difference of predominant frequencies for each shock. Therefore the difference in the station effects can be neglected when comparing the spectra of the shocks in the aftershock region.

The comparison of the estimated spectra for the same shock at two stations clearly shows with no exception that the predominant frequency at IRT (8-15 Hz) is lower than that at NHT (16-25 Hz) by 4-8 Hz (Figs. 10-12).

4. Discussions

Discussions are presented below on some problems concerning the spectral estimation.

4.1 Employment of the MEM spectral estimation

This has already been discussed in the introduction, however, a more detailed explanation is given below.

The spectra estimated for aftershocks #6, 13, 31 by the Blackman-Tukey method are shown in Fig. 14 with no correction due to the frequency characteristic of the system.

When the corrections due to the frequency characteristic of the system are made on these spectra, they will become meaningless. Thus the Blackman-Tukey spectral estimation cannot be applied to a short time series analysis. In general it has less resolvability and less power range than those by the MEM spectra which are generally characterized by sharp peaks and smooth spectra. For example, the spectra for NHT indicate similar phenomena to a stationary sinusoidal wave which is due to the extrapolation of ACF by the coefficients of the AR process fitting, $\alpha_1, \alpha_2, \ldots, \alpha_p$.

According to the theory of the MEM spectral estimation, the extrapolation is done as follows;

$$r_{p+k} = \sum_{m=1}^{p} r_{p-m+k} \cdot \alpha_m ; \quad k = 1, 2, \ldots, \infty \quad (9)$$
Fig. 1. Energy spectra of shocks #6, #13, and #31 by Blackman-Tukey method. Maximum lag time of the ACF—falls in the range of 0.7–1.0 s. Maximum energy frequency is not made. Predominant frequencies are not as distinct as those of MEM spectra.
where

\[ \text{ACF } r_0, \ldots, r_p \text{ and } \alpha_1, \ldots, \alpha_p \text{ are given.} \quad (9') \]

Though the seismic traces are not stationary, the extrapolation by (9) will not produce irrelevant peaks in the energy spectra, but make clear the difference between the predominant frequency for NHT and that for IRT.

Some spectra for IRT have a second peak in the high frequency range though their levels are low. As have been pointed out (Chen and Stegen, 1974; Ulrych and Bishop, 1975), the overestimation of the order \( p \) of an AR-process leads to the splitting of the true spectrum peak into two or more peaks which have almost the same level or differ by a few dB. However, the second peak that can be found in the spectra for IRT in Figs. 10–12 seems to have some meaning because the differences in the levels are about 10 dB and \( p \) of the first minimum FPE is adopted (Fig. 9).

4.2 Comparison of the absolute values of spectra at NHT and IRT

It is possible to discuss the effect of wave paths on the absolute value of the difference in the energy spectral density if we know the source mechanism, as we already know the sensitivity of the seismometer and the gain of the DAR playback amplifier and also the frequency characteristic of the system.

However, the determination of the source mechanism of aftershocks is very difficult, because the stations are all restricted to one side of the fault plane. Nevertheless, it is sufficient to compare relative spectra to find the effect of the aftershock region on the propagation of seismic waves, for the difference of the physical properties of the paths to the stations at equal epicentral distances should be reflected on the difference in the predominant frequencies on the assumption that the velocity spectrum at the source in the generation of a micro-earthquake is independent on the azimuth.

4.3 Correlation of the spectra and the loss mechanism in the aftershock region

We detected the effect of the aftershock region on the propagation of seismic body waves, and obtained the fact that for each of eleven shocks at equal epicentral distances from NHT and IRT, every spectrum at IRT showed a lower predominant frequency by 4–8 Hz. The aftershock region seems to play a role of a high-cut filter for the seismic waves. For more detailed studies of the loss mechanism like the \( Q \) estimation from amplitude spectra, more information from observation is needed. Therefore other methods such as estimating \( Q \) from the difference in predominant frequencies by numerical experiments will be effective.

We are now studying the \( Q \) values in the aftershock region from the
above standpoint and conducting an analysis to determine whether the change of physical properties in the aftershock region can be traced by studying the time variation of the predominant frequency using the aftershock data taken over period of six months.

We express our sincere appreciation to Professors Toshi Asada, Ryosuke Sato and Dr. Katsuhiko Ishibashi and all the members of the seismological section of the Geophysical Institute for their encouragement and comments. Drs. Takeo Hirasswa and Katsutada Kami-numa have kindly permitted us to use an A-D converter and Miss Miyoko Takizawa helped us greatly during its operation at the National Institute of the Polar Research.

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REFERENCES


