CORRELATION OF TSUNAMI AND SUB-OCEANIC RAYLEIGH WAVE AMPLITUDES
POSSIBILITY OF THE USE OF RAYLEIGH WAVE IN TSUNAMI WARNING SYSTEM

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In the present tsunami warning system, the predominance of the approaching tsunami is estimated, taking account of the earthquake magnitude and focal depth. However the predominance of tsunami also depends on other focal parameters, e.g. the rise time of the source time function, fault length etc. Accordingly it is quite natural that great tsunamis due to earthquakes with relatively small magnitudes have been sometimes observed.

For the improvement of the tsunami warning system, it is necessary to develop the method to find the focal parameters before the tsunami wave arrivals and to make clear the dependence of the generation of tsunami on the above parameters. Effects of the focal parameters on the generation of tsunami and Rayleigh waves are investigated in a previous and in the present paper respectively. If the focal parameters are pre-estimated by the use of early arrival Rayleigh waves, the predominance of tsunami can be predicted. Although some of the focal parameters have fairly complex effects on both waves, it is shown that there is some possibility of the improvement of the present warning system by the use of tectonic features and body waves in addition to sub-oceanic Rayleigh wave observation.

1. Introduction

There are some cases when an earthquake with a relatively small magnitude generates a great tsunami. The 1946 Aleutian earthquake generated an exceedingly large tsunami despite the moderate magnitude of 7.4. At Scotch Cap, the west point of Unimak Island (Aleutians), the waves demolished a large well-built lighthouse 45 feet above the sea level and surge to a height of over 100 feet. The highest wave levels on the Hawaiian islands were 30 to 60 feet (Richter, 1958). The 1896 Sanriku earthquake with a magnitude of 7.6 also generated a noteworthy tsunami in the history of tsunami disasters in Japan. Along the Sanriku coast of north eastern Japan, it was reported by Imamura (1937) that 10,617 houses were swept away, 2,456 houses partially
T. YAMASHITA and R. SATO

demolished, 27,122 persons killed, 9,247 persons injured and waves as high as 30 m were observed. A large earthquake again occurred off the Sanriku coast in 1933. The 1933 earthquake has a larger magnitude, 8.4, than that of the 1896 earthquake \((M=7.6)\) and the epicenter of the former was located close to that of the latter. But the intensity of the latter tsunami was much larger than that of the former and so the cause of an abnormally great tsunami with a relatively small magnitude cannot be attributed to only the bottom topography near the coast. KANAMORI (1972a) attributed these abnormal earthquakes to the abnormally large rise time of nearly 100 sec.

In the present system, a tsunami warning is issued when an shallow sub-oceanic earthquake occurs and the earthquake magnitude is larger than a certain threshold value. But as mentioned above, there are some cases when an earthquake with a relatively small magnitude generates a great tsunami and therefore a more improved warning system is now required. EWING et al. (1950) has found a remarkable correlation between the generation of tsunami and the T-phase and pointed out the possibility of applying the T-phase observation to a tsunami warning system. In that paper, however, the argument is somewhat qualitative and the physical mechanism is obscured. YAMASHITA and SATO in their paper (1974), which is hereafter referred to as Paper I, have pointed out that the generation of tsunami strongly depends on some focal parameters (fault length, dip-angle and focal depth). Hence, if we can successfully estimate these parameters before tsunami arrivals, we provide a tsunami warning system which is far more accurate than that based on the earthquake magnitude and the focal depth. In this paper the possibility of the application of Rayleigh wave to pre-estimate focal parameters will be discussed.

The numerical computation based on the same model as in Paper I is carried out and the solution is obtained by the far field approximation.

2. Sea Bottom Displacements Due to a Dislocation Model

2.1 Solution for a point source

At first a point fault which is the same as that in Fig. 2 in Paper I is considered and the sea bottom displacements are derived. The method of the derivation of a solution is almost the same as in Paper I. The \(r\)- and \(\varphi\)-component of the sea bottom displacements, \(u\) and \(w\), are obtained as

\[
\begin{align*}
  u &= \sum_{n=0}^{\infty} \int_{0}^{\infty} \frac{\gamma_{A} A_{n} e^{-\nu_{A} d} + A_{n} B_{n} e^{-\nu_{B} d}}{D(\xi)} \frac{\partial}{\partial r} F_{n}(r, \varphi) d\xi , \\
  w &= \sum_{n=0}^{\infty} \int_{0}^{\infty} \frac{\gamma_{A} A_{n} e^{-\nu_{A} d} + A_{n} B_{n} e^{-\nu_{B} d}}{D(\xi)} F_{n}(r, \varphi) d\xi ,
\end{align*}
\]

(1)
where

\[ \chi_0 = 2k^2 \Theta' \omega^2 \frac{\nu_a}{\mu} (g \nu'' \cos \nu'' h_0 - \omega^2 \sin \nu'' h_0) \]

\[ + 2 \nu_a \left\{ 2k^2 \nu_\rho + k^2 \frac{g}{\mu} (\rho - \rho') \right\} (g \nu'' \sin \nu'' h_0 + \omega^2 \cos \nu'' h_0), \]

\[ A_0 = 2k^2 \Theta' \omega^2 \frac{\nu_a}{\mu} (g \nu'' \cos \nu'' h_0 - \omega^2 \sin \nu'' h_0) \]

\[ + 2 \nu_a k^2 \left\{ 2k^2 - \nu_\alpha \frac{g}{\mu} (\rho - \rho') \right\} (g \nu'' \sin \nu'' h_0 + \omega^2 \cos \nu'' h_0), \]

\[ \begin{align*} 
A_1 &= 4 \nu_a \rho^2 k^2 \xi^2 (g \nu'' \sin \nu'' h_0 + \omega^2 \cos \nu'' h_0), \\
F_a(r, \varphi) &= \alpha_a J_n(\xi r), \\
\end{align*} \]

\[ \alpha_0 = \frac{1}{2} \sin \lambda \sin 2\delta, \quad \alpha_1 = -\cos \lambda \cos \delta \cos \varphi + \sin \lambda \cos 2\delta \sin \varphi, \]

\[ \alpha_2 = \alpha_0 \cos 2\varphi + \cos \lambda \sin \delta \sin 2\varphi, \]

\[ A_0 = \frac{M^*}{4\pi \rho \omega^2} (2\xi^2 + \xi^2) \frac{\xi}{\nu_\alpha}, \quad A_1 = \frac{M^*}{4\pi \rho \omega^2} 2 \text{sgn}(z-d) \xi^2, \]

\[ B_0 = \frac{M^*}{4\pi \rho \omega^2} \text{sgn}(z-d) \xi, \quad B_1 = \frac{M^*}{4\pi \rho \omega^2} 2 \xi^2 - k^2 \frac{\nu_\rho}{\nu_\alpha}, \]

\[ B_2 = \frac{M^*}{4\pi \rho \omega^2} \text{sgn}(z-d) \xi, \quad \nu_\alpha = \sqrt{\xi^2 - h^2}, \quad \nu_\rho = \sqrt{\xi^2 - k^2}, \]

\[ \nu_a'' = \sqrt{h^2 - \xi^2}, \quad h = \omega / V_r, \quad k = \omega / V_s, \quad h' = \omega / V', \]

\[ \Delta(\xi) = \left\{ (2\xi^2 - k^2)^2 - 4 \xi^2 \nu_\alpha \nu_\rho - k^2 \nu_\alpha \frac{g}{\mu} (\rho - \rho') \right\} \left\{ -g \nu'' \sin \nu'' h_0 - \omega^2 \cos \nu'' h_0 \right\} \]

\[ + \frac{\rho' \omega^2}{\mu \nu''^2} k \nu_\alpha \left\{ g \nu'' \cos \nu'' h_0 - \omega^2 \sin \nu'' h_0 \right\}. \]

\[
M^*, \text{ Fourier spectrum of the moment function;} \\
V_r, \text{ P wave velocity in the substratum;} \\
V_s, \text{ S wave velocity in the substratum;} \\
V, \text{ P wave velocity in the water layer;} \\
\rho, \text{ density of the substratum;} \\
\rho', \text{ density of the water layer;} \\
\mu, \text{ rigidity of the substratum;} \\
h_0, \text{ water depth;} \\
g, \text{ acceleration of the gravity;} \\
J_n(\xi r), \text{ Bessel function of the n-th order;} \\
\Delta(\xi) = 0 \text{ means a period equation of the sub-oceanic Rayleigh wave.} \]
Fig. 1 is shown the result of computation of the sub-oceanic Rayleigh wave dispersion curve. When carrying out numerical computations, we take $\rho' = 1 \text{ g/cm}^3$, $\rho = 2.67 \text{ g/cm}^3$, $V_p = 6.9 \text{ km/sec}$, $V_s = 4.0 \text{ km/sec}$, $g = 0.0098 \text{ km/sec}^2$, $V = 1.52 \text{ km/sec}$, $h_0 = 5.0 \text{ km}$, as in Paper I.

When $\xi r$ is large, $J_n(\xi r)$ in Eq. (2) is approximated as

$$J_n(\xi r) \rightarrow \frac{1}{2} \sqrt{\frac{2}{\pi \xi r}} e^{-\xi r + i(\frac{(2n+1)}{4})}.$$

As we are interested only in Rayleigh wave at present, the contribution from the residue is calculated. Then we have the solutions in the dimensionless forms as

$$u = \frac{M^*}{\mu h_0^2} \left[ \sum_{n=0}^{2} \alpha_n \left\{ \Gamma_n^0 e^{-\xi r + i(\frac{(2n+1)}{4})} + \Pi_n^0 e^{-\xi r + i(\frac{(2n+3)}{4})} \right\} e^{-i \xi r + i(\frac{(2n+1)}{4})} \right],$$

$$w = -\frac{M^*}{\mu h_0^2} \left[ \sum_{n=0}^{2} \alpha_n \left\{ \Gamma_n^1 e^{-\xi r + i(\frac{(2n+1)}{4})} + \Pi_n^1 e^{-\xi r + i(\frac{(2n+3)}{4})} \right\} e^{-i \xi r + i(\frac{(2n+1)}{4})} \right],$$

where

$$\Gamma_n^0 = \frac{1}{\sqrt{2\pi}} \left( \frac{h_0^4}{2V_0^6} \right)^{1/4} \left( KQ^4 \frac{\Omega}{\eta} \right)^{-1/4} \chi_0 A_n', \quad \Gamma_n^1 = -\frac{1}{\sqrt{2\pi}} \left( \frac{h_0^4}{2V_0^6} \right)^{1/4} \left( KQ^4 \frac{\Omega}{\eta} \right)^{-1/4} \chi_0 A_n', \quad \Pi_n^0 = \frac{1}{\sqrt{2\pi}} \left( \frac{h_0^4}{2V_0^6} \right)^{1/4} \left( KQ^4 \frac{\Omega}{\eta} \right)^{-1/4} A_n B_n', \quad \Pi_n^1 = -\frac{1}{\sqrt{2\pi}} \left( \frac{h_0^4}{2V_0^6} \right)^{1/4} \left( KQ^4 \frac{\Omega}{\eta} \right)^{-1/4} A_n B_n',$$

$$A_n' = (2\kappa_n^2 + 1)/\kappa, \quad A_n' = -2, \quad A_n' = 1/\kappa,$$

$$B_n' = -3, \quad B_n' = (2-\eta^2)/\kappa, \quad B_n' = -1.$$
\[ K = 4 \left\{ \left( 2 - \eta^2 - \eta_{a} \eta_{b} \right) \frac{\kappa_{a}}{\kappa_{b}} \right\} \zeta - \frac{(\rho - \rho')}{\rho} \frac{G \eta' \kappa''}{4 \kappa_{a}} \left[ - \frac{G \kappa'''}{\eta^3} \sin \zeta \cos \zeta \right] \]

\[ + \frac{i}{\eta^3} \left[ (2 - \eta^2)^2 - 4 \kappa_{a} \kappa_{b} \right] \zeta - G \eta' \kappa' \kappa'' (\rho - \rho')/\rho \left[ (G - \Omega^2) \sin \zeta/\zeta + G \cos \zeta \right] \]

\[ - i \frac{\rho'}{\rho} \frac{\eta^4}{\kappa''} \left( \kappa''_{a} + \kappa'_{b} \right) \left\{ - \frac{G \kappa''}{\eta^3} \cos \zeta + \zeta \sin \zeta \right\} , \]

\[ \nu = \kappa \cdot \kappa_{a} , \quad \nu_{p} = \kappa \cdot \kappa_{b} , \quad \nu''_{a} = \kappa \cdot \kappa''_{a} , \quad \Omega = \omega h_{0} / V_{a} , \quad \eta = c / V_{a} , \]

\[ \zeta = \Omega \kappa''_{a} / \eta , \quad G = g h_{0} / V_{a}^2 \quad \text{and} \quad \kappa \quad \text{is the root of} \quad \Delta (\xi) = 0 . \]

(6)

The source time function with rise time \( t_{0} \) and final dislocation amplitude \( D_{0} \) is taken, then

\[ M_{a} = \frac{h_{0} M_{0}}{V_{a}} \frac{2 \Omega \tau_{a} e^{-i(\Omega t_{a})}}{\tau_{a} \omega} \sin \frac{\Omega \tau_{a} e^{-i(\Omega t_{a})}}{2} \]

with \( M_{0} = \mu D_{0} \), \( \tau_{a} = V_{a} t_{a} / h_{0} \).

2.2 Solution for a moving source with dimensions

We take the fault model as is seen in Fig. 2 and obtain the solution in this case by making the same operation to Eq. (5) as 2.5 in Paper I as follows:

\[ u = \frac{D_{0}}{V_{a} h_{0}} \sqrt{\frac{\eta_{a}}{r}} \sum_{n=0}^{2} \alpha_{n} \frac{2}{\tau_{0} \omega} \sin \frac{\tau_{0} Q}{2} \sin \frac{\tau_{0} Q}{X} \sin \frac{\tau_{0} Q}{Y} \]

\[ \times \left\{ \sin \frac{\tau_{0} Q}{X} \frac{Y_{a} \tau_{0} Q e^{-i(\tau_{0} Q/2) - (\tau_{0} Q/2)}}{Y_{a} \tau_{0} Q e^{-i(\tau_{0} Q/2) - (\tau_{0} Q/2)}} \right\} e^{-i(\tau_{0} Q/2) - (\tau_{0} Q/2)} \]

\[ w = \frac{D_{0}}{V_{b} h_{0}} \sqrt{\frac{\eta_{b}}{r}} \sum_{n=0}^{2} \alpha_{n} \frac{2}{\tau_{0} \omega} \sin \frac{\tau_{0} Q}{2} \sin \frac{\tau_{0} Q}{X} \sin \frac{\tau_{0} Q}{Y} \]

\[ \times \left\{ \sin \frac{\tau_{0} Q}{X} \frac{Y_{a} \tau_{0} Q e^{-i(\tau_{0} Q/2) - (\tau_{0} Q/2)}}{Y_{a} \tau_{0} Q e^{-i(\tau_{0} Q/2) - (\tau_{0} Q/2)}} \right\} e^{-i(\tau_{0} Q/2) - (\tau_{0} Q/2)} \]

where

\[ z = -h_{0} \]

\[ x \]

\[ y \]

\[ z = d \]

\[ \delta , \text{dip-angle;} \lambda , \text{slip angle;} z = 0, \text{sea bottom;} z = -h_{0}, \text{sea surface.} \]
The waveform is obtained by the inverse Fourier transform of Eq. (5) or (7) with respect to $\Omega$, which is accomplished numerically.

The following numerical computations are carried out for a source model of a unilateral pure dip-slip reverse fault ($c_2=\infty$) which is the same as that in Paper I and the correlation of the vertical component of the displacement of Rayleigh wave and tsunami is investigated. Epicentral distance, $r$, is fixed at 250 km.

3. Azimuthal Distribution of Spectral Amplitudes

The azimuthal distribution of vertical component of spectral amplitudes for several periods are shown in Fig. 3. This figure indicates that the spectral amplitude of Rayleigh wave is largest at $\varphi=30^\circ-60^\circ$ and $\varphi=300^\circ-330^\circ$ in the period range from 9.0 to 67.1 sec considered. On the other hand, tsunamis in the direction of $\varphi=90^\circ$ and $\varphi=270^\circ$ (i.e. directions perpendicular to the strike of the fault plane) are much larger than those in the other directions. In this computation, $c_1=3$ km/sec and $L=100$ km are taken and therefore the total time duration of faulting, $t_r=L/c_1$, is 33.3 sec. Since the predominant period of tsunami is above several hundred seconds, tsunami is almost independent of $t_r$ and the radiation pattern of tsunami may only depend on the slip direction. On the contrary the predominant period of Rayleigh wave is several tens of seconds and so the azimuthal distribution of Rayleigh wave strongly depends on both $t_r$ and the slip direction. Then the difference in the azimuthal distribution between tsunami and Rayleigh wave may be attributed to that in the dependence on $t_r$ for both waves. Judging from the view-point of the prognosis of tsunami, the investigation of Rayleigh wave at $\varphi=90^\circ$ or $\varphi=270^\circ$ is most desirable, because tsunami is most violent in these directions. However Rayleigh wave spectral amplitudes are very complex in these directions as seen in Fig. 3. Numerical computations are carried out at $\varphi=45^\circ$ for the rest of this paper, with the intention of estimating the predominance of tsunami at $\varphi=90^\circ$ or $270^\circ$ by the observation of Rayleigh wave at $\varphi=45^\circ$. 

\[
\begin{align*}
X &= \frac{\Omega}{2} \cdot \frac{L}{h_0} \cdot \left( \frac{V_g}{c_1} \frac{\cos \varphi}{\eta} \right), \\
Y_\alpha &= \frac{\Omega}{2} \cdot \frac{W}{h_0} \cdot \left( \frac{V_g}{c_2} \frac{\cos \varphi}{\eta} + i \kappa_\alpha \frac{\sin \delta}{\eta} \right), \\
Y_\beta &= \frac{\Omega}{2} \cdot \frac{W}{h_0} \cdot \left( \frac{V_g}{c_2} \frac{\cos \varphi}{\eta} + i \kappa_\beta \frac{\sin \delta}{\eta} \right),
\end{align*}
\]
Fig. 3. Radiation patterns of spectral amplitudes of sub-oceanic Rayleigh waves (vertical component) for several periods in the case when $L=100$ km, $W=50$ km, $\delta=45^\circ$, $\lambda=90^\circ$, $\lambda_t=0.5$ sec, $t_0=0.5$ sec, $r=250$ km and the top of the fault nearly reaches the sea bottom.

The unit of the scale is $k_0 D_0 / \gamma$. 

Correlation of Tsunami and Sub-Oceanic Rayleigh Wave Amplitudes
4. Dependence of Rayleigh Wave on Focal Parameters

In this section, the effects of various focal parameters on sub-oceanic Rayleigh wave is examined.

4.1 Rise time

A factor associated with the rise time in the expression of the Rayleigh wave spectral amplitude, (5) or (7), is

\[ f(\tau_0) = \left| \sin \frac{\Omega \tau_0}{2} \right|, \]  

which is the same function as Eq. (20) in Paper I and the variation of \( f(\tau_0) \) with a period was shown in Fig. 9 in that paper. It is found from that figure that the contribution to the spectral amplitude decreases with the increase of the rise time. In the case of Rayleigh wave, the predominant period is ten to several tens of seconds in our model, and so the effect of the rise time variation is more sensitive for Rayleigh wave than for tsunami.

4.2 Focal depth

The variation of the spectral amplitudes and wave forms for vertical components with focal depth are shown in Figs. 4 and 5, respectively. We can notice from these figures that the spectral amplitude and the wave ampli-

![Fig. 4. Dependence of Rayleigh wave spectral amplitude (vertical component) on focal depth. Curve parameter is the depth from the sea bottom to the top of the fault plane. The computation is carried out at \( \varphi = 45^\circ \). For the other focal parameters \( (L, W, \delta, \lambda, c_1, c_2 \text{ and } \sigma_0) \), see the caption for Fig. 3. The unit of the vertical axis is \( h_0 D_0/V_0 \).]
Correlation of Tsunami and Sub-Oceanic Rayleigh Wave Amplitudes

Fig. 5. Dependence of the waveform on focal depth. Parameters are the same as in Fig. 4. The unit of the vertical axis is $D_0$ and the negative direction means an up motion.

The amplitude of Rayleigh waves decreases severely, especially in the short period range, as the focal depth increases.

4.3 Rupture velocity

Since only the unilateral pure dip-slip fault model as in Paper I is considered as mentioned above, only the effect of $c_1$ is discussed. The factor which is related to the rupture velocity, $c_1$, is

\[ g(c_1) = \left| \sin \frac{X}{X} \right|, \]  \hspace{1cm} (10)

where

\[ X = \frac{\Omega}{2} \frac{L}{h_0} \left( \frac{V_g}{c_1} - \frac{1}{\eta} \cos \varphi \right), \]

which is the same function as used in the discussion of the effect of the rupture velocity in Paper I. This function is similar to Eq. (9) and therefore the contribution of $g(c_1)$ to the spectral amplitude, Eq. (7), at $\varphi = 90^\circ$ and $270^\circ$ increases with the rupture velocity, $c_1$, although circumstances are somewhat complex in the other directions. As was investigated in Paper I, each large tsunamigenic earthquake has a rather similar rupture velocity ranging within 2 to 4.5 km/sec. Hence rupture velocity, $c_1$, is fixed at 3 km/sec in the following computations as in Paper I.

4.4 Fault length

A factor in the amplitude expression, (7), which is associated with the fault length, is

\[ h(L) = L \left| \sin \frac{X}{X} \right|, \]

where

\[ X = \frac{\Omega}{2} \frac{L}{h_0} \left( \frac{V_g}{c_1} - \frac{1}{\eta} \cos \varphi \right). \]
which is also the same function as Eq. (24) in Paper I. We show $|\sin X/X|$ as a function of the period at $\varphi=15^\circ$ and $45^\circ$ in Figs. 6 and 7 respectively. It is seen that as the fault length increases, the trough of $|\sin X/X|$ in the period range, 20 to 30 sec, becomes deep especially at $\varphi=45^\circ$. Moreover, since $h(L)$ is the product of $L$ and $|\sin X/X|$, the size of the spectral amplitude becomes large in all the period range as the fault length increases in addition to the formation of the trough.

Fig. 6. Numerical values of $|\sin X/X|$ in Eq. (11) at $\varphi=15^\circ$. Curve parameter is the fault length and $c_l=3$ km/sec.

Fig. 7. Numerical values of $|\sin X/X|$ in Eq. (11) at $\varphi=45^\circ$. Other conditions are the same as in Fig. 6.
4.5 Dip-angle

The wave forms and the spectral amplitudes which are numerically computed at $\phi = 45^\circ$ for various dip-angles are shown in Figs. 8 and 9 respectively. From both figures, the predominance of Rayleigh wave for small dip-angles is seen. In our numerical computation for the fault model with dimensions, the top of the fault plane nearly reaches the bottom surface and therefore the lowest part of the fault plane becomes shallow for small dip-angles and

![Fig. 8](image-url) Dependence of the waveform of Rayleigh wave (vertical component) on dip-angle. Focal parameters are the same as in Fig. 3. For the other conditions, see the caption for Fig. 5.

![Fig. 9](image-url) Dependence of the spectral amplitude of Rayleigh wave (vertical component) on dip-angle. Focal parameters are the same as in Fig. 3. The unit of vertical scale is $h_0D_0/V_s$. 
becomes deep for large dip-angles. Accordingly in the numerical computation of Figs. 8 and 9 the effect of the focal depth is also contained. The fault plane of the earthquake which generates a considerable tsunami will reach the sea bottom and it is quite natural to fix the top of the fault plane close to the bottom surface, which is the same model as in the discussion of the effect of the dip-angle on the generation of tsunami in Paper I.

5. Correlation of Tsunami and Rayleigh Wave

When we assume that we can pre-estimate the generation of tsunami by the use of Rayleigh wave, the problem becomes simpler, if every focal parameter has the same effect on both waves. But some focal parameters have the same effect and others have opposite effects on both waves. Hence it can be supposed that some earthquake causes a large tsunami in spite of the slight Rayleigh wave or a slight tsunami in spite of the large Rayleigh wave. The focal parameters which play important parts in this problem are the rise time, fault length, focal depth and dip-angle as discussed in the preceding section. The effect of the focal parameters on tsunami was discussed in Paper I.

First of all, the difference in the effect of each focal parameter on both waves is summarized as follows:

5.1 Rise time

The contribution of this factor to the spectral amplitude of tsunami is largest when the rise time is 0 (step time function). As the rise time increases, the contribution to the short period range decreases, but that to the long period range is not changed too much (see Fig. 9 in Paper I). We can take the rise time as 10 to 20 sec at the utmost (this will be accepted from studies on the rise times of various great earthquakes beneath the sea bottom, as was discussed in Paper I). Since the predominant period of tsunami is above several hundreds of seconds, this factor does not make an important contribution to tsunami. On the contrary, since the predominant period of Rayleigh wave is several tens of seconds, the spectral amplitude of Rayleigh wave becomes small, especially in the short period range, as the rise time increases.

5.2 Fault length

In the case of tsunami, as the fault length increases, short period components of spectral amplitude are cut off (see Fig. 14 in Paper I), but this factor has a somewhat different effect on Rayleigh wave. At \( \phi = 45^\circ \), as the fault length increases, the spectral amplitude becomes large over the entire period range and the trough of the spectral amplitude around 20 sec becomes deep relatively to the other peaks.
5.3 Dip-angle

This factor has the entirely opposite effect on both waves. As is illustrated in Fig. 9, the spectral amplitude of Rayleigh wave predominates for small dip-angles but that of tsunami does for large dip-angles (see Fig. 16 in Paper I).

5.4 Focal depth

The dependence of tsunami waveform on the focal depth is illustrated in Fig. 10. As tsunami is a wave phenomenon with an extremely long period, it should be closely related to the static deformation of the sea bottom. Static deformation due to the faults with focal depths corresponding to the examples in Fig. 10 are shown in Fig. 11. These computations are carried out by using a program by SATO and MATSU'URA (1974). The case in Fig. 11(a) yields the largest displacement among the examples in Fig. 11. However, in the case of (a), large displacement is restricted only to the region where the fault plane reaches the surface. Tsunami is efficiently excited by a wide range deformation rather than a local deformation. For the example of Fig. 10 which corresponds to Fig. 11(a), the short period component is predominant, comparing with the other examples, because in this case tsunami is excited by the local deformation. In cases of Fig. 11(b) and (c), the volume and the extent of the deformation is larger than the case of (a). Then the fault with a focal depth somewhat deeper than the sea bottom excites tsunami most efficiently; in other words, there is an optimal depth for tsunami generation. On the contrary, the spectral amplitude of sub-oceanic Rayleigh wave decreases monotonously, especially in the short period range, as the focal depth.
Fig. 11. Contour of vertical displacements which corresponds to each case in Fig. 10. The unit is $D_0$. Numerical computation is carried out only in the half plane because of the symmetry. The rectangular figure means the projection of the fault plane on the surface and the broken line is the top of the fault plane. (a) The depth from the sea bottom to the top of the fault plane is nearly 0 km. (b) 10 km, (c) 20 km, (d) 30 km, (e) 40 km.
increases. As is understood from Fig. 4, the peak of the Rayleigh wave spectral amplitude around 10 sec decreases rapidly as the focal depth increases.

6. Tsunami Warning System by the Use of Rayleigh Wave

As described in the introduction, the current tsunami warning system is based almost entirely on the earthquake magnitude. It is necessary to improve this warning system, because sometimes a large tsunami is generated in spite of the smallness of the earthquake magnitude. As is understood in Paper I, the predominance of tsunami depends considerably on some focal parameters. Accordingly, if these focal parameters can be pre-estimated by some means, it is possible to predict the predominance of tsunami. In this section the possibility of this prediction by means of Rayleigh wave is discussed, and the predominance of tsunami at $\varphi = 90^\circ$ or $270^\circ$ is estimated through the investigation of Rayleigh wave. The directivity of tsunamigenic energy is largest along the above directions and so we have a principal interest in those directions. Rayleigh wave at $\varphi = 45^\circ$ is employed as a typical case.

The conditions of a visitation of a large tsunami are (1) a large dip-angle, (2) the focal depth around the optimal depth, (3) a large fault length and (4) a short rise time. Some of them have sensitive effects and others do not, as was discussed in Paper I. For example, when the rise time increases to 20 or 30 sec, the decrease in the spectral amplitude of a tsunami is slight in the entire period range.

The spectral amplitude of Rayleigh wave due to a longer fault is characterized by larger values in all the period range and by a relatively deeper trough around 20 sec, in comparison with the shorter fault case.

The effect of the dip-angle is complicated because the dip-angle has opposite effects on both waves. If a shallow large earthquake beneath the sea bottom is caused by a subduction of the lithosphere, the dip-angle may be regarded as almost constant in each seismic zone. For example, both the dip-angles of the 1944 Tonankai earthquake ($M = 8.0; 33.7^\circ$N, $136.2^\circ$E) and the 1946 Nankaido earthquake ($M = 8.1; 33.0^\circ$N, $135.6^\circ$E) were estimated to be $10^\circ$ (KANAMORI, 1972b). Hence the dip-angle of the shallow large earthquake off the southern coasts of the Sıkoku district and the Kii Peninsula in Japan may be able to be fixed around $10^\circ$. Thus, for a tsunamigenic area, the dip-angle of the fault may be reasonably taken as constant, when discussing the spectral amplitude of Rayleigh wave to pre-estimate the generation of tsunami. For the focal depth shallower than the optimal depth for the tsunami generation, the factor of the focal depth has reverse effects on both waves, but for a deeper focal depth it has the same effect. Because of the above complex
circumstances, it is necessary to estimate the focal depth in a usual way, for example, by using body wave arrival times.

The rest is the problem of the rise time. With increasing rise time, the decrease of tsunamigenic energy is quite slight, but that of Rayleigh wave, especially in the short period range, is considerable. For the above reasons, mainly the long period range of Rayleigh wave should be investigated in the following.

From the above discussion, the predominance of tsunami can be pre-estimated by the use of Rayleigh wave observations, tectonic features and body wave travel times. As a consequence, the following warning system is proposed.

1) First of all it is necessary to find the predominant dip-angle in each seismic zone, where a great tsunamigenic earthquake occurred in the past and also is likely to occur in the future, by the study of the past earthquakes. According to the information up to now, it has been found that the dip-angle for the Sanriku area is $45^\circ$ and that for the Tonankai area $10^\circ$.

2) The combination of the fault length and focal depth, which yields the minimum tsunami damage, for example, a tsunami magnitude of 1, is obtained for a given dip-angle. An example of this procedure is illustrated in Fig. 12, where the computation is carried out for the dip-angle of $10^\circ$.

![Waveform of tsunami](image)

**Fig. 12.** Waveform of tsunami, in the direction of $\phi=90^\circ$ and with the epicentral distance of 250 km, for several focal depths in the case when the fault length has a threshold value for the tsunami disaster. The minimum tsunami damage is assumed to be caused by the wave-height of 0.8 to 0.9 $D_0$. The fixed focal parameters are as follows; $W=50$ km, $\delta=10^\circ$, $\lambda=90^\circ$, $c_1=3$ km/sec, $c_2=\infty$ and $t_0=0.5$ sec. $d$ in the figure means the depth from the sea bottom to the top of the fault plane. The unit of the vertical scale is $D_0$ and the negative direction means an up motion.
earthquake with the combination of fault length $L$ and focal depth $d$ shown in this figure causes a tsunami with a wave height of $0.8D_0$ to $0.9D_0$, $D_0$ being the final dislocation, at a distance of 250 km.

The spectrum of Rayleigh waves for each combination above around 20 sec is then computed and tabulated, which can be used as the threshold value. Figure 13 represents the spectral amplitude of Rayleigh waves corresponding to each case in Fig. 12. At a distance of $r$ km, the amplitude is simply obtained by multiplying by a factor $\sqrt{250/r}$.

The following procedure is carried out at the moment of the occurrence of a sub-oceanic earthquake.

3) At the moment of the occurrence of the large sub-oceanic earthquake, the focal depth is determined by the body wave observation. Then the threshold spectral amplitude of Rayleigh wave with the corresponding focal depth and dip-angle is picked out from the table prepared.

Secondly the comparison of the threshold spectral amplitude with the observed one is made. If the size of the latter is larger than the former, especially in the long period component, a tsunami warning should be issued.

7. Discussion

In this paper the dependence of sub-oceanic Rayleigh wave on focal parameters and the possibility of the use of this wave as a tsunami warning are discussed. Besides, it is elucidated that the use of this wave can improve the reliability of a tsunami warning system, which is mainly based on the earthquake magnitude at present.

The model which consists of a liquid and elastic half space is assumed in this paper. But this is a fairly simplified model. As a more realistic model, a layered structure which contains a soft sedimentary layer should be taken account of because Rayleigh wave depends on the layered structure consider-
Correlation of Tsunami and Sub-Oceanic Rayleigh Wave Amplitudes

Fig. 14. Dispersion curve of phase-(C) and group-(U) velocities of sub-oceanic Rayleigh wave corresponding to the model of Fig. 15 which has the effect of a layered structure.

Fig. 15. Model of the oceanic crust which consists of a layered structure. Shadow region represents the liquid layer and \( \rho \) is the density (in g/cm\(^3\)). \( V_P \) and \( V_S \) are P and S wave velocities respectively.

ably. Figure 15 is a model of the oceanic crust which has a layered structure and Fig. 14 is the dispersion curve of sub-oceanic Rayleigh wave corresponding to the model of Fig. 15. The dispersion curve in Fig. 14 is nearly the same as that in Fig. 1 above the period of about 10 sec. Then the rough character of the Rayleigh wave amplitude is not likely to be much changed, even if the layered structure is taken into consideration, since the discussion is restricted in the long period range. For example, it may be impossible that the Rayleigh wave amplitude predominates for larger dip-angles, when layered structure is taken into consideration (Rayleigh wave amplitude predominates for smaller dip-angles in case of the elastic half space model).

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