SINGLE ISOTROPIC SCATTERING MODEL INCLUDING WAVE CONVERSIONS SIMPLE THEORETICAL MODEL OF THE SHORT PERIOD BODY WAVE PROPAGATION

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The elastic energy propagation in a three dimensional elastic medium, in which scatterers are distributed homogeneously and randomly, is studied from the point of view of the scattering theory. The isotropic scattering is assumed and the scattering process is considered up to the first order. It is assumed that not only the P-wave to P-wave scattering and the S-wave to S-wave scattering but also the S-wave to P-wave conversion scattering and the P-wave to S-wave conversion scattering take place. Green functions corresponding to these single isotropic scatterings are obtained and their properties are exhibited when the elastic energy is radiated spherically from a source. A space-time distribution of the energy density of the single scattered elastic waves is obtained. This model is applied to the short period body wave propagation of a small local earthquake near a hypocenter in the earth, when heterogeneities are interpreted as the scatterers. The temporal development of the seismic energy density, which is closely related to the seismogram, may be explained qualitatively.

1. Introduction

In general, the seismogram of a small local earthquake recorded by a short period seismograph near a hypocenter consists mainly of the direct longitudinal wave (P-wave) and the direct transverse wave (S-wave). In addition to these clear phases, there remain wave trains (see Fig. 1) which will be interpreted as scattered seismic body waves by heterogeneities in the earth (cracks, faults, velocity and/or density anomalies, etc.), that is, there exists no clear phase in the wave trains. In this paper, these wave trains are studied from the point of view of the scattering theory.

Precisely, what we intend to study is not the temporal development of the amplitudes of the scattered seismic body waves themselves, but the temporal development of the energy density of the scattered seismic body waves.

The temporal development of the energy density of the scattered seismic body waves may be given as follows. The energy density distribution of the

Fig. 1. Example; The seismogram recorded by a short period seismograph.

scattered seismic body waves is of the form of a concave curve between the direct P-wave phase and the direct S-wave phase, and it has a minimum value just before the direct S-wave phase. It decreases monotonously after the direct S-wave phase.

The object of this paper is to construct a simple theoretical model which explains these properties.

Theoretical studies corresponding to elastic wave propagations in a heterogeneous medium have been made (CHERNOV, 1960; KNOPOFF and HUDSON, 1964). The mean square amplitudes for elastic waves scattered by fluctuations in a elastic medium were calculated.

Several studies of the seismic waves from the point of view of the scattering theory have been made (WESLEY, 1965; HUDSON and KNOPOFF, 1966; AKI, 1969, 1973; AKI and CHOUET, 1975; NAKAMURA, 1976; SATO, 1977).

The author assumed a simple model in his paper I (SATO, 1977), in which coda waves of a small local earthquake were interpreted as single isotropic scattered body waves by the heterogeneities in the earth. In the paper I, the scattering process including wave conversions such as P-wave to S-wave or S-wave to P-wave was not studied.

Now, more realistic assumptions are made in this paper. It is assumed that not only the P-wave to P-wave scattering and the S-wave to S-wave scattering but also the S-wave to P-wave conversion scattering and the P-wave to S-wave conversion scattering take place when elastic waves are scattered. We assume that the angular dependence of the scattering process is isotropic and the scattering is considered up to the first order. And it is assumed that the scatterers are distributed homogeneously and randomly.

On the basis of these assumptions, the energy density of the scattered elastic waves near a source is investigated when the elastic energy is radiated spherically from the source in a short duration. The temporal development of the energy density of the scattered elastic waves is investigated in detail.

2. Assumptions

We assume that the earth is a three dimensional infinite elastic medium and the heterogeneities (cracks, faults, velocity and/or density anomalies, etc.) are interpreted as scatterers.

We make the following assumptions for the medium, the source and the scatterer.

A. The medium

- (1) Three dimensional infinite elastic medium.
- (2) Characterized by the P-wave velocity V^p (cm/sec), the S-wave velocity V^s (cm/sec) and the mass density m (g/cm³).
- (3) Non-dispersive.
- (4) The energy transfer into heat does not occur ($Q = \infty$).

B. The source

- (1) Point source.
- (2) Spherical symmetric radiation.
- (3) The source duration time u (sec) is sufficiently small (pulse like radiation).
- (4) The energy radiations of the elastic waves per unit time within a unit angular frequency band around ω are given as

$$L_u^p(t|\omega) \text{ (erg/sec) for P-waves,}$$

$$L_u^s(t|\omega) \text{ (erg/sec) for S-waves.}$$

C. The scatterer

- (1) Elastic waves are scattered spherically (isotropic scattering).
- (2) The scattering cross sections are given by
 - $\sigma^{pp}(\omega)$ (cm²) for the P-wave to P-wave scattering,
 - $\sigma^{ss}(\omega)$ (cm²) for the S-wave to S-wave scattering,
 - $\sigma^{sp}(\omega)$ (cm²) for the S-wave to P-wave conversion scattering,
 - $\sigma^{ps}(\omega)$ (cm²) for the P-wave to S-wave conversion scattering.
- (3) The distribution of the scatterers is homogeneous, where the distribution densities are given by

$$n^{pp} \text{ (cm}^{-3}\text{) for the PP scatterers,}$$

$$n^{ss} \text{ (cm}^{-3}\text{) for the SS scatterers,}$$

$$n^{sp} \text{ (cm}^{-3}\text{) for the SP scatterers,}$$

$$n^{ps} \text{ (cm}^{-3}\text{) for the PS scatterers.}$$

- (4) The mean free paths are given by

$$l^p \equiv (n^{pp}\sigma^{pp} + n^{ps}\sigma^{ps})^{-1} \text{ for P-waves,}$$

$$l^s \equiv (n^{ss}\sigma^{ss} + n^{sp}\sigma^{sp})^{-1} \text{ for S-waves.}$$

- (5) The distribution of the scatterers is random, so that the scattered waves are mutually incoherent.
- (6) The scatterers are in the wave zone of the radiated waves and the observer is in the wave zone of the scattered waves,

$$a \ll r_1, r_2 \quad \text{and} \quad \lambda^p, \lambda^s \ll r_1, r_2$$

where a is a size of the scatterer, λ^p and λ^s are the wave-lengths, r_1 is the distance from the source to the scatterer and r_2 is the distance from the scatterer to the observer (short wave-length, that is, short period waves).

The frequency dependence and the angular dependence of the scattering process are interesting from the point of view of the elastic theory as mentioned in the paper I. Interesting investigations including conversion scatterings have been made (YAMAKAWA, 1962; KNOPOFF and HUDSON, 1964, 1967). But here, we do not intend to study this problem in detail. We simply assume C(1) and C(2).

The single scattering approximation holds true for $r \ll l^p, l^s$ and $t \ll l^p/V^p, l^s/V^s$, where r is the distance from the source to the observer and t is the time measured from the earthquake origin time.

3. Single Isotropic Scattering Model Including Wave Conversions

In the wave zone, a spherical wave can be regarded as a plane wave over small regions of the space. For propagating plane waves the elastic energy density $E(\mathbf{r}, t|\omega)$ (erg/cm³) within a unit angular frequency band around ω is twice the kinetic energy density; so we have

$$E(\mathbf{r}, t|\omega) = \sum_{\nu=1}^3 m\omega^2 P_{\nu}(\mathbf{r}, t|\omega), \quad (1)$$

where $P_{\nu}(\mathbf{r}, t|\omega)$ is the power spectrum of the ν -component of the displacement vector of the elastic medium at a co-ordinate $\mathbf{r}$ determined from a short sample around a given time t (AKI, 1969; AKI and CHOUET, 1975).

In this model, the scattered waves are mutually incoherent, so that the elastic energy density is an additive quantity. In the following, only the energy quantities are investigated.

The elastic energy density of the direct longitudinal wave E_d^p in the single isotropic scattering approximation is given by

$$\begin{aligned} E_d^p(\mathbf{r}, t|\omega) &= \frac{e^{-r/l^p}}{4\pi r^2 V^p} L_u^p\left(t - \frac{r}{V^p} \middle| \omega\right) \\ &\cong \frac{(1-r/l^p)}{4\pi r^2 V^p} L_u^p\left(t - \frac{r}{V^p} \middle| \omega\right) \end{aligned} \quad (2.1)$$

The elastic energy density of the direct transverse wave E_d^s is given by

$$\begin{aligned} E_d^s(r, t|\omega) &= \frac{e^{-r/l^s}}{4\pi r^2 V^s} L_u^s\left(t - \frac{r}{V^s} \middle| \omega\right) \\ &\cong \frac{(1-r/l^s)}{4\pi r^2 V^s} L_u^s\left(t - \frac{r}{V^s} \middle| \omega\right), \end{aligned} \quad (2.2)$$

where r is the distance from the source to the observer. We notice that E_d^p and E_d^s are functions of a scalar quantity r .

The energy density of the single scattered elastic wave E_s is given by

$$E_s = E_s^{pp} + E_s^{ss} + E_s^{sp} + E_s^{ps}, \quad (3)$$

where E_s^{pp} corresponds to the PP scattering, E_s^{ss} to the SS scattering, E_s^{sp} to the SP scattering and E_s^{ps} to the PS scattering. They are given as follows as derived in the paper I,

$$E_s^{pp}(r, t|\omega) = \frac{n^{pp}\sigma^{pp}(\omega)}{(4\pi)^2 V^p} \iiint_{-\infty}^{+\infty} \frac{1}{r_1^2 r_2^2} L_u^p\left(t - \frac{r_1}{V^p} - \frac{r_2}{V^p} \middle| \omega\right) d^3x, \quad (4.1)$$

$$E_s^{ss}(r, t|\omega) = \frac{n^{ss}\sigma^{ss}(\omega)}{(4\pi)^2 V^s} \iiint_{-\infty}^{+\infty} \frac{1}{r_1^2 r_2^2} L_u^s\left(t - \frac{r_1}{V^s} - \frac{r_2}{V^s} \middle| \omega\right) d^3x, \quad (4.2)$$

$$E_s^{sp}(r, t|\omega) = \frac{n^{sp}\sigma^{sp}(\omega)}{(4\pi)^2 V^p} \iiint_{-\infty}^{+\infty} \frac{1}{r_1^2 r_2^2} L_u^s\left(t - \frac{r_1}{V^s} - \frac{r_2}{V^p} \middle| \omega\right) d^3x, \quad (4.3)$$

$$E_s^{ps}(r, t|\omega) = \frac{n^{ps}\sigma^{ps}(\omega)}{(4\pi)^2 V^s} \iiint_{-\infty}^{+\infty} \frac{1}{r_1^2 r_2^2} L_u^p\left(t - \frac{r_1}{V^p} - \frac{r_2}{V^s} \middle| \omega\right) d^3x, \quad (4.4)$$

where x is the co-ordinate vector of the scattering point, r_1 is the distance from the source to the scattering point and r_2 is the distance from the scattering point to the observer (see Fig. 2). Here, we notice that E_s^{pp} , E_s^{ss} , E_s^{sp} and E_s^{ps} are functions of a scalar r , too. Equations (4.1–4) are rewritten in the integral form. Using Green's functions as integral kernels, they are given as follows:

$$E_s^{pp}(r, t|\omega) = n^{pp}\sigma^{pp}(\omega) \int_{-\infty}^{\infty} G_s^{pp}(r, t-t') L_u^p(t'|\omega) dt', \quad (5.1)$$

$$E_s^{ss}(r, t|\omega) = n^{ss}\sigma^{ss}(\omega) \int_{-\infty}^{\infty} G_s^{ss}(r, t-t') L_u^s(t'|\omega) dt', \quad (5.2)$$

$$E_s^{sp}(r, t|\omega) = n^{sp}\sigma^{sp}(\omega) \int_{-\infty}^{\infty} G_s^{sp}(r, t-t') L_u^s(t'|\omega) dt', \quad (5.3)$$

$$E_s^{ps}(r, t|\omega) = n^{ps}\sigma^{ps}(\omega) \int_{-\infty}^{\infty} G_s^{ps}(r, t-t') L_u^p(t'|\omega) dt'. \quad (5.4)$$

The Green's functions are defined as

$$G_s^{pp}(r, t) = \frac{1}{(4\pi)^2 V^p} \iiint_{-\infty}^{\infty} \frac{1}{r_1^2 r_2^2} \delta\left(t - \frac{r_1}{V^p} - \frac{r_2}{V^p}\right) d^3x, \quad (6.1)$$

$$G_s^{ss}(r, t) = \frac{1}{(4\pi)^2 V^s} \iiint_{-\infty}^{\infty} \frac{1}{r_1^2 r_2^2} \delta\left(t - \frac{r_1}{V^s} - \frac{r_2}{V^s}\right) d^3x, \quad (6.2)$$

$$G_s^{sp}(r, t) = \frac{1}{(4\pi)^2 V^p} \iiint_{-\infty}^{\infty} \frac{1}{r_1^2 r_2^2} \delta\left(t - \frac{r_1}{V^s} - \frac{r_2}{V^p}\right) d^3x, \quad (6.3)$$

$$G_s^{ps}(r, t) = \frac{1}{(4\pi)^2 V^s} \iiint_{-\infty}^{\infty} \frac{1}{r_1^2 r_2^2} \delta\left(t - \frac{r_1}{V^p} - \frac{r_2}{V^s}\right) d^3x. \quad (6.4)$$

Equations (6.1) and (6.2) are integrated in the paper I. Here, we define a quantity $\beta (\equiv V^p t / r)$, which is, the time normalized by the P-wave travel time at the distance r from the source. Equations (6.1) and (6.2) are integrated as

$$G_s^{pp}(r, t) = \frac{1}{4\pi r^2} K(\beta) \theta(\beta - 1), \quad (7.1)$$

$$G_s^{ss}(r, t) = \frac{1}{4\pi r^2} K\left(\frac{\beta}{\gamma}\right) \theta\left(\frac{\beta}{\gamma} - 1\right), \quad (7.2)$$

where $\gamma \equiv V^p / V^s (> 1)$ and β/γ is the time normalized by the S-wave travel time at the distance r from the source ($\beta/\gamma = V^s t / r$). The function K is defined in the paper I,

$$K(x) = \frac{1}{x} \ln \frac{x+1}{x-1} \quad \text{for } x > 1. \quad (8)$$

The step function θ is defined as

$$\theta(x) = \begin{cases} 1 & \text{for } x \geq 0, \\ 0 & \text{for } x < 0. \end{cases} \quad (9)$$

From equations (6.3) and (6.4), we obtain

$$G_s^{ps}(r, t) = \gamma G_s^{sp}(r, t). \quad (10)$$

Therefore, it is only necessary to integrate G_s^{sp} . The integral (6.3) is evaluated by transforming the cartesian co-ordinates into the prolate spheroidal co-ordinates (see the paper I, Appendix A). Let us place a source and an observer on the Z -axis $(0, 0, \mp C)$, which are the foci of the prolate spheroid as shown in Fig. 2. The co-ordinate transformation is given by

$$\begin{cases} x = C\sqrt{(\alpha^2 - 1)(1 - \xi^2)} \cos \zeta \\ y = C\sqrt{(\alpha^2 - 1)(1 - \xi^2)} \sin \zeta \\ z = C\alpha\xi, \end{cases} \quad (11.1)$$

where the domains of these new variables are given by

$$\begin{cases} 1 \leq \alpha < \infty \\ -1 \leq \xi \leq 1 \\ 0 \leq \zeta < 2\pi. \end{cases} \quad (11.2)$$

(see Fig. 3).

Fig. 2. The source and the observer are placed at the foci of the prolate spheroidal co-ordinate.

Fig. 3. The prolate spheroidal co-ordinate.

Equation (6.3) is rewritten by the new variables as

$$G_s^{sp}(2C, t) = \frac{1}{4\pi(2C)^2} \frac{2}{1+\gamma} \int_1^\infty d\alpha \int_{-1}^1 d\xi \frac{1}{\alpha^2 - \xi^2} \delta\left(\frac{2\beta - (\gamma-1)\xi}{1+\gamma} - \alpha\right), \quad (12)$$

where $\beta = V^*t/2C$.

The variable ξ is eliminated by

$$\alpha(\xi) = \frac{2\beta - (\gamma-1)\xi}{1+\gamma}. \quad (13)$$

Equation (13) represents a shell of the scatterers corresponding to β . The scatterers shells are illustrated in Fig. 4, where $\gamma = \sqrt{3}$. From Fig. 4, we see that the shells are spheroidal for the PP scattering and the SS scattering. But, these shells are oval for the SP scattering and the PS scattering.

Here, we notice that the integral domain is restricted by the condition $\alpha \geq 1$. We define ξ_a by $\alpha(\xi_a) = 1$ (see Fig. 5).

$$\xi_a = \frac{2\beta - (\gamma+1)}{\gamma-1}. \quad (14)$$

The integral domain does not exist for $\beta < 1$, and we obtain $G_s^{sp} = 0$. We can write G_s^{sp} in the form

$$G_s^{sp}(2C, t) = \frac{1}{4\pi(2C)^2} K_0(\beta) \theta(\beta-1). \quad (15)$$

Fig. 4. The scatterers shells corresponding to the PP, SS, SP and PS scattering are represented by the dotted curves, which are symmetric with respect to the Z-axis, where $\beta = V^0 t / 2C$.

Fig. 5. The α - ξ relation given by Eq. (13).

Fig. 6. The t - r diagram, where $\beta = V^0 t / r$.

The integral domain of G_s^{sp} is given by $[-1, \xi_a]$ for $1 < \beta < \gamma$ and $[-1, 1]$ for $\beta > \gamma$ (see Fig. 6). From Eqs. (12), (13), (14) and (15), the integral $K_c(\beta)$ is written as

$$K_c(\beta) = \begin{cases} \frac{\gamma+1}{2} \int_{-1}^{\xi_a} \frac{d\xi}{(\beta - \gamma\xi)(\beta + \xi)} & \text{for } 1 < \beta < \gamma, \\ \frac{\gamma+1}{2} \int_{-1}^1 \frac{d\xi}{(\beta - \gamma\xi)(\beta + \xi)} & \text{for } \beta > \gamma. \end{cases} \quad (16.1)$$

$$(16.2)$$

The integral K_e is evaluated, and we obtain

$$K_e(\beta) = \begin{cases} \frac{1}{2\beta} \ln \frac{\gamma + \beta}{\gamma - \beta} & \text{for } 1 < \beta < \gamma, \\ \frac{1}{2\beta} \ln \frac{(\beta + 1)(\beta + \gamma)}{(\beta - 1)(\beta - \gamma)} & \text{for } \beta > \gamma. \end{cases} \quad (17.1)$$

$$K_e(\beta) = \begin{cases} \frac{1}{2\beta} \ln \frac{(\beta + 1)(\beta + \gamma)}{(\beta - 1)(\beta - \gamma)} & \text{for } \beta > \gamma. \end{cases} \quad (17.2)$$

Here, we summarize the solutions obtained.

$$G_s^{sp}(r, t) = \frac{1}{4\pi r^2} K_e(\beta) \theta(\beta - 1), \quad (18.1)$$

$$G_s^{ps}(r, t) = \frac{\gamma}{4\pi r^2} K_e(\beta) \theta(\beta - 1), \quad (18.2)$$

where $\beta = V^p t / r$.

4. Properties of the Green's Functions

In the preceding section, we have obtained the Green's functions of the single isotropic scattering model including wave conversions.

The properties of these Green's functions are given below. The geometrical factor $4\pi r^2$ is common to all of them. Therefore, it is easy for our understanding to describe $4\pi r^2 G_s$'s. The function $4\pi r^2 G_s^{pp}$ and $4\pi r^2 G_s^{ss}$ are

Fig. 7. The solid curves represent $4\pi r^2 G_s^{pp}(r, t)$ and $4\pi r^2 G_s^{ss}(r, t)$, where $\beta = V^p t / r$. And also $K(\beta)$ is illustrated.

Fig. 8. The solid curves represent $4\pi r^2 G_s^{ps}(r, t)$ and $4\pi r^2 G_s^{sp}(r, t)$, where $\beta = V^p t / r$. And also $K_e(\beta)$ is illustrated.

shown in Fig. 7 and $4\pi r^2 G_s^{sp}$ and $4\pi r^2 G_s^{ps}$ are shown in Fig. 8, when $\beta=1$ and $\beta=\gamma$ correspond to the direct P-wave and the direct S-wave ($\gamma=\sqrt{3}$).

The function $K(\beta)$ and $K_e(\beta)$ are investigated. As shown in the paper I, we have

$$K(\beta) \rightarrow \log. \text{ div. as } \beta \rightarrow 1_+, \quad (19.1)$$

$$K(\beta) \cong \frac{2}{\beta^2} \quad \text{for } \beta \gg 1. \quad (19.2)$$

In the same manner, we obtain

$$K_e(1) = \frac{1}{2} \ln \frac{\gamma+1}{\gamma-1}, \quad (20.1)$$

$$K_e(\beta) \rightarrow \log. \text{ div. as } \beta \rightarrow \gamma_{\pm}, \quad (20.2)$$

$$K_e(\beta) \cong \frac{1+\gamma}{\beta^2} \quad \text{for } \beta \gg \gamma. \quad (20.3)$$

We show $K(\beta)$ and $K_e(\beta)$ in Fig. 7 and Fig. 8, where $\gamma=\sqrt{3}$. As shown by Eqs. (19.1) and (20.2), $K(1)$ and $K_e(\gamma)$ diverge logarithmically. These divergent quantities correspond to the energy density. The integration of the energy density over some region of the space is finite; the integral all over the space is finite as shown in the Appendix. But the scatterers shells appear at the foci of the prolate spheroid at $\beta=\gamma$ for the SP and PS conversion scatterings; and the shells converge to the foci at $\beta=1$ for the PP scattering and at $\beta=\gamma$ for the SS scattering. They are shown in Fig. 4. These situations correspond to $r_1=0$ and $r_2=0$, which contradict assumption C(6). Therefore, our model is not valid at $\beta \cong 1$ and $\beta \cong \gamma$.

5. Examples

It is well known that the energy density of the direct waves has peaks at $\beta=1$ (for P-wave) and $\beta=\gamma$ (for S-wave). The energy density of the direct waves is determined by the total radiated energies W_0^p and W_0^s and the source duration u .

In the following, the behaviour of the energy density of the single scattered waves is discussed. This model contains well known constants V^p , V^s and unfamiliar parameters $n^{pp}\sigma^{pp}$, $n^{ss}\sigma^{ss}$, $n^{sp}\sigma^{sp}$ and $n^{ps}\sigma^{ps}$. The energy densities of the single scattered waves for several sets of appropriate parameters are investigated, when u is sufficiently small. L_u^p and L_u^s are written as

$$L_u^p(t|\omega) = W_0^p(\omega) \delta(t), \quad (21.1)$$

$$L_u^s(t|\omega) = W_0^s(\omega) \delta(t). \quad (21.2)$$

We obtain

$$E_s(r, t|\omega) = [n^{pp}\sigma^{pp}(\omega)G_s^{pp}(r, t) + n^{ps}\sigma^{ps}(\omega)G_s^{ps}(r, t)]W_0^p(\omega) \\ + [n^{ss}\sigma^{ss}(\omega)G_s^{ss}(r, t) + n^{sp}\sigma^{sp}(\omega)G_s^{sp}(r, t)]W_0^s(\omega). \quad (22)$$

Here, we define the dimensionless quantity $\varphi(\beta)$ as

$$\varphi\left(\frac{V^p t}{r}\right) \equiv \frac{4\pi r^2 E_s(r, t|\omega)}{\left[\frac{W_0^p}{l^p(\omega)} + \frac{W_0^s}{l^s(\omega)}\right]}. \quad (23)$$

The dimensionless quantity $\varphi(\beta)$ is illustrated instead of E_s itself.

In Fig. 9, $\varphi(\beta)$ is shown in the case of $n^{ps}\sigma^{ps} = n^{sp}\sigma^{sp} = 0$ and $n^{pp}\sigma^{pp}W_0^p = n^{ss}\sigma^{ss}W_0^s$, when the PS and SP conversion scatterings do not happen.

It is very interesting to investigate the case of $n^{ps}\sigma^{ps} \neq 0$ and $n^{sp}\sigma^{sp} \neq 0$. In

Fig. 9. The solid curve represents $\varphi(\beta)$ in the case of $n^{ps}\sigma^{ps}W_0^p = n^{sp}\sigma^{sp}W_0^s = 0$ and $n^{pp}\sigma^{pp}W_0^p = n^{ss}\sigma^{ss}W_0^s$, where $\beta = V^p t/r$.

Fig. 10. The solid curve represents $\varphi(\beta)$ in the case of $n^{pp}\sigma^{pp}W_0^p = n^{ss}\sigma^{ss}W_0^s = n^{sp}\sigma^{sp}W_0^s = n^{ps}\sigma^{ps}W_0^p$, where $\beta = V^p t/r$.

Fig. 11. The solid curve represents $\varphi(\beta)$ in the case of $n^{pp}\sigma^{pp}W_0^p : n^{ss}\sigma^{ss}W_0^s : n^{sp}\sigma^{sp}W_0^s : n^{ps}\sigma^{ps}W_0^p = 4 : 4 : 1 : 1$, where $\beta = V^p t/r$.

Fig. 10, $\varphi(\beta)$ is shown in the case of $n^{pp}\sigma^{pp}W_0^p = n^{ss}\sigma^{ss}W_0^s = n^{sp}\sigma^{sp}W_0^s = n^{ps}\sigma^{ps}W_0^p$. And in Fig. 11, $\varphi(\beta)$ is shown in the case of $n^{pp}\sigma^{pp}W_0^p : n^{ss}\sigma^{ss}W_0^s : n^{sp}\sigma^{sp}W_0^s : n^{ps}\sigma^{ps}W_0^p = 4 : 4 : 1 : 1$.

It should be noted that the function $\varphi(\beta)$ traces a concave curve in $1 < \beta < \gamma$ in Figs. 10 and 11, and it decreases monotonously for $\beta > \gamma$ in every case.

Although we have investigated only three cases as illustrated in Figs. 9, 10 and 11, it seems that the temporal development of the energy density of the scattered short period body waves of a small local earthquake near a hypocenter may be explained qualitatively by these behaviours of $\varphi(\beta)$ when appropriate parameters ($n\sigma$'s) are chosen.

6. Summary

The obtained results are summarized here.

The elastic energy propagation in a three dimensional elastic medium with heterogeneities distributed homogeneously and randomly is studied from the point of view of the scattering theory. The elastic medium is characterized by the wave propagation velocities V^p and V^s and the heterogeneities are interpreted as the scatterers. The isotropic scattering is assumed and the scattering process is considered up to the first order. And we assume not only the PP and SS scatterings but also the SP and PS conversion scatterings. Then, we obtain the space-time distribution of the energy density of the single scattered elastic waves when the elastic energy is radiated spherically from a source in a short duration. The Green's functions corresponding to these single isotropic scatterings are obtained and their properties are exhibited. If appropriate parameters ($n\sigma$'s) are chosen, the temporal development of the energy density of the scattered short period body waves of a small local earthquake near a hypocenter may be explained qualitatively by the single isotropic scattering model.

The quantitative determination of the parameters $n^{pp}\sigma^{pp}$, $n^{ss}\sigma^{ss}$, $n^{sp}\sigma^{sp}$ and $n^{ps}\sigma^{ps}$ from the observation remains.

APPENDIX

Energy conservation law

In the single isotropic scattering model, the total energy of elastic waves within a unit angular frequency band around ω , $W(t|\omega)$ (erg), is given by

$$W(t|\omega) = W^p(t|\omega) + W^s(t|\omega), \quad (\text{A.1})$$

where W^p and W^s are given by

$$W^p(t|\omega) = \int_0^\infty [E_d^p + E_s^{pp} + E_s^{ps}] 4\pi r^2 dr, \quad (\text{A.2.1})$$

$$W^s(t|\omega) = \int_0^\infty [E_d^s + E_s^{ss} + E_s^{sp}] 4\pi r^2 dr. \quad (\text{A.2.2})$$

Now, we integrate these equations.

$$\int_0^\infty E_d^p(r, t|\omega) 4\pi r^2 dr = \int_0^\infty \left[1 - \frac{V^p}{l^p}(t-t') \right] L_u^p(t'|\omega) dt'. \quad (\text{A.3})$$

The integration of E_s^{pp} is given by (see the paper I)

$$\int_0^\infty E_s^{pp}(r, t|\omega) 4\pi r^2 dr = n^{pp} \sigma^{pp} V^p \int_0^\infty (t-t') L_u^p(t'|\omega) dt'. \quad (\text{A.4})$$

The integration of E_s^{ps} is tedious but simple. We obtain

$$\int_0^\infty E_s^{ps}(r, t|\omega) 4\pi r^2 dr = n^{ps} \sigma^{ps} V^p \int_0^\infty (t-t') L_u^p(t'|\omega) dt'. \quad (\text{A.5})$$

The following equation is obtained.

$$W^p(t|\omega) = \int_0^\infty L_u^p(t'|\omega) dt' = \int_0^u L_u^p(t'|\omega) dt' = W_0^p(\omega) \quad (= \text{const.}). \quad (\text{A.6.1})$$

The integral $W^s(t|\omega)$ is evaluated in the same manner.

$$W^s(t|\omega) = \int_0^u L_u^s(t'|\omega) dt' = W_0^s(\omega) \quad (= \text{const.}). \quad (\text{A.6.2})$$

Thus, $W^p(t|\omega)$ and $W^s(t|\omega)$ are constants, which are independent of time, that is,

$$W(t|\omega) = W_0^p(\omega) + W_0^s(\omega) \quad (= \text{const.}). \quad (\text{A.7})$$

The radiated energy is then conserved in the single isotropic scattering model including the wave conversions.

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