INVERSION OF GEODETIC DATA. PART II. OPTIMAL MODEL OF CONJUGATE FAULT SYSTEM FOR THE 1927 TANGO EARTHQUAKE

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Permanent changes in displacement due to the 1927 Tango earthquake ($M=7.5$) are analyzed by using the method of "geodetic data inversion" developed in a previous paper (Part I; MATSU'URA, 1977), and a set of optimal source parameters is obtained for a system of conjugate faults, the Gomura and the Yamada.

Deformation fields are modelled by a multiple-fault system composed of several dimensional faults in an elastic half-space, and the optimization of model parameters is accomplished through a process of successive iteration of inversions for 85 triangulation and 25 levelling data around the seismic faults.

The optimal fault parameters obtained are D (dislocation)=3.7 m, δ (dip-angle)= 64° E, λ (slip direction)= -8° , L (length)=33 km, W (width)=19 km, d (depth)=0.4 km and θ (strike of fault)=N 24° W for the Gomura fault, and $D=1.8$ m, $\delta=58^\circ$ N, $\lambda=120^\circ$, $L=15$ km, $W=9$ km, $d=0.2$ km and $\theta=N77^\circ$ E for the Yamada fault, where the slip direction is measured counterclockwise from horizon on the fault plane. These results indicate that the two seismic faults constitute a system of conjugate faults caused by a single stress system of NW-SE compression.

A computer program for the geodetic data inversion is completed, by which a set of optimal source parameters of seismic and aseismic events can be obtained from the observed static displacement data.

1. Introduction

This is the second paper of a series of papers dealing with the "geodetic data inversion" for source parameters of seismic and aseismic events. A practical method of finding the optimal fault-plane solutions and evaluating the degree of their reliabilities from a set of inaccurate geodetic data has been developed in Part I (MATSU'URA, 1977) of this series. In the present paper, the method of geodetic data inversion is applied to the observed static displacement data associated with the 1927 Tango earthquake.

The Tango earthquake ($M=7.5$) occurred in the Japan Sea side of Honshu

island on March 7, 1927. Detailed investigations have been made on the deformation of the earth's crust by Tsuboi (1930a, b, 1931, 1932), and on the aftershock distributions by Nasu (1929) caused by this earthquake. At the time of the earthquake, two remarkable seismic faults, the Gomura and the Yamada, appeared in the central area of the Tango district.

For the Gomura fault, several fault-plane studies have been made to interpret the horizontal displacement data near the northwest coastline of the Tango peninsula by using highly idealized models (Kasahara, 1957, 1958; Chinnery, 1961; Walsh, 1969). These authors pointed out that the Gomura fault is a predominantly left-lateral slip on a nearly vertical (steeply dipping westward) plane with a width of about 15 km. On the other hand, the Yamada fault has received less attention except for the investigation by Walsh (1969). From levelling data, he estimated that the faulting is a reverse type on a plane dipping northward with an angle of about 40° . This agrees with a result based on the study of aftershocks by Nasu (1929), but differs from that based on the field observations, such as a normal faulting with a dip-angle of 85° S, by Tsuboi (1930a).

All of these fault-plane solutions were obtained by searching for a set of the best fitting parameters to the observations in some least squares sense, under the assumption that the data are contaminated by some kind of systematic errors, e.g. the effect of the rotation of the base line by the accumulation of tectonic strain or the anelastic deformation of the northeast side block. When estimating the source parameters of a seismic event from geodetic data, it is not avoidable that the resulting solutions are involved in some degrees of non-uniqueness, because the data are finite in number and contain random errors, and more fundamentally, because there exist some deficiencies in the mathematical model concerned. Therefore, it is needed to evaluate the degree of reliability of the result as well as to find the optimal solution in a reasonable and systematic fashion.

Recently, several attempts to apply the inversion technique to the analysis of geodetic data have been made to estimate the source parameters of seismic events. For example, Alewine and Jordan (1973) analyzed the vertical displacements due to the occurrence of the 1971 San Fernando earthquake by taking only the amount of the slip along a complex fault system as a variable parameter. For a point dislocation source model, Jovanovich (1975) has developed an inversion method to estimate the source parameters of seismic events from static strain data, and applied it to the cases of the 1971 San Fernando and the 1969 Gifu earthquake.

However, for the analysis of geodetic data in the near-field, it is necessary that the deformation fields are modelled by a dimensional fault with various variables, such as the magnitude and direction of the slip, the length,

width, depth and location of the fault, the dip-angle and strike direction. In the present study, the method of geodetic data inversion is applied to the case of the 1927 Tango earthquake, and the nature of the conjugate faults, the Gomura and the Yamada, is discussed in detail.

2. *Observations and Initial Model*

Concerning the 1927 Tango earthquake, a large amount of measurements of permanent changes in displacement around two remarkable seismic faults, the Gomura and the Yamada, has been reported by Tsuboi (1930a, b, 1931, 1932). Among them, the data used in the present analysis are summarized in Fig. 1.

The Gomura fault consists of several parallel cracks which extend out into the Sea of Japan. The western block of the fault moved southward about 2.5 m and was elevated 0.7 m relative to the eastern block. The motion is predominantly left-lateral. On the other hand, the Yamada fault seems to stretch discontinuously from the base of the Tango peninsula to the northeast. The amount of the slip along this fault is somewhat smaller than that along the Gomura fault, that is, the southern side of the fault subsided by about 1.0 m and moved 0.7 m westward relative to the northern side. The horizontal motion is right-lateral.

Horizontal displacement data (1927—1884~1889) were obtained for over 270 triangulation points (6 first order, 41 second order and 226 third order triangulation points) in a series of post-seismic surveys. The gross feature of the observed horizontal displacement field suggests that the Gomura fault is a nearly vertical, predominantly left-lateral slipping which extends over a distance of 30 km in a direction of N30°W. On the other hand, little information on the nature of the Yamada fault is obtained from the pattern of the horizontal displacement field alone, except that the slip has a right-lateral component.

Relative changes in the vertical displacement (1927—1888) were measured at about 50 bench marks along the levelling route which extends to a region more than 30 km from the epicenter where the changes in the displacement gradient were slight. The results of levelling indicate that the Yamada fault is not a normal, but a reverse type of faulting with a comparatively large dip-slip component of about 1.0 m on a plane dipping somewhat loosely to the north.

The distribution of aftershocks also gives us useful information on the extension of the main faulting. The aftershocks associated with the Tango earthquake are roughly divided into two groups which correspond to two remarkable seismic faults, the Gomura and the Yamada, respectively. On a

Fig. 1. Static deformations due to the 1927 Tango earthquake. Only 85 triangulation and 25 levelling data are shown, which are used in the present analysis. Each station number corresponds to that in Table 2.

Fig. 2. Distribution of aftershocks for March 12, 1927–July 31, 1928 (reproduced from NASU (1929) after some abbreviation). (a) Projection of seismic foci on the earth's surface. (b) Projection of seismic foci on the vertical plane normal to the Gomura fault. (c) Projection of seismic foci on the vertical plane normal to the Yamada fault.

reasonable assumption that the region accompanied with the occurrence of aftershocks almost coincides with the extension of the main faulting, we are able to expect the gross aspects of two main faultings from the aftershock distributions as follows:

The Gomura fault seems to extend out into the Sea of Japan over a length of 30 km in the direction of $N30^{\circ}W$ (Fig. 2(a)). For the Yamada fault, the area of aftershocks spreads from the base of the Tango peninsula to the west (Fig. 2(a)). Projections of the aftershocks on the vertical plane normal to each fault suggest that the Gomura fault is nearly vertical (Fig. 2(b)), and the dip-angle of the Yamada fault is about $50^{\circ}N$ (Fig. 2(c)).

Table 1. Adopted initial model (Model I) and the fault-plane solutions estimated by the previous investigators.

Gomura fault

	D (m)	δ	λ	W (km)	d (km)	a_0	b_0	a_1	b_1
Model I	3.0	80° E	20°	20	0.3	-1.85	3.30	-0.35	0.70
TSUBOI (1930a)	2.5	64° W	20°	—	0.0	$\theta = \text{N}30^\circ \text{W}$			
NASU (1929)	—	70° W	—	$\leq 25^*$	—	$\theta = \text{N}30^\circ \text{W},^* \quad L \geq 30 \text{ km}^*$			
KASAHARA (1957)	3.0	90°	0°	15	0.0	$\theta = \text{N}30^\circ \text{W}, \quad L = 30 \text{ km}$			
CHINNERY (1961)	3.0	90°	0°	10~12	0.0	$\theta = \text{N}30^\circ \text{W},^* \quad L = 36 \text{ km}$			
WALSH (1969)	3.0	70°~75° W	—	—	0.0	$\theta = \text{N}30^\circ \text{W}^*$			

Yamada fault

	D (m)	δ	λ	W (km)	d (km)	a_0	b_0	a_1	b_1
Model I	1.5	70° N	120°	20	0.1	-0.85	-0.30	0.55	0.50
TSUBOI (1930a)	1.4	85° S	-120°	—	0.0	$\theta = \text{N}55^\circ \text{E}$			
NASU (1929)	—	40°~50° N	—	$\leq 20^*$	—	$\theta = \text{N}60^\circ \text{E}^*$			
WALSH (1969)	1.3	40° N	—	—	0.0	$\theta = \text{N}60^\circ \text{E}^*$			

Notes: D ; magnitude of dislocation. δ ; dip-angle. λ ; slip direction measured counter-clockwise from horizon on the fault plane. W ; fault width. d ; depth of upper fault edge. (a_0, b_0) and (a_1, b_1) ; location of the upper-right and -left fault corner on the X - Y coordinates with a unit of 10 km, where the origin is fixed to a point (35.53°N, 135.11°E), and the X - and Y -axis are taken parallel to the east- and north-direction respectively. L ; fault length. θ ; strike direction. * ; rough estimation.

On the basis of these investigations about the observed data, an initial model of conjugate fault system (referred to as Model I hereafter) is adopted as shown in Table 1, in which the fault-plane solutions estimated by the previous investigators are also presented for comparison with Model I. It is noted that we do not assume any kind of systematic error associated with the observed data in the selection of the initial model. Of course, the possibility of a systematic contamination of the data cannot be denied, but we do not have any kind of reliable information for the data to remove such contamination (MATSUZAWA, 1964). Therefore, it is reasonable to assume that the errors associated with data, such as observational errors, some systematic contaminations caused by the occurrence of the events beyond our concern and the deficiencies in the adopted mathematical model, are non-singular, random and finite.

In the present study, only 85 horizontal and 25 vertical displacement data shown in Fig. 1 are used, because of the occurrence of a faulting due to the Tazima earthquake (35.7°N, 134.8°E; $M=7.0$; May 23, 1925) in the western part close to the area under consideration, and more practically, because of a restriction on the memory capacity of the computer. Each observed datum

Table 2.1. Horizontal displacements of triangulation points in the Tango district (1927-1884~1889).

No.	Station	Displacement (m)		Position ($H=10$ km)	
		U_X	U_Y	X/H	Y/H
1	Setoyama	-0.45	0.43	1.030	0.115
2	Hamagasira	-0.33	0.25	1.425	0.020
3	Yosidu	-0.55	0.43	0.530	0.130
4	Dogatani	-0.50	0.46	1.190	0.410
5	Kurosaki	-0.56	0.49	1.285	0.680
6	Minamitani	-0.58	0.52	0.555	0.540
7	Nisitani	-0.50	0.56	0.840	0.760
8	Yasuke	-0.60	0.43	1.070	1.030
9	Sengokuyama	-0.58	0.37	1.220	1.360
10	Tsudumigatake	-0.52	0.52	0.635	0.850
11	Tsunoduki	-0.50	0.31	0.945	1.485
12	Moriage	-0.50	0.43	1.070	1.750
13	Miemura	0.12	0.92	0.150	0.275
14	Mititani	-0.48	0.68	0.320	0.600
15	Yanaganaru	-0.60	0.56	0.600	1.045
16	Dake	-0.60	0.46	0.705	1.320
17	Inunegasira	-0.63	0.43	0.685	1.555
18	Taikoyama	-0.47	0.34	0.840	1.835
19	Kasayama	-0.48	0.40	0.800	2.140
20	Takao	-0.73	0.65	0.365	1.130
21	Kongosan	-0.60	0.68	0.280	1.435
22	Gamatani	-0.75	0.49	0.400	1.570
23	Kubo	-0.68	0.43	0.550	1.960
24	Nimatu	-0.55	0.43	0.610	2.265
25	Kidumiyama	-0.55	0.80	0.070	0.760
26	Mitinoku	-0.96	0.80	0.020	1.865
27	Ooisi	-0.91	0.59	0.270	1.970
28	Itigao	-0.87	0.63	0.245	2.260
29	He	-0.75	0.62	0.410	2.405
30	Misakayama	-0.43	0.89	-0.020	0.420
31	Yosizawa	-0.75	0.80	-0.045	1.130
32	Naya	-0.88	0.83	-0.120	1.510
33	Ootani	-0.98	0.92	-0.265	1.810
34	Oohira	-0.96	0.71	-0.165	2.175
35	Takase	-0.76	0.96	-0.200	0.800
36	Yada	-0.93	1.08	-0.325	1.310
37	Mineyama	-0.91	1.23	-0.420	0.965
38	Akasaka	-0.88	0.83	-0.570	1.125
39	Asago	-1.26	1.05	-0.570	1.490
40	Obama	-0.55	1.17	-0.675	1.885
41	Kaitanidake	0.05	0.06	-0.155	0.205
42	Ainome	0.03	-0.99	-0.660	0.820
43	Nisiyama	-0.10	-1.36	-0.805	1.030
44	Arai	-0.03	-1.23	-0.930	1.240

Table 2.1. (continued)

No.	Station	Displacement (m)		Position ($H=10$ km)	
		U_X	U_Y	X/H	Y/H
45	Simooka	0.53	-1.13	-0.945	1.600
46	Tukinami	0.03	-1.20	-1.175	1.520
47	Byobuyama	-0.10	-0.56	-0.420	0.495
48	Kidu	-0.10	-1.05	-1.160	1.125
49	Mizono	-0.08	-0.86	-1.405	1.185
50	Hiziyama	0.00	-0.89	-0.935	0.680
51	Nyoo	-0.07	-0.81	-1.230	0.845
52	Yakedosi	0.08	-0.71	-0.790	0.440
53	Sano	0.13	-0.68	-1.075	0.555
54	Nonaka	0.10	-0.56	-1.400	0.610
55	Isanagosan	0.02	-0.48	-0.655	0.200
56	Kaya	0.05	-0.52	-1.000	0.305
57	Hutamata	0.00	-0.43	-1.160	0.240
58	Itinono	0.18	-0.43	-1.420	0.225
59	Nakahudi	-0.23	-0.34	-0.895	-0.040
60	Simomura	0.02	-0.18	-1.260	-0.080
61	Oogatani	-0.40	0.25	-0.270	-0.090
62	Iwaya	-0.18	-0.19	-0.605	-0.145
63	Nanmeizan	0.00	-0.19	-1.155	-0.270
64	Okuhudi	-0.15	-0.15	-0.780	-0.375
65	Kayaoku	-0.15	-0.01	-0.550	-0.430
66	Akahanamura	-0.08	0.00	-0.960	-0.615
67	Arikuma	-0.25	0.19	-0.300	-0.460
68	Egasayama	-0.10	0.09	-0.545	-0.885
69	Dannosaka	-0.20	0.00	-0.300	-0.760
70	Akaisidake	-0.13	0.25	-0.225	-0.950
71	Kanase	-0.20	0.28	-0.060	-0.470
72	Sendyogatake	-0.18	0.25	-0.035	-0.855
73	Ooeyama	-0.25	0.34	0.125	-0.675
74	Tyuda	-0.25	0.31	0.245	-0.470
75	Sorayama	-0.08	0.43	0.440	-0.855
76	Toisigatake	-0.13	0.37	0.750	-0.910
77	Hukotoge	-0.20	0.43	0.525	-0.575
78	Isikawamura	-0.41	0.43	0.380	-0.225
79	Odamura	-0.28	0.46	0.610	-0.365
80	Seihozi	-0.30	0.31	0.870	-0.730
81	Dinusi	-0.35	0.28	1.085	-0.580
82	Okuyama	-0.40	0.37	0.910	-0.195
83	Yuragatake	-0.39	0.30	1.320	-0.225
84	Kanawaridani	-0.50	0.46	0.505	-0.060
85	Waki	-0.43	0.31	1.230	-0.085

Origin of the X - Y coordinates; Oidi (35.53°N, 135.11°E).

No.; corresponds to Fig. 1.

Table 2.2. Vertical displacements of bench marks along E-W levelling route from Kumihama to Maizuru (1927–1888).

No. of bench mark	Displacement (m)	Position ($H=10$ km)	
		X/H	Y/H
1222	0.01	−1.310	0.570
1223	0.04	−1.130	0.495
1224	0.09	−1.000	0.510
1225	0.12	−0.905	0.520
1226	0.19	−0.710	0.565
1227	0.23	−0.615	0.720
1228	0.40	−0.540	0.875
1229	−0.35	−0.475	1.045
1230	−0.16	−0.415	0.905
1231	0.01	−0.250	0.735
1232	0.09	−0.170	0.590
1233	0.35	−0.125	0.365
1234	0.56	0.015	0.300
1235	0.88	0.005	0.110
1236	−0.26	0.140	0.155
1237	−0.19	0.355	0.240
1238	−0.12	0.485	0.300
1239	−0.04	0.630	0.325
1240	−0.04	0.720	0.160
1241	−0.06	0.740	0.010
1376	−0.07	0.890	0.130
1375	−0.02	1.000	0.205
1374	−0.03	1.180	0.085
1373	−0.01	1.315	0.045
1372	−0.03	1.470	−0.065

Origin of the X - Y coordinates; Oidi (35.53°N , 135.11°E).
No.; corresponds to Fig. 1.

which corresponds to the number in Fig. 1 is listed in Table 2.1 (horizontal displacement) or 2.2 (vertical displacement). The origin of the coordinates is fixed to the triangulation point, Oidi (35.53°N , 135.11°E), located in the center of the Tango district. The X - and Y -axis are taken parallel to the east- and north-direction respectively.

3. Optimizing Process and Resulting Solutions

In the previous paper (Part I), the method of geodetic data inversion has been formulated mathematically. Following the definitions in Part I, we reproduce three quantities, “resolution” of the model parameter, “expected error” of the estimated value and “effective data number” in the estimation, as the measures of the reliability of the resulting solution.

Consider a $n \times m$ linear system which is already standardized by "predicted errors" of the data and "maximum allowable deviations" of the model parameters,

$$\Delta y_i = \sum_{j=1}^m A_{ij} \Delta x_j \quad (i=1, \dots, n), \quad (1)$$

where A_{ij} is the ij element of a known coefficient matrix A , and Δy_i and Δx_j are the correction term of the data and that of an unknown model parameter respectively. Defining the matrix elements, U_{ik} , V_{jk} and λ_k , by the following equations (decomposition theorem for an arbitrary $n \times m$ matrix; refer to section 2.3 in Part I),

$$A_{ij} = \sum_{k=1}^q U_{ik} V_{jk} / \lambda_k \quad (\lambda_k: \text{positive eigenvalue}), \quad (2)$$

the resolution r_j , expected error s_j and effective data number N_j are written as

$$r_j^2 = \sum_{k=1}^m \left(\sum_{l=1}^q V_{jl} V_{kl} - \delta_{jk} \right)^2 \quad (j=1, \dots, m), \quad (3.1)$$

$$s_j^2 = \sum_{k=1}^q (V_{jk} / \lambda_k)^2 \quad (j=1, \dots, m), \quad (3.2)$$

$$N_j = (1 - r_j) \sum_{i=1}^n e_{ji} / e_{j\max} \quad (j=1, \dots, m), \quad (3.3)$$

with

$$e_{ji} = \left| \sum_{k=1}^q V_{jk} U_{ik} / \lambda_k \right| \quad (j=1, \dots, m, i=1, \dots, n), \quad (4)$$

where δ_{jk} , $e_{j\max}$ and q are the Kronecker's symbol, maximum value of e_{ji} ($i=1, \dots, n$) and "effective number" of the degree of freedom in the data respectively. It is noted that the effective data number N_j is an indicator of how many data contribute effectively to determining the j -th model parameter.

To interpret the observed deformation fields, we take a multiple-fault system composed of several dimensional faults in an elastic half-space, e.g.; two-fault model in 3.1, which consists of two independent rectangular faults, and six-fault model in 3.2, where the model consists of two complex-faults, and each of them is composed of three rectangular faults jointed to each other at the fault corners. In either case, the predicted errors of data σ_i are defined as

$$\sigma_i = \sigma_0 |o_{\max}| (1 + |o_i / o_{\max}|) / 2 \quad (i=1, \dots, n), \quad (5)$$

where σ_0 is called the "error coefficient", and $o_{\max}$ is the maximum displacement of the observed data o_i ($i=1, \dots, n$). Formally, the error coefficient is guessed both from the effect of deficiencies in the model and from the accuracy of observed data. However, as discussed in section 2.4 of Part I, in practice the appropriate value of σ_0 should be determined through the analysis of real data, because we have no reliable information about the effect of deficiencies in the model.

3.1 Two-fault model

The two-fault model (Fig. 3) is uniquely prescribed by 18 independent model parameters (2×9 fault parameters). As the fault parameters, we take the following quantities; the dip-angle (δ), slip direction (λ), magnitude of dislocation (D), width (W), depth of upper fault edge (d) and the location of the upper-left and -right corners of the fault plane ($a_0, b_0; a_1, b_1$), where the slip direction is measured counterclockwise from horizon on the fault plane. As to "presupposed conditions" for analysis, the initial fault model is already given in Table 1, and others, i.e. the error coefficient and the maximum allowable deviations, are presented in Table 3, where the capital letters, G and Y, denote the Gomura and the Yamada fault respectively. It is noted that the maximum allowable deviations are determined apriori only from our demand for certainty in the model, but in the present analyzing method the resulting solutions are not significantly affected by some variations in these values.

Fig. 3. Geometry of two-fault model. The rectangles indicate the projection of the starting fault model on the X - Y plane, where the unit of length (H) is 10 km.

Table 3. Presupposed conditions for analysis (Two-fault model).

		δ	λ	D (m)	W (km)	d (km)	a_0	b_0	a_1	b_1
t_j	G	5.0°	5.0°	0.20	3.0	0.10	0.20	0.20	0.05	0.05
	Y	5.0°	5.0°	0.20	3.0	0.10	0.10	0.10	0.10	0.10
Error coefficient; $\sigma_0=0.15$										

t_j ; maximum allowable deviations.

Through a successive iteration of inversions, the model parameters are simultaneously optimized step by step as shown in Fig. 4. At each step of the iteration, the resolution r_j and expected error s_j in the estimation are indicated by the width and the length of the rectangle respectively. If r_j is small enough, information from the data is distinctly reflected in the estimation of the j -th parameter. Conversely, if r_j is nearly equal to 1, we can only obtain a meaningless conclusion that the j -th parameter is indefinite under such presupposed conditions. When the estimate is accompanied by large values both in the resolution and expected error, it can be said that the observed data contain little information on its parameter.

The effective data number N_j indicates the number of data which contribute effectively to determining the j -th model parameter. If N_j is large enough, the estimate is reliable. When the effective data number is not so large, the estimate is not reliable, because the resulting solution may be remarkably changed by adding a new effective datum to the previous data set. In the present case (two-fault model), it is noticed from Fig. 4 that the estimation with a high degree of reliability (large N_j) is always accompanied with a good resolution (small r_j). Such a relation between N_j and r_j holds, if and only if the system is strongly over-constrained (refer to 3.2).

The optimal fault parameters obtained from the analysis of triangulation data are listed in Table 4. For the Gomura fault, the slip motion is predominantly left-lateral ($\lambda = -8^\circ$) along a plane dipping eastward with an angle of 64° . This fault extends out into the Sea of Japan in the direction of $N26^\circ W$, but its location at the northwestern end is indefinite, because of the lack of data. The average dislocation is 3.7 m on the fault plane with dimensions of about 33 km (length) $\times$ 19 km (width). For the Yamada fault, the dip-angle and the slip direction are estimated to be $58^\circ N$ and 120° respectively, that is, the slip motion is a right-lateral reverse type. This result almost agrees with that by WALSH (1969). Contrary to the expectation from visible cracks, this fault stretches from the base of the Tango peninsula to the west ($N103^\circ W$) with dimensions of 15 km $\times$ 9 km. The average dislocation is about 1.8 m. The principal axes of compression calculated respectively by these two fault models almost agree (MATSU'URA *et al.*, 1976).

From these results, we can obtain some conclusions as follows: i) the extension of two main faultings almost coincides with the region accompanied

Table 4. Optimal solution (Two-fault model).

		δ	λ	D (m)	W (km)	d (km)	a_0	b_0	a_1	b_1
$\hat{x}_j$	G	$64^\circ E$	-8°	3.70	19.0	0.40	-1.82	3.71	-0.43	0.74
	Y	$58^\circ N$	120°	1.80	9.0	0.20	-0.94	-0.21	0.50	0.16

$\hat{x}_j$; optimal model parameters.

Fig. 5. Horizontal and vertical displacements calculated by Model II (optimal two-fault model). The residuals are given by the error vectors for triangulation data and the error bars for levelling data respectively.

with the occurrence of aftershocks, and ii) these seismic faults constitute a system of conjugate faults caused by a single stress system of NW-SE compression.

Theoretical displacements calculated by the optimal two-fault model (referred to as Model II) are shown in Fig. 5 with the residual errors. The Model II fits well to the observed triangulation data, except to those at the southwestern area of the Gomura fault, where the deformation fields seem to be disturbed by the effect of faulting due to the Tazima earthquake in 1925. The goodness of fit of the optimal model to the observations is represented

by a weighted mean error, which is defined by $\sigma_e = \sqrt{\frac{1}{n} \sum_{i=1}^n (o_i - \hat{y}_i)^2 / \sigma_i^2}$, where $\hat{y}_i$ ($i=1, \dots, n$) are the theoretical data. For the Model II, the value of σ_e is 0.7 and satisfies the inequality for the appropriateness in the selection of σ_e , $1/2 < \sigma_e < 1$. This shows that the presupposed error coefficient (0.15) is appropriate.

For the levelling data, which are not used in the present analysis, we cannot obtain satisfactory interpretation by the two-fault model (Fig. 5). The discrepancies between the observed and the theoretical data are remarkable at the bench marks close to the faults. This indicates that the deformation fields should be modelled by a more complex fault system to interpret both the triangulation and the levelling data, because the variation in the strike along each seismic fault cannot be represented by a simple rectangular fault (two-fault model).

3.2 Six-fault model

Producing a new data set by adding 25 levelling data (Table 2.2) to the previous set of triangulation data, more detailed analysis is carried out for the two seismic faults. Consider a six-fault model as shown in Fig. 6 to take account of the variation in the strike along the faults, where each complex-fault consists of three rectangular faults jointed to each other at the fault corners. This model is prescribed by a set of 46 independent model parameters produced from 6×9 fault parameters with 8 restrictions among them.

The initial values of model parameters are determined from the results obtained in 3.1. These values are presented in Table 5, with other presupposed conditions, where the capital letters, G_1, G_2, G_3, Y_1, Y_2 and Y_3 , denote the corresponding elemental faults (refer to Fig. 6) respectively. For the present model, the error coefficient is set up a priori as 0.10, because the degree of deficiencies in the six-fault model should be lower than that in the two-fault model. As to the maximum allowable deviations, it is not necessary to pay a great deal of attention to selecting the appropriate values, since the method of geodetic data inversion includes a process of re-standardization of the model parameters.

Fig. 6. Geometry of six-fault model. The rectangles indicate the projection of the starting fault model on the X - Y plane, where the unit of length (H) is 10 km. The capital letters, G_1 , G_2 , G_3 , Y_1 , Y_2 and Y_3 denote the corresponding elemental fault respectively.

Table 5. Presupposed conditions for analysis (Six-fault model).

	δ	λ	D (m)	W (km)	d (km)	a_0	b_0	a_1	b_1	
x_j^0	G_1	65°E	-10°	3.60	18.0	0.40	-1.90	3.70	-1.20	2.30
	G_2	65°E	-10°	3.60	18.0	0.40	-1.20	2.30	-0.80	1.50
	G_3	65°E	-10°	3.60	18.0	0.40	-0.80	1.50	-0.40	0.70
	Y_1	60°N	120°	1.80	8.0	0.20	-0.90	-0.20	-0.50	0.10
	Y_2	60°N	120°	1.80	8.0	0.20	-0.50	-0.10	-0.10	0.05
	Y_3	60°N	120°	1.80	8.0	0.20	-0.10	0.05	0.50	0.30
t_j	G_1	3.0°	3.0°	0.20	2.0	0.10	0.20	0.20	0.10	0.10
	G_2	3.0°	3.0°	0.20	2.0	0.10	0.10	0.10	0.05	0.05
	G_3	3.0°	3.0°	0.20	2.0	0.10	0.05	0.05	0.05	0.05
	Y_1	3.0°	3.0°	0.10	1.0	0.10	0.10	0.10	0.05	0.05
	Y_2	3.0°	3.0°	0.10	1.0	0.10	0.05	0.05	0.05	0.05
	Y_3	3.0°	3.0°	0.10	1.0	0.10	0.05	0.05	0.10	0.10
Error coefficient; $\sigma_0=0.10$										

x_j^0 ; initial value of model parameters.

t_j ; maximum allowable deviations.

The optimizing process of the model parameters is completed, in the present computation, at the ninth step of the successive iteration of inversions. The final estimates are given in Table 6, with the corresponding resolutions r_j , expected errors s_j and effective data numbers N_j . The system is not strongly over-constrained, unlike that in the case of the two-fault model. It should be

Table 6. Optimal solution (Six-fault model).

		$\bar{\theta}$	λ	D	W	d	a_0	b_0	a_1	b_1
$\hat{x}_j$	G_1	72° E	−3°	4.1 m	25 km	0.3 km	−1.65	4.82	−1.02	2.12
	G_2	76° E	−16°	3.1	22	0.3	−1.02	2.12	−0.88	1.64
	G_3	90° V	−14°	4.2	2	0.3	−0.88	1.64	−0.20	0.37
	Y_1	54° N	118°	1.9	6	0.1	−0.88	−0.05	−0.58	−0.13
	Y_2	51° N	122°	2.1	9	0.1	−0.58	−0.13	−0.12	0.05
	Y_3	66° N	111°	2.2	11	0.1	−0.12	0.05	0.18	0.19
s_j	G_1	1.2°	0.8°	0.07 m	0.6 km	0.05 km	0.08	0.05	0.03	0.03
	G_2	0.5°	0.6°	0.07	0.3	0.05	0.03	0.03	0.02	0.02
	G_3	2.3°	1.6°	0.05	0.2	0.05	0.02	0.02	0.01	0.02
	Y_1	0.9°	0.8°	0.03	0.4	0.05	0.02	0.03	0.04	0.01
	Y_2	1.3°	1.7°	0.04	0.6	0.05	0.04	0.01	0.02	0.01
	Y_3	1.1°	1.2°	0.02	0.3	0.05	0.02	0.01	0.03	0.02
r_j	G_1	0.82	0.95	0.75	0.90	0.37	0.26	0.87	0.32	0.92
	G_2	0.98	0.97	0.88	0.99	0.37	0.32	0.92	0.33	0.63
	G_3	0.64	0.14	0.94	0.06	0.37	0.33	0.63	0.15	0.16
	Y_1	0.95	0.96	0.98	0.88	0.20	0.06	0.09	0.42	0.08
	Y_2	0.80	0.79	0.93	0.67	0.20	0.42	0.08	0.80	0.37
	Y_3	0.87	0.90	0.97	0.95	0.20	0.80	0.37	0.15	0.08
N_j	G_1	5	2	9	4	12	30	5	9	2
	G_2	1	1	2	1	12	9	2	15	12
	G_3	2	10	2	11	12	15	12	8	9
	Y_1	10	14	6	13	10	6	1	1	1
	Y_2	22	17	1	17	10	1	1	18	18
	Y_3	5	4	2	6	10	18	18	13	12

$\hat{x}_j$; optimal model parameters. s_j ; expected errors. r_j ; resolutions.

N_j ; effective data numbers.

noted that two quantities, r_j and N_j , are the measures of different types in the physical meaning as pointed out in 3.1. In general, the fault-plane solution can be regarded as a representation of the gross aspect of the actual complex faulting. From such a point of view, resolution r_j is not taken as a reasonable measure of reliability, unless the system is strongly over-constrained.

As a whole, results of the present analysis are not so reliable except for several parameters, such as the dip-angles of Y_1 and Y_2 , the slip directions of G_3 , Y_1 and Y_2 , the widths of G_3 , Y_1 and Y_2 , etc. (Table 6). However, it is possible to point out some interesting feature of each faulting from these results. The Gomura fault seems to consist of two types of faulting, that is a normal and predominantly left-lateral slip on a plane dipping eastward with a width of about 25 km (G_1 and G_2 ; northwestern part) and left-lateral motion along a nearly vertical plane with a width of only 2 km (G_3 ; southeastern part). The strike also varies from N13°W (G_1) to N28°W (G_3) along the fault. For the

Fig. 7. Horizontal and vertical displacements calculated by Model III (optimal six-fault model). The residuals are given by the error vectors for triangulation data and the error bars for levelling data respectively.

Yamada fault, a gradual increase in width is from the western part to the eastern part ($Y_1W=6$ km, $Y_2W=9$ km, $Y_3W=11$ km). The strike of fault is $N103^\circ E$ at the western part and $N64^\circ E$ at the eastern part. The changes both in dip-angle and in slip direction are not remarkable along this faulting.

The horizontal and vertical displacements calculated by the optimal six-fault model (referred to as Model III) are shown in Fig. 7 with the residual errors. The observed deformation fields are almost completely interpreted by the Model III within the predicted errors. The weighted mean error σ_e takes a value of 0.8, and satisfies the inequality, $1/2 < \sigma_e < 1$, that is, the presupposed error coefficient (0.10) is appropriate for the six-fault model. From this result, it can be said that the total effect of various deficiencies in the mathematical model on the surface deformation fields is less than 10% for the case of a multiple-fault system composed of several rectangular faults in an elastic half-space.

4. Computer Program

On the basis of the mathematical formulation given in Part I, a computer program for the geodetic data inversion has been developed, by which a set of optimal fault parameters of seismic and aseismic events can be obtained from observed static displacement data. In the present section, a block diagram of this program is presented with several comments for actual use.

The program consists of three parts, BLOCKs I, II and III. All information, such as observed data and presupposed conditions needed for the inversion analysis, are inputted to the first part (BLOCK I). The BLOCK II is a subroutine program for producing the coefficient matrix of a linear system, in which the theoretical displacements at data points and their partial derivatives with respect to each fault parameter are calculated for the starting model at each step of the iteration. In the last block, which is a subroutine program of general linear inversion, optimization of the model parameters is carried out at each step of iteration, through a process of factorization of the coefficient matrix into the eigenvalues and eigenvectors.

The detailed explanation of each part of program is given item by item in the following part of this section, where the number for the item corresponds to that in the block diagram.

BLOCK I (READ IN INFORMATION FOR ANALYSIS)

- a-1: An arbitrary quantity, H (in km), is taken as a unit of length.
- a-2: Read in the number of triangulation points and bench marks respectively.
- a-3: Each of stations is represented as a point of the Cartesian coordinates with unit of H on the earth's surface.
- a-4: One of the displacement components at each station corresponds to one data parameter o_i ($i=1, \dots, n$).

- b-1: For a multiple-fault system composed of N elemental faults, the number of independent model parameters is given by $m=9N-r$, where r is the number of restrictions existing among fault parameters.
- b-2: Appropriate value of the error coefficient, σ_0 , is guessed apriori from the degree of deficiencies in the model.
- b-3: Selection of an adequate initial model is made by an inspection of patterns of deformation fields (MATSU'URA and SATO, 1975).
- b-4: Relationships between a fault plane and others are determined by this matrix.
- b-5: The maximum allowable deviations of model parameters, t_j ($j=1, \dots, m$), are determined apriori from our requirement for the accuracy of estimates.

BLOCK II (COEFFICIENT MATRIX)

- 1: Theoretical displacements at the data points, $\hat{y}_i$ ($i=1, \dots, n$), are calculated for the starting model, $x^0=(x_1^0, \dots, x_m^0)$, at each step of iteration.
- 2: Partial derivatives of displacements with respect to each fault parameter are calculated for the starting model at each step.

BLOCK III (GENERAL LINEAR INVERSION)

- 1: Linear system, $\Delta y_i = \sum_{j=1}^m A_{ij} \Delta x_j$, is constructed by using the constitution matrix (I-b-4), where $\Delta y_i = o_i - \hat{y}_i$, $\Delta x_j = x_j - x_j^0$ and A_{ij} is the ij element of the coefficient matrix A .
- 2: Standardization of the linear system by t_j ($j=1, \dots, m$) and σ_i ($i=1, \dots, n$), where σ_i is the predicted error defined by $\sigma_0 |o_{\max}| (1 + |o_i/o_{\max}|)/2$.
- 3: Matrix A is decomposed into the product of three matrices, i.e. $A = UAV^T$, where U and V are semi-orthonormal matrices constituted from eigenvectors, and A is a diagonal matrix with nonzero elements of positive eigenvalues λ_k ($k=1, \dots, p$).
- 4: The maximum integer which satisfies the following conditions: $\sigma(\Delta \hat{x}_j) \leq 1$ ($j=1, \dots, m$), is taken as the effective number of the degree of freedom in the data, q , where $\sigma^2(\Delta \hat{x}_j) = \sum_{k=1}^q (V_{jk}/\lambda_k)^2$.
- 5: Linear system is re-standardized by a new weighting matrix $W^{-1/2}$, where W is a covariance matrix for model parameters defined by $W = VA^{-2}V^T$.
- 6: For the re-standardized system, the decomposition theorem (III-3) is applied again, and a pseudo inverse matrix H is produced as $H = VA^{-1}U^T$.
- 7: Optimal solution at this step is written by $\hat{x} = x^0 + \Delta \hat{x}$, where $\Delta \hat{x}$ is defined by $\Delta \hat{x} = H \Delta y$.
- 8: Reliability of the resulting solution is evaluated by using the quantities, such as the resolution, expected error and effective data number, which are defined in section 3.

If every perturbation of model parameters satisfies the condition, $|\Delta \hat{x}_j| \leq \sigma(\Delta \hat{x}_j)$, the successive iteration of inversions is accomplished at this step. When the condition for convergence is not satisfied, the process is returned back to the beginning of BLOCK II, after replacing the starting model by the optimal solution of the preceding step.

PROGRAM OF GEODETIC DATA INVERSION

The maximum size of a linear system which can be treated by this program is $300 \text{ (data)} \times 90 \text{ (fault parameters)}$, and it depends only on the memory capacity of the computer. All of the computations involved were made on the HITAC 8700, 8800 at the Computer Center, University of Tokyo. The computational time for the case of the six-fault model is about one hour to accomplish a successive iteration of inversions, which converged at the ninth

step. The computer program for the geodetic data inversion is now available to all for use.

5. Summary

This study demonstrates that the method of geodetic data inversion developed in Part I provides us a useful tool to estimate the optimal fault-plane solutions of seismic events and evaluate their reliabilities from the static displacement data, through the analysis of crustal deformations due to the occurrence of the 1927 Tango earthquake.

In geophysical problems, it is particularly needed that the estimated results are quasi-stable against the variations of the presupposed conditions for analysis, such as the starting model, error coefficient and maximum allowable deviations. As to the present method, such a requirement is satisfied by introducing a process of re-standardization of a linear system using a predicted covariance matrix of model parameters.

Two types of measures, the resolution and effective data number, are used for evaluating the reliability of the resulting solution. It is not proper to take the resolution as a measure of reliability, unless the system is strongly over-constrained. When the system is not strongly over-constrained, it is very frequent that the parameter determined only from a few critical data is accompanied with a good resolution, which may be remarkably changed by adding a new effective datum to the previous data set. On the other hand, the effective data number, which indicates the number of effective data contributing to the determination of a model parameter, acts as a good measure of reliability, even in such a case.

In case of the Tango earthquake, adequate values of error coefficient are about 0.15 for the two-fault model and 0.10 for the six-fault model respectively. This suggests that the total effect of various deficiencies in the mathematical model on the surface deformations is less than 10%, i.e., the actual displacement fields caused by seismic events are well interpreted within the residual error of 10% by the multiple-fault model composed of several rectangular faults in an elastic half-space.

On the basis of the analyzing results, two interesting conclusions are obtained:

- i) Extension of two main faultings, the Gomura and the Yamada, almost coincides with the aftershock area.
- ii) These two seismic faults constitute a system of conjugate faults caused by a single stress system of NW-SE compression.

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