SCATTERING OF RAYLEIGH WAVES IN AN ELASTIC QUARTER SPACE

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This study investigates the generation of waves due to the incidence of the Rayleigh wave upon the corner of an elastic quarter space. The Rayleigh wave is incident from infinity and travels along one surface of a right-angled wedge. Integral equations are derived by use of the Fourier transform technique and are solved by deforming their integration path along which the integrands vary smoothly in magnitude. Expressions for the energy fluxes of the Rayleigh waves along two free surfaces and the scattered body waves are obtained. Partition of energy fluxes and directivities of the scattered P and S waves are discussed. The scattered S waves are then found to be composed primarily of four kinds of waves as if they were generated from different wave sources. The orbital motions of the particle of the elastic medium are depicted along two free surfaces. The orbit form on the second surface tends to that of the Rayleigh wave more rapidly than on the first surface where the incident Rayleigh wave exists. The comparison with experimental results previously obtained by several authors shows considerably good agreement.

1. Introduction

The phenomena which take place when the Rayleigh wave is incident upon a corner are of special interest to both pure and applied seismology as a preliminary step to understanding the phenomena involved when a surface wave travelling in one medium continues into another. A number of authors have studied the problem of Rayleigh wave propagation in elastic wedges (de Bremaecker, 1958; Knopoff and Gangi, 1960; Lapwood, 1961; Kane and Spence, 1963; Hudson and Knopoff, 1964; Pilant et al., 1964; Mal and Knopoff, 1966; Satô, 1972) by both the theoretical and experimental means. The transmission coefficients derived theoretically are approximations which are consistently smaller than those obtained by careful experiments. The discrepancy is probably caused by the fact that there is considerable difficulty in computing the diffracted waves near a corner. In order to overcome the above difficulty, we have derived integral equations by use of the Fourier transform technique. These integral equations are solved by deforming the integration path along which the integrands are relatively smooth in variation. In solving the equations, the diffracted waves around the vertex are taken into account with an accuracy sufficient for the
purpose. Many interesting features have been exposed as a result of the investigation.

2. Theory

2.1 Model used and equations

The model used in this work is a homogeneous elastic quarter space as illustrated in Fig. 1. One face of the elastic quarter space runs along the x-axis (using Cartesian coordinates) and the other along the z-axis. Incident Rayleigh waves then travel along the x-axis toward the apex (x=z=0). It is assumed that the displacements in the incident wave lie in a plane perpendicular to the apex of the right-angled wedge. The problem is, therefore, two-dimensional. Let (u, w) be the displacement components of the medium in the (x, z) directions. The equations of motion of the medium in case of periodic motion are expressed by

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \left( \frac{h^2}{k^2} \right) \right) \{ \phi \} = 0 ,
\]

\[
u = \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial z}, \quad w = \frac{\partial \phi}{\partial z} - \frac{\partial \phi}{\partial x},
\]

\[
h = \sigma \sqrt{\frac{\lambda + 2\mu}{\rho}} \quad \text{and} \quad k = \sigma \sqrt{\frac{\mu}{\rho}},
\]

where \( \rho \): density of the elastic medium, \( \sigma \): angular frequency in the time factor \( e^{i\sigma t} \) (t: variable of time) and \( \lambda, \mu \): Lamé's constants. In a later development of the theory, the Cartesian coordinates in (1a) is appropriately transformed to the polar ones (r, \( \theta \)) with \( x = r \cos \theta \) and \( z = r \sin \theta \).

\[\text{Fig. 1. Used model and definitions of coordinates.}\]

2.2 Stress conditions and incident wave

On the surfaces z=0 (along the x-axis) and x=0 (along the z-axis), conditions of shear and normal stresses (\( X_s, Z_s \)) and (\( Z_s, X_s \)) are given by

\[
\begin{align*}
X_s &= \mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) = 0 \\
Z_s &= \lambda \frac{\partial u}{\partial x} + (\lambda + 2\mu) \frac{\partial w}{\partial z} = 0
\end{align*}
\]

(2a)
The incident Rayleigh wave which satisfies Eq. (1a) and conditions (2a) is expressed by
\[ u_{R}^{\text{inc}} = \text{constant}. \]

\[ w_{R}^{\text{inc}} = \text{constant}. \]

where \((u_{R}^{\text{inc}}, w_{R}^{\text{inc}})\): \((u, w)\) components for the incident Rayleigh wave, and \(C\): constant. In the above, the time factor \(e^{\text{i}\omega t}\) is omitted. This convention is followed hereafter.

2.3 Formal expressions for waves

Using the Fourier transform technique, the potentials \(\phi_{j}, \psi_{j}\) \((j = 1, 2)\) satisfying Eq. (1a) are expressed by
\[ \phi_{j} = \sqrt{\frac{2}{\pi}} \int_{0}^{\infty} A_{j} e^{\text{i}k_{j}z} \cdot \exp \left\{ -\frac{\alpha_{j} k_{j} \xi_{j}}{\sin k_{j} \eta_{j}} \right\} \cdot \text{d}k_{j} \quad (j = 1, 2) \]

with \(\alpha_{j} = \sqrt{k_{j}^{2} - h^{2}}, \quad \beta_{j} = \sqrt{k_{j}^{2} - k^{2}}, \quad \{A_{j}, B_{j}\}: \text{unknown coefficients and } \{\xi_{j} = z, \eta_{j} = x\}. \) The branch cuts of \(\alpha_{j}\) or \(\beta_{j}\) run from \(k_{j} = \mp h\) or \(\mp k\) to \(\pm i\infty\) parallel to the imaginary axis of \(k_{j}\) on the complex \(k_{j}\)-plane.

Substituting (4a) into (1b), we have the displacements \((u_{j}, w_{j})\) associated with the potentials \(\phi_{j}\) and \(\psi_{j}\) \((j = 1, 2)\):
where \( \{u_1, u_2, w_1, w_2\} \) are given by (4b) and \( \{u^{inc}_{R}, w^{inc}_{R}\} \) are the displacements of the incident Rayleigh wave given in (3a).

### 2.4 Integral equations

In this section, the integral equations which determine the unknown factors \( A_j, B_j \) in (4b) are introduced.

Substituting the displacements \( (u, w) \) in (4c) into stress conditions (2a) at the \( x \)-surface, we have

\[
X_z = X^{(1)}_z + X^{(2)}_z = 0 \tag{5a}
\]

with

\[
X^{(1)}_z = \sqrt{2} \pi \int_0^\infty \mu (2\kappa_1 \alpha_1 + (\beta_1^2 + \kappa_1^2) \beta_1) \sin \kappa_1 x d\kappa_1 \quad \text{(from } u_1, w_1) \text{,}
\]

and

\[
X^{(2)}_z = 0 \quad \text{owing to the presence of } \sin \kappa_2 z (z = 0) \quad \text{(from } u_2, w_2) \text{,}
\]

and

\[
Z_z = Z^{(1)}_z + Z^{(2)}_z = 0 \tag{5b}
\]

with

\[
Z^{(1)}_z = \sqrt{2} \pi \int_0^\infty \left[ (-\lambda \kappa_1^2 + (\lambda + 2\mu) \kappa_1^2) A_1 + 2\mu \kappa_1 B_1 \right] \cos \kappa_1 x d\kappa_1 \quad \text{(from } u_1, w_1) \text{,}
\]

\[
Z^{(2)}_z = \sqrt{2} \pi \int_0^\infty \left[ (2\alpha_2^2 - (\lambda + 2\mu) \kappa_2^2) A_2 e^{-\beta z} - 2\mu \kappa_2 \beta_2 B_2 e^{-\beta z} \right] d\kappa_2 \quad \text{(from } u_2, w_2) \text{.} \tag{5c}
\]

In (5a) and (5b), stresses due to the incident Rayleigh wave vanish, since the incident wave is advancing along the \( x \)-axis.

Taking the Fourier sine and cosine inverse transforms of (5a) and (5b) respectively, these expressions are reduced to

\[
2\kappa_1 \alpha_1 A_1 + \kappa_1 B_1 = 0 \tag{6a}
\]

\[
\mu \kappa_1 A_1 + 2\mu \kappa_1 \beta_1 B_1 = Z_x \tag{6b}
\]

with

\[
\kappa_1 = 2\kappa_2^2 - \kappa^2
\]

and

\[
Z_x = -\sqrt{2} \pi \int_0^\infty Z^{(2)}_x \cos \kappa_1 x d\kappa_1 \tag{6c}
\]

with \( Z^{(2)}_x \) given by (5c).

Solving (6a) and (6b), the expressions for \( A_1 \) and \( B_1 \) are obtained:
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\[ A_1 = \frac{\vec{K}_1}{F(k_1)} \cdot \frac{Z_1}{\mu}, \]  
\(7a\)

\[ B_1 = \frac{-2k_1\alpha_1}{F(k_1)} \cdot \frac{Z_1}{\mu}, \]  
\(7b\)

where

\[ F(k_1) = \vec{K}_1 - 4k_1^2\alpha_1\beta_1, \]  
\(7c\)

and further reduction of \(Z_1\) is made later (given in \(10a\)).

Likewise, the expressions for \(A_2\) and \(B_2\) are obtained by substituting \(4c\) into stress conditions \(2b\) at the \(z\)-surface (following the reductions for \(A_1\) and \(B_1\)):

\[ A_2 = \frac{A_f}{F(k_2)} \quad \text{with} \quad A_f = \vec{K}_2 \left( \frac{X_f}{\mu} + \frac{X_R}{\mu} \right) - 2k_2\beta_2 \frac{Z_R}{\mu}, \]  
\(8a\)

\[ B_2 = \frac{B_f}{F(k_2)} \quad \text{with} \quad B_f = \vec{K}_2 \frac{Z_R}{\mu} - 2k_2\alpha_2 \left( \frac{X_f}{\mu} + \frac{X_R}{\mu} \right), \]  
\(8b\)

where

\[ \vec{K}_2 = 2k_2^2 - k^2, \quad F(k_2) = \vec{K}_2 - 4k_2^2\alpha_2\beta_2, \]  
\(8c\)

\[ X_f = -\sqrt{\frac{2}{\pi}} \int_0^\infty X_\alpha^{(1)} \cos k_2 z dz \]  
\(8d\)

(\(X_\alpha^{(1)}\) given in \(11\))

\[ X_R = -\sqrt{\frac{2}{\pi}} \int_0^\infty X_\alpha^{(R)} \cos k_2 z dz, \]  
\(8e\)

\[ Z_R = -\sqrt{\frac{2}{\pi}} \int_0^\infty Z_\alpha^{(R)} \sin k_2 z dz, \]  
\(8e\)

where \(\{X_\alpha^{(R)}, Z_\alpha^{(R)}\}\) are the stress conditions \(\{X_\alpha, Z_\alpha\}\) for the incident wave on the \(z\)-surface. Using the expressions \(3a\) for the incident wave, \(\{X_R, Z_R\}\) in \(8e\) are reduced to the following expressions:

\[ X_R = -\sqrt{\frac{2}{\pi}} \left\{ B_{in} \cdot G(\alpha_R, k_2) + C_{in} \cdot G(\beta_R, k_2) \right\}, \]  
\(8f\)

\[ Z_R = -\sqrt{\frac{2}{\pi}} \left\{ -G(k_2, \alpha_R) + G(k_2, \beta_R) \right\}, \]  
\(8g\)

with

\[ G(\xi, \eta) = \xi / (\xi^2 + \eta^2) \]  
\(8h\)

(\(\xi, \eta\): dummy variables)
\[
A_{in} = i \cdot 2 \mu \alpha_{n} \kappa_{n} \vec{K} \cdot C, \quad B_{in} = -(\vec{\alpha}^{2} + 2 \mu \kappa_{n}^{2}) \vec{K} \cdot C, \quad C_{in} = 4 \mu \alpha_{n} \beta_{n} \kappa_{n}^{2} \cdot C. \quad (8i)
\]

Now, further reductions of \(Z_{f}\) in (6c) and \(X_{f}\) in (8d) are made here. Integrals (6c) and (8d) can then be integrated with respect to \(x\) and \(z\) respectively:

\[
\begin{align*}
Z_{f}(k_{1}) &= \frac{2}{\pi} \int_{0}^{\infty} \left[ E_{f} \cdot A_{z} \cdot G(\alpha_{z}, k_{1}) - 2 \mu \kappa_{z} \beta_{z} \cdot B_{z} \cdot G(\beta_{z}, k_{1}) \right] dk_{2}, \quad (9a) \\
X_{f}(k_{2}) &= \frac{2}{\pi} \int_{0}^{\infty} \left[ E_{f} \cdot A_{z} \cdot G(\alpha_{z}, k_{2}) - 2 \mu \kappa_{z} \beta_{z} \cdot B_{z} \cdot G(\beta_{z}, k_{2}) \right] dk_{1}, \quad (9b)
\end{align*}
\]

with

\[
E_{f} = \lambda \alpha_{f}^{2} - (\lambda + 2 \mu) k_{f}^{2} \quad (j = 1, 2).
\]

In the expressions (8a, b), the unknown coefficients \(A_{z}\) and \(B_{z}\) on the \(z\)-surface are coupled with \(A_{1}\) and \(B_{1}\) on the \(x\)-surface since \(X_{f}\) in (8a, b) are expressed in terms of \(A_{1}\) and \(B_{1}\), as found in (9b). In these couplings, \(A_{z}\) and \(B_{z}\) can be explicitly expressed in terms of \(A_{1}\) and \(B_{1}\). Therefore, if \(A_{z}\) and \(B_{z}\) in (8a, b) are substituted into (9a), \(Z_{f}\) in (9a) is expressed in terms of the unknown coefficients \(A_{1}\) and \(B_{1}\):

\[
Z_{f}(k_{1}) = H_{1}(k_{1}) + F^{1n}(k_{1}), \quad (10a)
\]

\[
H_{2}(k_{2}) = \frac{4}{\pi} \int_{0}^{\infty} \left[ \frac{1}{\mu} \cdot I_{f}(k_{2}) \cdot A_{2}(\gamma) - 2 \eta \beta_{2} \cdot I_{f}^{*}(k_{2}) \cdot B_{2}(\gamma) \right] d\gamma \quad (j = 1), \quad (10b)
\]

\[
F^{1n}(k_{1}) = \frac{2}{\pi} \int_{0}^{\infty} \left[ K_{f}(k_{1}) \cdot X_{n} - J_{f}(k_{1}) \cdot Z_{n} \right] d\zeta \quad (10c)
\]

\[
(\text{\(X_{n}, Z_{n}\) are given in (8f, g)}),
\]

\[
E_{\gamma} = \lambda \alpha_{\gamma}^{2} - (\lambda + 2 \mu) \gamma^{2}, \quad \alpha_{\gamma} = \sqrt{\gamma^{2} - h^{2}}, \quad \beta_{\gamma} = \sqrt{\gamma^{2} - k^{2}} \quad (10d)
\]

\[
I_{f}(k_{2}) = \left\{ \frac{\tilde{K}_{f}(k_{2})}{F(\zeta)} \cdot G(\gamma, \zeta) d\zeta \quad (j = \alpha, \beta) \right\} \quad (10e)
\]

\[
K_{f}(k_{2}) = \vec{K}_{f} \cdot E_{f} \cdot G(\alpha_{f}, k_{2}) + 4 \mu \zeta^{2} \alpha_{f} \beta_{f} \cdot G(\beta_{f}, k_{2}) \quad (10f)
\]

\[
J_{f}(k_{2}) = 2 \zeta \beta_{f} \cdot E_{f} \cdot G(\alpha_{f}, k_{2}) + 2 \mu \zeta \beta_{f} \cdot \vec{K}_{f} \cdot G(\beta_{f}, k_{2}) \quad \text{with} \quad \vec{K}_{f} = 2\zeta^{2} - k^{2}.
\]

Likewise, in the expressions (7a, b), the unknown coefficients \(A_{1}\) and \(B_{1}\) are coupled with \(A_{2}\) and \(B_{2}\) since \(Z_{f}\) in (7a, b) are expressed in terms of \(A_{2}\) and \(B_{2}\), as shown in (9a). In these couplings, \(A_{1}\) and \(B_{1}\) can be explicitly expressed in terms of \(A_{2}\) and \(B_{2}\). Therefore, if \(A_{2}\) and \(B_{2}\) in (7a, b) are substituted into (9b), \(X_{f}\) in (9b) is expressed in terms of the unknown coefficients \(A_{2}\) and \(B_{2}\):

\[
X_{f}(k_{2}) = H_{2}(k_{2}), \quad (11)
\]

where \(H_{2}(k_{2})\) is given by \(H_{2}(k_{2})\) in (10b) with the substitution of \(j = 2\).

Substitution of (10a) into (7a, b) yields integral equations in terms of \(A_{1}\) and \(B_{1}\):
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\[
F(\kappa_1) \cdot A_1(\kappa_1) - \overline{K}_1 \cdot \frac{H_1(\kappa_1)}{\mu} = \overline{K}_1 \cdot \frac{F^{1n}(\kappa_1)}{\mu}, 
\]

\[
F(\kappa_1) \cdot B_1(\kappa_1) + 2\kappa_1\alpha_1 \cdot \frac{H_1(\kappa_1)}{\mu} = -2\kappa_1\alpha_1 \cdot \frac{F^{1n}(\kappa_1)}{\mu}. 
\]

(12a)  

(12b)

The two expressions above are the equations which determine the unknown coefficients \(A_1\) and \(B_1\); these equations do not have the unknown coefficients \(A_2\) and \(B_2\). The effect of the coupling of \((A_1, B_1)\) with \((A_2, B_2)\) are then incorporated into the function \(H_1(\kappa_1)\) which is produced by the process of elimination of \((A_2, B_2)\) by use of (8a, b).

Likewise, substitution of (11) into (8a, b) yields integral equations in terms of \(A_2\) and \(B_2\):

\[
F(\kappa_2) \cdot A_2(\kappa_2) - \overline{K}_2 \cdot \frac{H_2(\kappa_2)}{\mu} = \overline{K}_2 \cdot \frac{X_R}{\mu} - 2\kappa_2\beta_2 \cdot \frac{Z_R}{\mu}, 
\]

\[
F(\kappa_2) \cdot B_2(\kappa_2) + 2\kappa_2\alpha_2 \cdot \frac{H_2(\kappa_2)}{\mu} = \overline{K}_2 \cdot \frac{Z_R}{\mu} - 2\kappa_2\alpha_2 \cdot \frac{X_R}{\mu}, 
\]

(13a)  

(13b)

where \(H_2(\kappa_2)\) is given by (10b) with the substitution of \(j=2\), and \((X_R, Z_R)\) are described in (8f, g).

If virtual viscosity \(\bar{\mu}\) is introduced, the zero points of \(F(\kappa_j)\) are located at \(\kappa_j = -k_n \pm i\bar{\mu}\) or within the second and fourth quadrants. In the reduction of \(G(\xi, \eta)\) in (9a, b), virtual viscosity also affects the influence upon \(G(\xi, \eta)\) through the exponential factors in (5c) and (8d), i.e., \(e^{-\gamma_3} - \beta_3\), \(e^{-\gamma_3} - \beta_3\), \(e^{-\gamma_3} - \beta_3\), \(e^{-\gamma_3} - \beta_3\). The expression for \(G(\xi, \eta)\) including \(\bar{\mu}\) is then

\[
\left\{1/(\xi + \bar{\mu} - i\eta) + 1/(\xi + \bar{\mu} + i\eta)\right\}/2 \quad (\bar{\mu} \to 0).
\]

Evaluation of the above expression in the case of \(\xi = \alpha_j\) or \(\beta_j\) shows that the poles of \(G(\xi, \eta)\) in terms of \(\eta\) are expressed as

\[
\eta = \pm i\gamma_j \pm i\bar{\mu} \quad (\gamma = \alpha, \beta) \quad \text{for real } \gamma_j
\]

and

\[
\eta = \mp |\gamma_j| \pm i\bar{\mu} \quad \text{for imaginary } \gamma_j;
\]

the former of the above \(\eta\)-values are located along the imaginary axis on the \(\eta\)-plane and the latter along the real axis within the second and fourth quadrants. The path of the integral \(I_\zeta(\xi)\) in (10e), therefore, runs from \(\zeta = 0\) to \(\infty\) on the upper side of the real axis on the complex \(\zeta\)-plane. Computation of the integral \(I_\zeta(\xi)\) is made by use of Simpson's formula. The same discussion can be applied to the integration path of the integrals \(H_j(\kappa_j)\) in (10b) on the complex \(\eta\)-plane: the integrals \(H_j(\kappa_j)\) involve the integrals \(I_\zeta(\kappa_j)\) \((\gamma = \alpha, \beta)\) which possess poles in \(G(\gamma_\eta, \zeta)\) (see (10e)). Therefore, the path of \(H_j(\kappa_j)\) must be deformed on the upper side of the real axis on the complex \(\eta\)-plane; this deformed path is designated by (c) beneath the integration symbol \(\int\) in (10b) as \(\int_c\). In the next section,
the deformed path \( c \) is determined numerically by a number of trials of numerical computations.

2.5 Infinite system of simultaneous equations

The integral equations (12a, b) and (13a, b) are so complicated that they cannot be solved analytically. Numerical procedure is therefore employed by introducing some new device.

First of all, the path \( c \) in the integrals \( H_j(k_j) \) in (10b) is described here. On the complex \( \gamma \)-plane, the path \( c \) runs from \( \gamma = 0 \) (origin) to \( \gamma = (2k, 2k) \), \( k \) being the wave number of the S wave, and, further, the path extends from \( \gamma = (2k, 2k) \) to \( \gamma = (-\infty, 2k) \) parallel to the real axis of \( \eta \). Determination of the turning point \((2k, 2k)\) is fairly arbitrary; if this point is nearer to the real axis, the rapid variation of the function \( F(k_j) \) or \( G(\gamma, \zeta) \) around the zero points renders the accuracy of the solution uncertain and hence in order to obtain a better accuracy the number of infinite system of simultaneous equations (which are introduced later) increases rapidly. In order to solve Eqs. (12a, b) and (13a, b), the values of the unknown factors \( \{A_j(k_j), B_j(k_j)\} \) \( (j=1, 2) \) must be taken along the path \( c \) on the \( k_j \)-plane instead of the \( \gamma \)-plane. The reasons why the path \( c \) is chosen as stated above are that: (i) along the path \( c \), the function \( F(k_j) \) \( (j=1, 2) \) is fairly smooth without any zero points, which are located along the real axis on the complex \( k_j \)-plane (though this function is more smooth along the imaginary axis, the reason (ii) described below makes the path along the imaginary axis impossible), and (ii) the functions \( G(\gamma, \zeta) \) for \( \gamma = \alpha \) or \( \beta \), which occur in the expressions \( I_{\alpha}(k_j) \) and \( I_{\beta}(k_j) \) in (10b) have poles along both the real and imaginary axes on the \( \gamma \)-plane. The path determined in the above way is employed in this work, though other better alternatives might be considered.

Even though the path \( c \) is assumed, the solutions of the integral equations cannot be obtained so easily. The path \( c \) on the complex \( k_j \)-plane is divided into two parts, i.e., path \( c_1 \): \( k_j \) from 0 to \((2k, 2k)\), and, path \( c_2 \): \( k_j \) from \((2k, 2k)\) to \((\infty, 2k)\). The path \( c_1 \) is evenly divided into \( M \) divisions; the length of each division is \( 2\delta s_0 \) with a reference point at its midpoint. On the other hand, the path \( c_2 \) need not be divided into divisions with equal intervals, since the variation of the unknown factors \( \{A_j(k_j), B_j(k_j)\} \) becomes more smooth and smaller with increase of \( k_j \) along the real axis. Let \( 2\delta s_m \) \( (m=1, 2, 3, \text{ etc.}) \) be intervals divided successively from the point \( k_j=(2k, 2k) \) to \((\infty, 2k)\) along the path \( c_2 \) and \( s_m \) the real values of the midpoints of the above intervals \( 2\delta s_m \). The determination of \( \delta s_m \) is made in the following way. Suppose that \( |A_j(k_j)| \) or \( |B_j(k_j)| \) are decreasing functions like \( s^{-n+1} \) for large real \( s \) where \( n \) is a positive value (>1) not necessarily restricted to an integer and \( s \) the real part of \( k_j \) along the path \( c_2 \), we then have

\[ \frac{\partial |A_j(k_j)|}{\partial s} = s^{-n} \delta s. \]

Using this expression, the divided intervals \( \delta s_m \) are determined so as to make \( \partial |A_j(k_j)| \) or \( \partial |B_j(k_j)| \) constant; this procedure implies that, if the values of \( A_j(k_j) \)
or $B_j(\kappa_j)$ are approximated by the values at the midpoint in the above-mentioned intervals, errors due to this approximation could be kept within the same order throughout all intervals of $\kappa_j$. From the above expression, under the assumption $\partial|A_j(\kappa_j)|$ or $\partial|B_j(\kappa_j)|=\text{constant}$, we have

$$s_m^n - s_m = \text{const} = s_1^n - s_1,$$

i.e.,

$$\partial s_m = (s_m/s_1)^n \partial s_1 \quad (m=1, 2, 3, \ldots, N),$$

(14)

where $N$ is a large positive number and $\{s_m, \partial s_m\}$ are the initial values of $\{s_m, \partial s_m\}$ near $\kappa_j = (2k, 2k)$. Putting $\partial s_1 = \partial s_0$, the interval $\partial s_m$ on path $c_2$ is determined successively by use of (14). In the actual computation of $\partial s_m$ in (14), $s_m$ is approximated by

$$s_m \approx (s_{m-1} + \partial s_{m-1}) + \partial s_{m-1},$$

where the expression in the parentheses denotes the value of the upper-side end of the interval $2\partial s_{m-1}$ and the last $\partial s_{m-1}$ is an approximate value extended toward the next interval $\partial s_m$ so as to anticipate its midpoint; exactly speaking, $\partial s_{m-1}$ must be replaced by $\partial s_m$, while $s_m$ is an unknown value in an undetermined interval. In (14), the value of $n$ has not yet been determined. In our formulation of the theory, the value $n$ which denotes the descending rate of the factor $|A_j(\kappa_j)|$ or $|B_j(\kappa_j)|$ cannot be found analytically. The determination of $n$ is made by numerical computations of an infinite system of simultaneous equations later.

Let us now consider the formulation of the infinite system of simultaneous equations. Let $\kappa_{jl}$ $(l=1, 2, 3, \ldots)$ be complex values of $\kappa_j$ at the midpoints of the $l$-th complex intervals $2\partial \kappa_{jl}$ along path $c$ $(l=1 \sim M$ on path $c_1$ and $l=M+1 \sim M+N$ on path $c_2)$ which are determined by (14). The values of $\kappa_j$ in Eqs. (12a, b) and (13a, b) are then replaced by the discrete values $\kappa_{jl}$ defined above. Multiplying Eqs. (12a, b) and Eqs. (13a, b) by $2\partial \kappa_{1l}$ and $2\partial \kappa_{2l}$ respectively, these equations are reduced to the following:

$$F(\kappa_{1l}) \cdot \{A_1(\kappa_{1l}) \cdot 2\partial \kappa_{1l} - \overline{K}_{1l} - \frac{H_1(\kappa_{1l})}{\mu} \cdot 2\partial \kappa_{1l} = \overline{K}_{1l} \cdot \frac{F_{1n}(\kappa_{1l})}{\mu} \cdot 2\partial \kappa_{1l},$$

(15a)

$$F(\kappa_{1l}) \cdot \{B_1(\kappa_{1l}) \cdot 2\partial \kappa_{1l} + 2\kappa_{1l} \alpha_{1l} \cdot \frac{H_1(\kappa_{1l})}{\mu} \cdot 2\partial \kappa_{1l} = -2\kappa_{1l} \alpha_{1l} \cdot \frac{F_{1n}(\kappa_{1l})}{\mu} \cdot 2\partial \kappa_{1l},$$

(15b)

and

$$F(\kappa_{2l}) \cdot \{A_2(\kappa_{2l}) \cdot 2\partial \kappa_{2l} - \overline{K}_{2l} - \frac{H_2(\kappa_{2l})}{\mu} \cdot 2\partial \kappa_{2l} = \left\{ \frac{\overline{K}_{2l} \cdot X_{R} - 2\kappa_{2l} \beta_{2l} \cdot \frac{Z_{R}}{\mu}}{\overline{K}_{2l} \cdot \overline{K}_{2l}} \right\} \cdot 2\partial \kappa_{2l},$$

(16a)

$$F(\kappa_{2l}) \cdot \{B_2(\kappa_{2l}) \cdot 2\partial \kappa_{2l} + 2\kappa_{2l} \alpha_{2l} \cdot \frac{H_2(\kappa_{2l})}{\mu} \cdot 2\partial \kappa_{2l}$$

$$= \left\{ \frac{\overline{K}_{2l} \cdot X_{R} - 2\kappa_{2l} \beta_{2l} \cdot \frac{X_{R}}{\mu}}{\overline{K}_{2l} \cdot \overline{K}_{2l}} \right\} \cdot 2\partial \kappa_{2l},$$

(16b)
with
\[ H_\ell(k_{j\ell}) = \frac{4}{\pi^2} \sum_{i} \text{along } \eta \left[ -\frac{E_\eta \cdot I_\eta(k_{j\ell})}{\mu} \cdot (A_\eta(\eta_i) \cdot 2\delta \eta_i) \right. \]
\[ \left. -\frac{4}{\pi^2} \sum_{i} [2\eta \beta_\eta \cdot I_\eta(k_{j\ell})]_{\eta=\eta_i} \cdot (B_\eta(\eta_i) \cdot 2\delta \eta_i) \right] \quad (j=1, 2), \] (17)

where \( l \) ranges from 1 to \( M+N \) in Eqs. (15a, b) and (16a, b), \( \sum_{i} \text{along } \eta \) in (17) denotes the sum \( \sum_{i=1}^{M+N} \) along path \( \eta \) on the complex \( \eta \)-plane, and \( \{\alpha_{j\ell}, \bar{\alpha}_{j\ell}\} \) are \( \{\alpha_j, \bar{\alpha}_j\} \) for \( k_j = \bar{k}_{j\ell} \quad (j=1, 2) \). Solving Eqs. (15a, b) and (16a, b), the unknown factors \( \{A_j(\bar{k}_{j\ell}), B_j(\bar{k}_{j\ell}) \cdot 2\delta \bar{k}_{j\ell}\} \quad (j=1, 2) \) are obtained.

In the expression (14), the factor \( n \) has not yet been determined. Solving the two systems of simultaneous equations (15a, b) and (16a, b) with a variety of \( n \) ranging from 1 to 5 (not necessarily restricted to integers), a value of \( n=1.6 \) or 1.7 is found the most appropriate one which makes the unknown factors \( \{A_j(\bar{k}_{j\ell}), B_j(\bar{k}_{j\ell}) \cdot 2\delta \bar{k}_{j\ell}\} \) normalized.

### 2.6 Expressions for scattered waves near the corner

In order to evaluate the waves near the vertex numerically, some modification of the expressions is required. Waves \((u_{\alpha\alpha}, w_{\alpha\alpha})\) scattered from the part near the vertex are expressed as
\[
\begin{align*}
[u_{\alpha\alpha} = u_1 + u_2 \\
w_{\alpha\alpha} = w_1 + w_2
\end{align*}
\] (18)

where \( u_{\alpha\alpha} \) and \( w_{\alpha\alpha} \) are, respectively, the first two expressions of \( u \) and \( w \) in (4c) and \( \{u_1, u_2, w_1, w_2\} \) are given in (4b). In integrals (4b), the integration path is deformed into path \( c \) determined in the foregoing section. Values of \( A_1 \) and \( B_1 \) in (4b) are computed by the use of expressions (7a, b) with \( Z_\ell \) given below, while values of \( A_2 \) and \( B_2 \) are obtained from (8a, b) with the help of \( X_\ell \) given below.

Since the unknown factors \( A_j(\bar{k}_{j\ell}) \) and \( B_j(\bar{k}_{j\ell}) \quad (j=1, 2) \) obtained along path \( c \), the expressions for \( Z_\ell \) and \( X_\ell \) (in (9a, b)) must be modified into
\[
\begin{align*}
Z_\ell(k_{j\ell}) &= -\frac{2}{\pi} \sum_{i} \text{along } \ell \left[ (E_\ell \cdot G(\alpha_\ell, k_{j\ell}))_{\ell=\ell_1} \cdot (A_\ell(k_{j\ell}) \cdot 2\delta k_{j\ell}) \right. \\
&\left. -2\mu \cdot (k_{j\ell} \beta_{j\ell} \cdot G(\beta_{j\ell}, k_{j\ell}))_{\ell=\ell_1} \cdot (B_\ell(k_{j\ell}) \cdot 2\delta k_{j\ell}) \right] \quad (19a) \\
X_\ell(k_{j\ell}) &= -\frac{2}{\pi} \sum_{i} \text{along } \ell \left[ (E_\ell \cdot G(\alpha_\ell, k_{j\ell}))_{\ell=\ell_1} \cdot (A_\ell(k_{j\ell}) \cdot 2\delta k_{j\ell}) \right. \\
&\left. -2\mu \cdot (k_{j\ell} \beta_{j\ell} \cdot G(\beta_{j\ell}, k_{j\ell}))_{\ell=\ell_1} \cdot (B_\ell(k_{j\ell}) \cdot 2\delta k_{j\ell}) \right] \quad (19b)
\end{align*}
\]

The actual computation of integrals (4b) is carried out by use of Simpson's formula.
2.7 Expressions for generated Rayleigh waves
Substituting (7a, b) and (8a, b) into (4b) and after some reductions, we have

\[ u_1 = \frac{1}{i} \sqrt{\frac{2\pi}{i}} \int_{-\infty}^{\infty} \left( -K_1^{(\alpha)} e^{-\alpha x z} \right) \cdot \frac{(Z_1/\mu)}{F(k_1)} \cdot e^{i\theta z} \, dz, \tag{20a} \]

\[ w_1 = \frac{1}{i} \sqrt{\frac{2\pi}{i}} \int_{-\infty}^{\infty} \left( -K_1^{(\beta)} e^{-\beta x z} \right) \cdot \frac{(Z_1/\mu)}{F(k_1)} \cdot e^{i\theta z} \, dz, \tag{20b} \]

and

\[ u_2 = \frac{1}{i} \sqrt{\frac{2\pi}{i}} \int_{-\infty}^{\infty} \left( -\alpha_z A_1 e^{-\alpha_2 z} - \beta_z B_1 e^{-\beta_2 z} \right) \cdot \frac{1}{F(k_2)} \cdot e^{i\theta z} \, dz, \tag{21a} \]

\[ w_2 = \frac{1}{i} \sqrt{\frac{2\pi}{i}} \int_{-\infty}^{\infty} \left( -\alpha_z A_1 e^{-\alpha_2 z} - \beta_z B_1 e^{-\beta_2 z} \right) \cdot \frac{1}{F(k_2)} \cdot e^{i\theta z} \, dz, \tag{21b} \]

where \( A_1, B_1 \) and \( Z_1 \) are given by (8a, b) and (19a) respectively. If our consideration is limited to the behavior of waves travelling away along the free surfaces from the corner to infinity, these waves are given by pole contributions of integrals (20a, b) (along x-surface) and of (21a, b) (along z-surface).

Let \((u_R^x, w_R^x)\) and \((u_R^z, w_R^z)\) be the \((x, z)\) components of the displacement of the Rayleigh waves along the \(x\)- and \(z\)-surface (super-suffices \(x, z\) denote the surface referred to) respectively. After some reductions, these Rayleigh waves are described as

\[ u_R^x = -A_R^x(z) \cdot C_1 \cdot e^{-ik_R z} \] (along the x-surface) \tag{22a}

with

\[ C_1 = \frac{\sqrt{2\pi} i}{\mu \delta F} \bigg|_{\theta = -k_R}, \tag{22b} \]

and

\[ u_R^z = -A_R^z(x) \cdot C_2 \cdot e^{-ik_R z} \] (along the z-surface) \tag{23a}

with

\[ C_2 = \frac{\sqrt{2\pi} i}{\mu \delta F} \left( X_R + X_R^{(2)} \frac{2K_2}{k_R} Z_R \right) \bigg|_{\theta = -k_R}, \tag{23b} \]

where \( \delta F = \{dF(\xi)/d\xi\}_t = k_R \) and \( \{A_R^x(x or z), A_R^z(x or z)\} \) are given in (3b).

2.8 Expressions for scattered body waves at a great distance
Changing the variable in the integrand of integrals (4b) from \( k_j \) \((j=1, 2)\) to \( \varphi \) by use of expression \( k_j = \{h or k\} \cdot \sin \varphi \) for \( \{P \) or \( S\} \) waves, respectively, and further, using polar coordinates \( \{x=r \cos \theta, z=r \sin \theta\} \), integrals (4b) are reduced to
\[ u_j = u_j^p + u_j^s, \quad w_j = w_j^p + w_j^s, \quad \{j = 1, 2\} \]

with

\[ u_j^{\text{p or s}} = \int G_{uj}^{\text{p or s}} \cdot \exp \{i(h, k)r \cdot f_j(\varphi)\} d\varphi, \]
\[ w_j^{\text{p or s}} = \int G_{wj}^{\text{p or s}} \cdot \exp \{i(h, k)r \cdot f_j(\varphi)\} d\varphi, \]
\[ f_1(\varphi) = \sin(\varphi - \theta) \quad \text{and} \quad f_2(\varphi) = -\cos(\varphi + \theta), \]

where the path of the integrations is \(-\pi/2 - i\infty\) to \(+\pi/2 + i\infty\) and \(\{G_{uj}^{\text{p or s}}(\varphi), G_{wj}^{\text{p or s}}(\varphi)\}\) are the integrands after the change of variables. Applying the saddle point method to the above integrations for large \(r\), we can obtain the expressions of displacements \(\{u_j, w_j\}\) at \(r = \infty\) in Cartesian coordinates. Since the expressions obtained in such a way are described in Cartesian coordinates, it is necessary to convert the coordinates to the polar ones by using the relations \(\{u_r = -u_j \sin \theta + w_j \cos \theta, u_\theta = u_j \cos \theta + w_j \sin \theta\}\). After these reductions, we have the azimuthal and radial displacements \(\{u_\theta, u_r\}\) of the scattered waves at \(r = \infty\):

\[ u_\theta = u_{\theta 1} + u_{\theta 2} = \frac{k^2}{\sqrt{kr}} \sin \theta \cdot B_1 \big|_{x_1 = -k \sin \theta} \cdot e^{-(h + r i \pi/4)}, \quad (24a) \]
\[ u_r = u_{r 1} + u_{r 2} = -\frac{ih^2}{\sqrt{kr}} \sin \theta \cdot A_1 \big|_{x_1 = -k \sin \theta} \cdot e^{-(h + r i \pi/4)}, \quad (24b) \]

where \(\{A_1, A_2, B_1, B_2\}\) are given by \((7a, b)\) and \((8a, b)\) with \(Z_f^c R\) and \(X_f^c R\) in \((19a, b)\). In \((24a, b)\), the suffices \((s, p)\) are added to the square brackets for reference on the occasion of the reduction of the expression of energy to be given later.

### 2.9 Expressions for energy fluxes

Energy flux \(E\), which is transmitted over one cycle in time through the surface of unit width along the \(y\)-axis, is expressed as

\[ E = \int_{s_0} d\tau \int_{j_0}^{T} \left( X_e \frac{du_e}{dt} + Z_e \frac{dw_e}{dt} \right), \quad (25) \]

where \(c_0\): path depending on only \((x, z)\) or \((r, \theta)\), \(ds\): infinitesimal along \(c_0\), \(T\): period, \(X_e\) and \(Z_e\): normal and shear stresses on path \(c_0\), \(u_e\) and \(w_e\): normal and tangential displacements on path \(c_0\) (the axis of \(Z_e\) and \(w_e\) are positive in the clockwise direction to the axis of \(X_e\) and \(u_e\)).

Consider first the energy flux of the incident Rayleigh wave. The path \(c_0\) is then \(z\) from 0 to \(\infty\) and \(\{X_e, Z_e\}\) are given by

\[ \{(\lambda + 2\mu)\partial u_0 / \partial x + \lambda \partial w_0 / \partial z, \mu (\partial w_0 / \partial x + \partial u_0 / \partial z)\} \]

with \(\{u_0, w_0\} = \text{Re} \{u_0^{\text{inc}} e^{i\omega t}, w_0^{\text{inc}} e^{i\omega t}\}\). Using \((25)\) and the expressions \(\{u_0^{\text{inc}}, w_0^{\text{inc}}\}\).
in (3a), the energy flux of the incident Rayleigh wave \( E_{\text{inc}}^R \) becomes

\[
E_{\text{inc}}^R = C^a k_R \pi \bar{K} H_R
\]  

(26a)

with

\[
H_R = \left[ \left( \frac{\bar{K}}{2 \alpha_R} - \frac{2 \zeta_R \beta_R}{\alpha_R + \beta_R} \right) (4 \mu k_R^2 + \lambda h^2) - \mu h^2 \bar{K} \right]_{\alpha_R} + \left[ 2 \mu \alpha_R \left( \frac{\bar{K}}{2 \alpha_R} + k_R^2 \frac{1}{\alpha_R + \beta_R} \right) \right]_{\alpha_R},
\]  

(26b)

where the expressions in the first and second square brackets, \([ ]_n\) and \([ ]_s\), are derived from the expression related with the normal (\(X_c\)) and shear (\(Z_c\)) stresses in the integral (25).

Following the procedure mentioned above, the normalized energy fluxes over one cycle in time of the Rayleigh waves (22a) on the \(x\)-surface and (23a) on the \(z\)-surface, \(E_x^R\) and \(E_z^R\) respectively, with \(E_{\text{inc}}^R\) as the normalization factor, are expressed as

\[
E_x^R = |C_1|^2, \quad E_z^R = |C_2|^2,
\]  

(26c)

where the substitutions are made that \(\{u_c, w_c\} = \text{Re} \{u_{cR} e^{i \omega t}, w_{cR} e^{i \omega t}\}\) on path \(c_0\) from \(z = 0\) to \(\infty\) for \(E_x^R\) and \(\{u_c, w_c\} = \text{Re} \{w_{zR} e^{i \omega t}, u_{zR} e^{i \omega t}\}\) on path \(c_0\) from \(x = 0\) to \(\infty\) for \(E_z^R\).

Consider next the energy flux over one cycle in time of the body waves scattered into the inside of the quarter space. Path \(c_0\) is then \(\theta\) from 0 to \(\pi/2\) at \(r = \infty\) and \(\{X_c, Z_c\}\) are given by

\[
\{(\lambda + 2 \mu) \partial u_c / \partial r, \mu \partial w_c / \partial r\} \quad \text{for} \quad r = \infty
\]

with \(\{u_c, w_c\} = \text{Re} \{u_{eR} e^{i \omega t}, u_{\theta R} e^{i \omega t}\}\), where \(\{u_r, u_\theta\}\) are given in (24a, b). Using (25) and the above-mentioned stresses and displacements, the normalized (with \(E_{\text{inc}}^R\)) energy flux of the scattered body waves, \(E_{\text{sc}}\), becomes

\[
\begin{align*}
\bar{E}_{x_{\text{sc}}} &= \bar{E}_{x_{\text{sc}}} + \bar{E}_{y_{\text{sc}}}, \\
\bar{E}_{y_{\text{sc}}} &= \int_0^{\pi/2} \delta \bar{E}_{y_{\text{sc}}} d\theta, \\
\bar{E}_{x_{\text{sc}}} &= \int_0^{\pi/2} \delta \bar{E}_{x_{\text{sc}}} d\theta,
\end{align*}
\]  

(27a)

(27b)

with

\[
\begin{align*}
\delta \bar{E}_{x_{\text{sc}}} &= \frac{(\lambda + 2 \mu) h^4}{k_R \bar{K} H_R} \left[ \left( 1 / |C|^2 \right) \right], \\
\delta \bar{E}_{y_{\text{sc}}} &= \frac{\mu k^4}{k_R \bar{K} H_R} \left[ \left( 1 / |C|^2 \right) \right].
\end{align*}
\]  

(27c)
where \((\bar{E}_{sc}^p, \bar{E}_{sc}^s)\) denote the normalized energy fluxes scattered into the quarter space by \(\{P, S\}\) waves over one cycle in time, \(\{\delta \bar{E}_{sc}^p, \delta \bar{E}_{sc}^s\}\) the energy densities of \(\{\bar{E}_{sc}^p, \bar{E}_{sc}^s\}\) and \([ [ ]_p, [ ]_s]\) the expressions of the square bracket included in (24a, b).

Conservation of energy is then described as
\[
\bar{E}_{total} = \bar{E}_{\bar{R}^p} + \bar{E}_{\bar{R}^s} + \bar{E}_{sc^p} + \bar{E}_{sc^s} = 1,
\]
\(\bar{E}_{total}\) being the total energy.

3. Numerical Computations and Discussions

3.1 Accuracy

Following the procedure described above, numerical computations are carried out. \(k\) (the wave number of \(S\) waves) is then used as the normalization factor for length. First of all, the accuracy of the computation will be examined. If the divided number \(M\) of path \(c_1\), which is a straight line in the range \(k_j\) from 0 to \((2k, 2k)\) on the complex \(k_j\)-plane, is 21 and 23 in the range of \(\lambda/\mu\) from 0.1 to 1.0 and \(\lambda/\mu\) from 1.0 to 10.0 respectively, the accuracy of energy fluxes \(\bar{E}_{\bar{R}^s}, \bar{E}_{\bar{R}^s}, \bar{E}_{sc^p}\) and \(\bar{E}_{sc^s}\) is of the order of 0.0001 and the error of the total energy \(|\bar{E}_{total} - 1|\) is also found to be 0.0002. From the accuracy mentioned above, it can be stated that the results obtained in this work are almost exact solutions, though the approximation by discrete values of \(k_j\) along path \(c\) is employed in section 2.5.

3.2 Energy fluxes

Variations of energy fluxes \(\bar{E}_{\bar{R}^s}, \bar{E}_{\bar{R}^s}, \bar{E}_{\bar{R}^s} + \bar{E}_{sc^p}, \bar{E}_{sc^p}\) and \(\bar{E}_{sc^s}\) are depicted in Fig. 2 (the first three and the last three are shown, respectively, in the upper and lower figures in Fig. 2); the ordinate and abscissa denote the values of energy fluxes and ratio \(\lambda/\mu\) respectively; the scale of \(\lambda/\mu\) ranging from 0 to 1.0 differs from that in the range from 1.0 to 10.0 (the former is ten times the latter).

These figures reveal the following. The transmitted energy \(\bar{E}_{\bar{R}^s}\) is the largest of the four energies \(\{\bar{E}_{\bar{R}^p}, \bar{E}_{\bar{R}^s}, \bar{E}_{sc^p}, \bar{E}_{sc^s}\}\). This is due to the direct arrival of the exponential part of the incident Rayleigh wave at the \(z\)-surface; along this surface the incident energy is trapped directly as a transmitted Rayleigh wave. Roughly speaking, among the four energy fluxes, the energies \(\bar{E}_{\bar{R}^s}\) of the transmitted Rayleigh wave and \(\bar{E}_{sc^s}\) of the scattered \(S\) wave are nearly independent of \(\lambda/\mu\)—the curves are seen to run nearly horizontally—while energy flux \(\bar{E}_{\bar{R}^s}\) of the reflected Rayleigh wave increases and \(\bar{E}_{sc^p}\) of the scattered \(P\) wave decreases with increase of \(\lambda/\mu\). For the behavior of \(\bar{E}_{sc^s}\) mentioned above, definite physical interpretation cannot be given, while for the behavior of \(\bar{E}_{\bar{R}^s}\) some interpretation is given below. As \(\lambda/\mu\) increases with consequent increase of \(\alpha_{\bar{R}^s}\), the amplitude, normal to the free surface, of the dilatational part of the incident Rayleigh wave increases while both amplitudes, normal and parallel to the free surface, of the dilatational part decrease rapidly with increase of \(z\) owing to the presence of \(e^{-\alpha_{\bar{R}^s}z}\) (see (3b)).
Fig. 2. Variations of energy fluxes $E^x_R$, $E^z_R$, $E_{psc}^x$ and $E_{ssc}^x$ versus $\lambda/\mu$. Br., Kn. and Pil. in the figure denote de Bremaecker’s, Knopoff et al.’s and Pilant et al.’s experimental data respectively.

The reflection of the incident waves, therefore, takes place on the z-surface close to the vertex; these reflected waves are propagated in the medium near the x-surface. This behavior leads to a better generation of the reflected Rayleigh wave ($E^x_R$ increases with $\lambda/\mu$ in Fig. 2); in due course, this behavior leads to a decreases of energy $E_{sc}$ of the scattered body waves which, in the present case, occurs in P waves.

In Fig. 2, a few experimental data are inserted. Four of the data of ($E^x_R$, $E^z_R$) and ($E_{psc}^x$, $E_{ssc}^x$) are cited from page 263 and the abstract of de Bremaecker’s work (1958); the other data of $E^x_R$ are from Knopoff et al.’s (1960) and Pilant et al.’s (1964) works. The transmission and reflection of the Rayleigh wave round corners has been the subject of theoretical studies by Lapwood (1961), Kane and Spence (1963), Hudson-Knopoff (1964) and Mal-Knopoff (1966); the predicted trans-
mission coefficients are consistently less than those obtained by experiments, especially for wedges with small interior angles including 90°. This discrepancy is attributed to the approximated theories in which the effect of diffracted waves from the nearby part of the corner is not definitely taken into account, while in our theory precise mathematical treatment is done for the above diffracted waves. According to Fig. 2, the agreements between experiments and our theory are fairly good; unfortunately, experimental data are very few, since most of the experiments were done for the change of wedge angle instead of $\lambda/\mu$.

In the conclusion of De Bremaecker's paper (1958), he stated that about 50 percent of the energy incident as surface wave is converted into body waves, this being divided approximately equally between P and S. According to our result, his conclusion is true only for $\lambda/\mu$ of the order of 0.515 or plate Poisson's ratio 0.17; de Bremaecker employed an elastic plate of this ratio in his experiment. As seen from Fig. 2, partition of the energy of body waves becomes quite a different one from de Bremaecker's with increase of $\lambda/\mu$: for example, with the total energy of body waves ($\bar{E}_b$)≈0.3, energies of the scattered P and S ($\bar{E}_s^P$ and $\bar{E}_s^n$)≈0.05 and 0.25 respectively for a large $\lambda/\mu$ such as 9.0.

3.3 Phase shifts of transmitted and reflected Rayleigh waves

De Bremaecker (1958) and Pilant et al. (1964) gave experimental data of the phase shift of a transmitted Rayleigh wave; the values of the phase shift obtained by them are 90° (≈0.25 circle; visual observation by de Bremaecker) and 0.208 circle (Pilant et al.). The phase shifts obtained by the above two authors are apparent as explained below. These phase shifts correspond to arg $C_2$ given in (23b) for the transmitted Rayleigh wave. As found in (22a), for the reflected wave, and (23a), for the transmitted one, the expressions $u_R^t$ and $w_R^t$ have a negative sign while the incident waves $u_{inc}^t$ and $w_{inc}^t$ in (3a) both have a positive sign. In order to avoid the above negative signs, new axes ($X_R, Z_R$) (for the reflected wave) and ($X_T, Z_T$) (for the transmitted) are set up on the $x$- and $z$-surfaces as shown in Fig. 3a which we shall call associated coordinates. The displacements of the reflected and transmitted Rayleigh waves on the associated coordinates, $(u_{R}^{ass}, w_{R}^{ass})$ and $(u_{T}^{ass}, w_{T}^{ass})$, are then expressed as

\[
\begin{align*}
(u_{R}^{ass}, w_{R}^{ass}) &= (-u_R^t, w_R^t) \text{ on the } x\text{-surface} \\
(u_{T}^{ass}, w_{T}^{ass}) &= (-w_R^t, u_R^t) \text{ on the } z\text{-surface}
\end{align*}
\]

with the help of (22a) and (23a) respectively, where the first and second expressions in the parentheses denote the displacements along and normal to the free surface on each of the associated coordinates. From (28) and \{(22a), (23a)\}, the displacements $(u_{R}^{ass}, w_{R}^{ass})$ and $(u_{T}^{ass}, w_{T}^{ass})$ on the associated coordinates are found to have the same sign (+); $C_1$ and $C_4$ in the expressions \{(22a), (23a)\} then become the complex reflection and transmission coefficients of the Rayleigh waves on the associated coordinates. The absolute values \{|$C_1/C|$ | $C_2/C$|\} denote the changes of the intensity of the reflected Rayleigh wave to the incident and the
transmitted Rayleigh wave to the incident, respectively; these values therefore have definite physical meaning. However, some attention is required for the physical interpretation of \{\arg C_1, \arg C_2\}. As shown in Fig. 3a, the two associated coordinates are independent of the axes in absolute space. Therefore, the phase shifts \{\arg C_1, \arg C_2\} are only apparent and their physical meaning is very ambiguous. At any rate, the value \arg C_2 (for \lambda/\mu=1) obtained in our computation is 1.409 radian (=0.224 circle; the middle value of de Bremaecker's and Pilant et al.'s). That is to say, the transmitted Rayleigh wave undergoes an apparent phase shift of approximately 90° as stated by de Bremaecker and Pilant et al.

In order to avoid the above-mentioned ambiguity in the phase shifts of the transmitted and reflected Rayleigh waves, some other procedure is considered here. For this purpose, consider the differences between the phases of the dis-
placements \( \{u_R^x, w_R^x\} \) with \( \xi = x \) and \( z \) for the reflected and transmitted Rayleigh waves, respectively, and those of \( \{u_R^{inc}, w_R^{inc}\} \) for the incident Rayleigh wave; these phase differences are defined as

\[
\begin{align*}
\delta \ \text{arg} \ u_R^x &= \text{arg} \ u_R^x - \text{arg} \ u_R^{inc} \\
\delta \ \text{arg} \ w_R^x &= \text{arg} \ w_R^x - \text{arg} \ w_R^{inc}
\end{align*}
\]  

(29)

Variations of the phase lags \( \{\delta \ \text{arg} \ u_R^x, \delta \ \text{arg} \ w_R^x\} \) (ordinate) are shown in Fig. 3b against \( \lambda/\mu \) (abscissa). Inspection of this figure reveals the following: (i) the displacements \( \{u_R^x, w_R^x\} \) of the transmitted Rayleigh wave undergo little phase retardation; it is nearly in phase instead of \( \pi/2 \) which is the value evaluated as the apparent phase shift (see the curves \( \delta \ \text{arg} \ u_R^x, \delta \ \text{arg} \ w_R^x \), (ii) the displacement \( u_R^x \) of the reflected Rayleigh wave is subject to a phase shift of approximately \( \pi \) as seen from the curve \( \delta \ \text{arg} \ u_R^x \), while the displacement \( w_R^x \) undergoes only a slight phase retardation, but the retarded amount is a little larger than that of the transmitted wave (see from the curve \( \delta \ \text{arg} \ w_R^x \)), (iii) the phase lags \( \{\delta \ \text{arg} \ u_R^x, \delta \ \text{arg} \ w_R^x\} \) of the reflected Rayleigh wave increase rapidly as \( \lambda/\mu \) gets small.

Physical interpretations of the above mentioned facts and comparison with other author's results will now be given. A numerical experiment on wave propagation in the elastic quarter space was done by SATO (1972) with the technique of the finite difference method (this work will be referred to as paper S in the following). The input wave in his work is of a type of impulse which is given within a finite dimension on one surface near the vertex. Though his experiment was made for the case of an aperiodic input wave, the results obtained by SATO, will be found to be in good agreement with the facts obtained from Fig. 3b in our work which is the case of periodic wave incidence. According to Figs. 3A and 3B in paper S, the Rayleigh wave \( R_1 \) impinging on the corner along the x-surface with positive \( u \) and negative \( w \) (No. 30 in Fig. 3A) appears on the z-surface nearly in phase, i.e., with positive \( u \) and negative \( w \) (No. 30 in Fig. 3B) (feature (i) obtained in our work). In Fig. 3A of paper S, the displacement \( u \) of the incident Rayleigh wave \( R_1 \) is reflected with a phase shift of \( \pi \) toward positive \( x \), while the displacement \( w \) does not undergo such a phase shift but is nearly in phase, (feature (ii) obtained in our work). The resultant displacement \( u \) of the incident and reflected waves is therefore of a form of \( u \sim \sin kRx \), though the amplitude of the reflected wave is small. As for the feature (iii), the dilatational part of the incident wave is closely related with the variation of \( \{\delta \ \text{arg} \ u_R^x, \delta \ \text{arg} \ w_R^x\} \).

As will be found in Fig. 5b showing the variation of phase of dilatational waves, the emitting point E.P. of these waves is located on the z-surface at a point slightly removed from the corner and waves retrograding from the point E.P. toward the vertex 0 are found. As \( \lambda/\mu \) becomes small, the dilatational part of the incident wave is more extended to the deep inside of the elastic medium, and, as a result, the waves reflected from the z-surface contribute to the generation of the reflected wave with increased retarded phase. Precisely speaking, a few differences are
found between our results and Satô's, but these differences seem to be due to the difference that our theory and Satô's experiment are for periodic and aperiodic waves, respectively.

### 3.4 Directivity of scattered body waves

Variations of the density of energy fluxes, \( \{ \delta \tilde{E}_{\theta}^{P}, \delta \tilde{E}_{\theta}^{S} \} \) (ordinate), given by (27c) are shown in Fig. 4a versus azimuth \( \theta \) (abscissa). This figure reveals that (i) the variation of \( \delta \tilde{E}_{\theta}^{P} \) (P wave) is relatively moderate with only one maximum in the direction of \( \theta \) a little less than \( \pi/4 \), and (ii) the variation of \( \delta \tilde{E}_{\theta}^{S} \) (S wave) has two maxima, more precisely four maxima for large \( \lambda/\mu = 5 \) or 9 if small maxima are taken into account, each pair of the four maxima being respectively in the ranges \( 0 < \theta < \pi/4 \) and \( \pi/4 < \theta < \pi/2 \). Before trying to give physical interpretations for the above features, we show the variation of phase of the scattered P and S body waves at a great distance in Fig. 4b for \( \lambda/\mu = 5.0 \); the phases are expressed by \( \arg u_{\theta} \) (P wave) and \( \arg u_{\theta} \) (S wave) respectively, where \( u_{\theta} \) and \( u_{\theta} \) are given by (24a, b), the factor \( e^{-ikr_1} \) or \( e^{-ikr_2} \) being excluded.

De Bremaecker (1958) evaluated the variation of the scattered energy along a quarter circle (Fig. 10 in his work). His experiment was made for \( \lambda/\mu = 0.515 \) (Poisson's ratio = 0.17); this value corresponds nearly to the middle value of \( \lambda/\mu = 0.1 \) and 1.0 which are used in the presentation of Fig. 4a. De Bremaecker's figure (Fig. 10 in his work) shows clearly two typical features, i.e., the deviation of a maximum of the \( \delta \tilde{E}_{\theta}^{P} \)-curve toward the x-surface, similar to the feature (i) in our work, and the presence of two maxima—more precisely speaking, the first maximum in the range \( 0 < \theta < \pi/4 \) is divided into two crests in the \( \delta \tilde{E}_{\theta}^{S} \)-curve—, similar to the feature (ii) in our work.

For the features (i) and (ii), some physical interpretations are given here. With respect to the feature (i)—the deviation of the maximum of P toward the x-surface—the waves reflected from the z-surface have a decisive influence. As seen in Figs. 5a and 5b concerning the spatial variations of the energy density and phase of the scattered dilatational waves, which will be considered later, a strong wave packet is emitted toward the positive x from the z-surface a little apart from the vertex. As for the feature (ii)—two or four maxima of the scattered S waves—, physical interpretation is not so easy. In order to interpret the aforementioned behavior, Fig. 4b showing the phase variation of the scattered waves are referred to; this figure is given for large \( \lambda/\mu = 5.0 \) where the typical four maxima appear. If the \( \delta \tilde{E}_{\theta}^{S} \)-curve in Fig. 4a (case of \( \lambda/\mu = 5.0 \)) is compared with \( \arg u_{\theta} \)-curve for the scattered S waves in Fig. 4b, it is found that the typical four maxima \( \{ A, B, C, D \} \) of the former correspond to four unsmooth curves \( \{ A', B', C', D' \} \), with discontinuity at \( \theta = \pi/4 \), of the latter. This unsmooth or discontinuous behavior of \( \arg u_{\theta} \) denotes that the four maxima have different wave sources.

To begin with, the discontinuity of \( \arg u_{\theta} \) along \( \theta = \pi/4 \) in Fig. 4b is explained. The phase shift between curves \( B' \) and \( C' \) along \( \theta = \pi/4 \) is found to be about \( \pi \). This phase shift of \( \pi \) is produced by the scattered S waves which are closely related.
with the generated transmitted and reflected Rayleigh waves; the energy of these scattered S waves is concentrated along the zones named breathing zone which extends in the direction of about 45° to the free surface. This breathing zone plays the role of a “reservoir” of energy to the produced immature (still undeveloped) Rayleigh wave through interchanging the energy between the scattered S wave and immature Rayleigh wave; for this behavior, a detailed discussion will be given later in Figs. 6a, b concerning the spatial variations of the energy density and phase of distortional waves. We have now two immature Rayleigh waves (reflected and transmitted) in our model and hence two breathing zones as shown in Fig. 4c. These breathing zones partly overlap, especially along $\theta = \pi/4$. As shown in Fig. 6b (phase variation of S), the phases in two breathing zones on the two sides of the B.B.Z. line, abbreviation for boundary of breathing zone, have a phase shift of $\pi$. This $\pi$-phase shift leads to the vanishing of $\delta E_{sc}^P$.
curve along $\theta = \pi/4$ as illustrated in Fig. 4c. The maxima B and C in Fig. 4a are therefore due to two breathing zones.

An inspection of the phase curve in Fig. 4b shows that the phase curve $D'$, corresponding to the maximum $D$ in Fig. 4a, is more advanced than the phase curve $A'$, corresponding to the maximum $A$. The advancement of the phase curve $D'$ is due to the fact that the waves scattered from the $z$-surface, where the incident waves arrive directly, are more advanced in the direction along the $z$-surface than along the $x$-surface. The above behavior leads to the appearance of the maxima $A$ and $D$ in Fig. 4a.

As for the variation of the phase of $P$ in Fig. 4b, a feature similar to that for $S$ is also found. That is to say, the phase of $P$ is more advanced along the $z$-surface than along the $x$-surface, though the amount is small. For this behavior of $P$, the physical interpretation is similar to that for $S$.

Now, there are actually small upheavals of $\delta E_{pc}$ near $\theta = 0$ and $\pi$, namely, along the free surfaces (designated by the arrow $\uparrow$ in the figure); the amounts are so small that the curves in Fig. 4a do not show the variation. As will be discussed later in section 3.7, these small amounts of $\delta E_{pc}$ are very important in the interpretation of the behavior of the waves along the free surfaces near the corner.

### 3.5 Spatial variations of energy density and phase of scattered waves near the corner

In order to examine the behavior of the scattered waves, the energy density and phase of dilatational and distortional scattered waves near the corner are shown in Figs. 5a, b and Figs. 6a, b respectively for a specified value of $\lambda/\mu = 1$. The energy densities used here are defined as

$$\delta \varepsilon^j_{pc} = \rho \left[ \frac{1}{2} \left( \frac{du^j}{dt} \right)^2 + \left( \frac{dw^j}{dt} \right)^2 \right] dt$$

$$\delta \varepsilon^j_{pc} = \frac{\rho}{2} \left( |u^j|^2 + |w^j|^2 \right)$$  \hspace{1cm} (30a)

$(j = p$ and $s$ for dilatational and distortional waves respectively),

where $\{u^j, w^j\}$ are dilatational and distortional wave components of $\{u_{pc}, w_{pc}\}$ in (18), i.e., $\{u_{pc} = u^p + u^s, w_{pc} = w^p + w^s\}$ and only the real part are considered here. In the presentation of Fig. 5a and 6a, the normalized expressions

$$\delta \varepsilon^j = \delta \varepsilon^j \left( \frac{\rho}{2} \right) = \frac{|u^j|^2 + |w^j|^2}{2}$$  \hspace{1cm} (30b)

are used. On the other hand, the contours of the phase of dilatational and distortional waves in Figs. 5b and 6b are given by arg $\phi$ and arg $\phi$, respectively.

In Figs. 5a and 6a, two zones of high energy density, which are shaded in the figures, run from two free surfaces near the vertex in the direction of 45° to the free surfaces. The two zones in Fig. 5a with respect to dilatational waves
Fig. 5a. Variation of the energy density of the scattered dilatational waves ($\epsilon^p$) in the quarter space for $\lambda/\mu = 1.0$. Numerals along the curves denote the values of $\epsilon^p$.

Fig. 5b. Variation of the phase (broken line) of the scattered dilatational waves ($\arg \phi$) in the quarter space for $\lambda/\mu = 1.0$. E.P. in the figure stands for Emitting Point of the dilatational waves. Arrows denote the directions of wave propagation.
Fig. 6a. Variation of the energy density of the scattered distortional waves ($\delta\varepsilon^s$) in the quarter space for $\lambda/\mu = 1.0$. Numerals along the curves denote the values of $\delta\varepsilon^s$. G.R.W. in the figure denotes the places where the generated Rayleigh waves appear.

Fig. 6b. Variation of the phase (broken line) of the scattered distortional waves (arg $\phi$) in the quarter space for $\lambda/\mu = 1.0$. Numerals along the contours of the phase denote the principal values of arg $\phi$. B.B.Z. stands for Boundary of Breathing Zone.
are actually unified into one zone owing to the nearness of the two; Fig. 5b for
the phase of dilatational waves shows that each one originates on the x- and z-
surfaces as shown respectively by the arrows A and B in the figure. From Figs. 5b
and 6b with respect to phase, the waves are found to advance along these zones
of high energy density. In Fig. 6a (energy density of distortional waves), a
definite separation of two zones (advancing toward 45° direction) are seen; these
two zones are closely related with the generation of the transmitted and reflected
Rayleigh waves. In Fig. 8 showing the displacements along the x- and z-surface,
the amplitudes of the displacement of the scattered waves normal to the free
surface, namely, $|\mu_{sc}|$ on the z-surface and $|\omega_{sc}|$ on the x-surface, tend to those of
the transmitted and reflected Rayleigh waves, respectively, at $z \approx 8$ and at $x \approx 15$.
In Fig. 6a, two zones toward 45° direction originate just below these values of
z and x (designated as G.R.W. or generation of Rayleigh wave in Fig. 6a).
These zones are named breathing zone from their generation process. That is
to say, the still undeveloped immature Rayleigh wave (abbreviated as I.R.W.)
appears first near the free surface and the end of I.R.W. merges into the scat-
tered S waves just under it without extending deep into the interior of the elastic
body, since the influence of the surface condition cannot reach the deep interior
owing to the embryo state of I.R.W. As I.R.W. is leaving away from the corner
of the quarter space, its form grows up to that of an ideal Rayleigh wave, pro-
duced along the free surface without any irregularity, interchanging (breathing)
the energy between I.R.W. and the scattered S waves trapped along the 45° zone.

As for the phase variation of S toward the 45° directions (refer to Fig. 6b),
a conspicuous feature is found that the S waves have a phase difference of $\pi$ along
the boundary of two breathing zones (designated as B.B.Z. in Fig. 6b) and hence
the S waves in the 45° direction offset each other. As leaving away from the
corner, the energy in the breathing zone for the transmitted Rayleigh wave is partly
transmitted to that for the reflected Rayleigh wave as seen from the arg $\phi$-curve.

In Figs. 5a and 6a, zones of high energy density run along the z- and x-
surfaces; these zones denote the generation of the transmitted and reflected
Rayleigh waves.

3.6 Double sources of the scattered waves

As shown in Figs. 5a and 6a, the energy densities of dilatational and
distortional waves are very high near the vertex, around which a kind of resonance
appears. In order to examine the behavior of the resonance, orbital motions of
dilatational and distortional waves are shown in Fig. 7a and Fig. 7b respec-
tively. In these figures, the orbits of dilatational and distortional waves at
$x \approx 0, z = 25$ are also given and as will be found later, these orbits at $x \approx 0, z = 25$
correspond nearly to those of the Rayleigh wave transmitted away along the z-
surface and hence these orbits are simply named O.R on z, i.e., orbit of the dilatational
and distortional wave components of the Rayleigh wave on the z-surface.
In Figs. 7a, b, the scale of O.R. on z is twenty times that of the orbits in other
Fig. 7a. Orbits of the scattered dilatational waves on the free surfaces near the vertex for the specified value of $\lambda/\mu = 1.0$. In this figure, $x$ and $z$ denote $kx$ and $kz$ respectively. The scale of the orbit at $(x=0, z=25)$ is twenty times that at other places. Numerals on the orbits denote a sequence of passing time, i.e., 1: $at=0$, 2: $at=\pi/2$, 3: $at=\pi$ and 4: $at=3\pi/2$ (these conventions are followed in Fig. 7b).

Fig. 7b. Orbits of the scattered distortional waves on the free surfaces near the vertex for the specified value of $\lambda/\mu = 1.0$. In this figure, $x$ and $z$ denote $kx$ and $kz$ respectively. The scale of the orbit at $(x=0, z=25)$ is twenty times that at other places. For the numerals on the orbits, the caption of Fig. 7a should be referred to.

places. In Figs. 7a, b, very strong motions (resonance) are found to occur around the vertex, that is to say, at the very point of the vertex, motions occur which are about twenty or thirty times that of O.R. on $z$. Comparison of Fig. 7a and Fig. 7b reveals that the dilatational and distortional waves around the vertex move nearly with $\pi$-phase difference and that these motions are nearly along a straight line or a very elongated elliptical orbit which is a form completely different from that of O.R. on $z$ which is almost circular. Now, we have arrived at the conclusion that the motion of the dilatational or distortional waves in the nearby part of the vertex is a resonance which is different from the motion of the dilatational or distortional part of the Rayleigh wave. The waves trapped near the vertex, as resonant waves, behave like wave sources for the scattered dilatational and distortional waves; these sources are designated by high energy C in Figs. 5a and 6a and by the arrows C, showing the direction of propagation of waves, in Figs. 5b and 6b. Though the above sources around the vertex are apparent, as being caused by the corner trapping of waves, the real sources run from the vertex to infinity along the $z$-surface due to the arrival of the exponential part of the incident Rayleigh wave; these sources are shown by high energy A in Figs. 5a and 6a and by the arrows A in Figs. 5b and 6b. The
double sources lead to a vortex-like behavior of arg $\phi$ or arg $\psi$ near the vertex in Fig. 5b or 6b.

3.7 Displacement and orbit of scattered waves on the free surfaces

Let us consider first the variations of the amplitudes of the displacements of the transmitted and reflected waves ($u_{sc}, w_{sc}$ in (18)) along both surfaces. These variations are given in Fig. 8 for a specified value of $\lambda/\mu=1$, in which the upper and lower figures denote the variations along the $x$- and $z$-surfaces, respectively. Typical features in these figures are that: (i) the transmitted Rayleigh waves are produced earlier than the reflected Rayleigh waves and (ii) the displacements normal to the free surface, $|w_{sc}|$ on the $x$-surface and $|u_{sc}|$ on the $z$-surface, tend more rapidly to those of Rayleigh waves than the displacements parallel to the free surfaces, $|u_{sc}|$ on the $x$-surface and $|u_{sc}|$ on the $z$-surface. The cause of the feature (i) is attributed to the property of the incident Rayleigh wave. That is to say, the Rayleigh waves incident upon the $z$-surface have a long tail (the exponential part) which arrives at the $z$-surface; the waves arriving at the

![Fig. 8. Variations of the amplitudes of the displacements on the free surfaces near the vertex for $\lambda/\mu=1.0$. The upper and lower figures show the variations along the $x$- and $z$-surfaces, respectively. Arrows (in the horizontal direction) accompanied with $|u_{sc}|$ and $|w_{sc}|$ denote the amplitudes of the displacements ($u$, $w$) of the generated Rayleigh waves along the $x$- (for $\xi=x$) and $z$- (for $\xi=z$) surfaces, respectively.](image)
Fig. 9. Orbits of the scattered waves (resultant of the dilatational and distortional waves) on the free surfaces for $\lambda/\mu = 1.0$. In this figure, $x$ and $z$ stand for $kx$, and $kz$, respectively. Numerals on the orbits denote a sequence of passing time, i.e., 1: $t=0$, 2: $t=\pi/2$, 3: $t=\pi$ and 4: $t=3\pi/2$. 
z-surface are converted there as transmitted Rayleigh waves—effect of the direct arrival of the incident Rayleigh wave. On the other hand, the reflected waves advance along the x-surface and produce the reflected Rayleigh waves being more retarded than the transmitted Rayleigh waves—effect of the secondary arrival of the incident wave. For the feature (ii) obtained above, the cause is attributed to a packet of the P waves being propagated along the free surface; these P waves at a great distance are shown in Fig. 4a by being designated as "P a little up" near $\theta = 0$ and $\pi/2$ (the free surfaces) and considered much stronger in the part near the vertex. These radiating P waves are primarily displacements parallel to the free surface, in the direction of the propagation, and, hence, cause the fluctuating behavior of the amplitude of the displacement parallel to the free surface by superimposing on the generated Rayleigh waves, namely, the feature (ii) revealed above.

Consider next the particle orbits of the reflected and transmitted waves along the x- and z-surfaces, respectively; these variations are shown in Fig. 9. Comparison of the orbits on the x- and z-surfaces shows that the particle orbit of the transmitted wave tends to that of the Rayleigh wave much more rapidly (nearly at $z=10$) than the orbit of the reflected wave does (nearly at $x=20$); a similar feature is also revealed in Fig. 8 showing the amplitude on the free surfaces. Around the vertex of the wedge in Fig. 9, resonance of the resultant wave of dilatational and distortional waves appears. For each of the waves shown in Figs. 7a and 7b, a similar resonance is found around the vertex in a much more remarkable form. The motion of the resultant wave at the vertex runs in the direction of $45^\circ$ to both surfaces. If the orbital motions near the corner of the wedge in our work (Fig. 9) are compared with those in SATO'S work (1972) (the figure for $t=42\Delta t$ or $46\Delta t$ in Fig. 1 in his work), the agreement of the orbital motions is found to be fairly good.

4. Summary and Conclusions

Scattering of the Rayleigh wave in the elastic quarter space is discussed. Important features obtained are described here. Detailed features are given in the text. The treatment used in this work is almost exact (Sec. 3.1).

Among the various energies, the energy of the transmitted Rayleigh wave is the greatest. Energy fluxes of the scattered P and S have typical directivity: the former (P) has a maximum in the direction a little deviated toward the free surface along which the incident wave is entering and the latter (S) has four maxima with a minimum (disappearance) in the direction of $45^\circ$ to the free surface. The four maxima for S are produced by completely different wave sources. Two maxima existing inside among the four and the minimum between these two maxima are caused by breathing zones (Sec. 3.4) which run under the generated Rayleigh waves in the direction of $45^\circ$ to the free surfaces acting as a kind of a reservoir for them. The phases of the scattered S waves in two breathing zones
have $\pi$-phase difference which causes the above minimum of $S$ in the 45° direction. For dilatational and distortional scattered waves, there exists a wave source which extends from the vertex to infinity along the second ($z$-) surface; this wave source is caused by the direct arrival of the exponential part of the incident Rayleigh wave. On the other hand, resonance appears near the vertex of the wedge; this resonance behaves like a wave source for the scattered waves as a result of the trapping of energy. The above wave sources, of which the former is real and the latter apparent, behave as if there existed a double source for the scattered waves. The above double source causes the waves near the vertex to rotate.

As for the transmitted and reflected Rayleigh waves, two interesting features are revealed: the transmitted Rayleigh wave is produced earlier than the reflected Rayleigh wave and the displacements normal to the free surfaces tend more rapidly to those of the Rayleigh wave than the displacements parallel to the free surfaces.

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