QUASISTATIC CRACK EXTENSIONS IN AN INHOMOGENEOUS VISCOELASTIC MEDIUM—PROPOSAL OF A UNIFIED SEISMIC SOURCE MODEL FOR THE DIVERSE EARTHQUAKE RUPTURE PROCESSES

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(Received February 16, 1981; Revised July 1, 1981)

A unified seismic source model is proposed which can explain diverse earthquake rupture processes, an earthquake faulting consisting only of aseismic slip, an aseismic fault slip preceded and/or followed by seismic faulting, mainshock-aftershocks sequence, multiple-rupture event, an earthquake swarm and the like.

We analyse, as a mathematical model, quasistatic extension of longitudinal shear cracks in a viscoelastic medium. A Dugdale model is adopted as a fracture model. An inhomogeneously distributed stress drop is assumed on the crack surface. It is shown that the viscoelastic properties of the medium, the fracture strength at the crack tip and the inhomogeneity of stress drop distribution play important roles in the occurrence of the diversity of earthquake rupture processes.

1. Introduction

Analysis of seismic focal process has been extensively made in this decade with the aid of the elasticity theory of dislocations. The source studies revealed a great diversity of seismic focal process, including slow earthquakes (e.g., KANAMORI and STEWART, 1979), slow precursory fault slip (e.g., KANAMORI and CIPAR, 1974) and multiple-rupture events (e.g., WYSS and BRUNE, 1967). Aseismic faultings are also considered to exist (e.g., FUJII, 1976). It is well known that the occurrence of foreshocks, aftershocks and earthquake swarm has a strong regionality (MOGI, 1963; UTSU, 1970). This diversity may be a manifestation of the regionality of the tectonic state. If the causes that result in the diversity are successfully understood, knowledge of the regionality of tectonic state may be greatly increased. Although a number of specific source models have been proposed to explain individual rupture phenomenon, a seismic source model is now required to account for the diversity of seismic focal process in a unified fashion. If such a source model is successfully constructed, the tectonic field associated with occurrence of earthquakes can be viewed from a unified standpoint.

YAMASHITA (1980b) proposed a seismic source model in which the stress drop and the fracture strength are assumed to be inhomogeneously distributed, and the viscoelasticity is introduced. His paper is hereafter referred to as paper I. The fracture strength stands for a critical crack tip opening displacement, in this paper and in paper I. It was shown that aseismic faultings preceding and/or following seismic faultings and those unaccompanied by any seismic faulting are explained, in a unified fashion, by his model. In this paper it will be shown that the proposed source model can explain, in addition to the above rupture phenomena, other kinds of earthquake rupture phenomena also, including multiple-rupture events, earthquakes accompanied by foreshocks and/or aftershocks, and earthquake swarm. It will also be shown that the diversity of seismic focal process is attributed to the difference in viscoelastic property and/or distributions of stress drop and fracture strength. We confine our attention especially to the stress drop distribution, and we will obtain its distribution associated with the generation of each earthquake rupture phenomenon.

2. Outline of Seismic Source Model

A seismic source model assumed in this paper will be briefly outlined below. This source model is essentially the same as that in paper I. However the analysis is more general in this paper than that in paper I in the sense that general form is assumed for the distribution of stress drop in the present paper. The construction of the source model is much owing to the work by KOSTROV *et al.* (1970).

We consider a straight crack of length $2a$ in an unbounded linearly viscoelastic medium under the condition of antiplane strain (Fig. 1). Since the tectonic field is much disturbed by the generation and the extension of crack, it is reasonable to assume the primary creep to describe the mechanical property of the medium. ITÔ and SASAJIMA (1979) observed in rock experiments that the viscosity of the primary creep is a few orders lower than that of the secondary creep. The secondary creep is associated with the steady accumulation of tectonic stress.

Fig. 1. Antiplane strain Dugdale model. Yielded zones are developed on the segments of the x -axis, $a < |x| < l$. The crack tips are located at $|x| = a$.

Fig. 2. Three-parameter solid and its non-dimensional creep compliance function. μ_0 is the instantaneous rigidity, η is the viscosity, σ_0 is the applied stress and ε is the strain.

The role of the primary creep is therefore very important for earthquake ruptures, which is a phenomenon of very short time-scale compared with usual geological phenomena. The three-parameter solid (Fig. 2) is assumed to model the primary creep. Since our main interest is directed to the quasistatic crack tip extension, the correspondence principle (LEE, 1955) is employed to introduce the viscoelastic property. If the corresponding static elasticity problem is solved, therefore, the solution of the viscoelasticity problem is easily obtained with the aid of integral transform. A Dugdale model (DUGDALE, 1960) is adopted as a fracture model. The Dugdale model is characterized by the existence of yielded zones at the crack tips. In this paper the segments defined by $a < |x| < l$ denote the yielded zones (Fig. 1). In this fracture model the stress is finite around the margin of the yielded zones. The small-scale yielding is assumed in the same manner as in paper I, which may be plausible for many shallow earthquakes. The stress drop on the slip plane of the crack is assumed to be an even function of x .

The solution of the corresponding static elasticity problem is obtained if the distribution of stress drop on the crack surface and the yielding stress is known. The integral transform method (SNEDDON, 1951) or the complex function theory (MUSKHELISHVILI, 1953) may be used as a mathematical tool. The length of yielded zone is obtained as a function of crack length using the condition of finite stress around the margin of yielded zone. Thus in the corresponding static elasticity problem, the displacement discontinuity is generally written in the form (see paper I)

$$[u_z(x)] = \varphi(x, a)/\mu, \quad (2.1)$$

where

$$\varphi(x, a) = \frac{4}{\pi} \int_x^l \frac{r}{\sqrt{r^2 - x^2}} dr \int_0^r \frac{\Delta\sigma(s)}{\sqrt{r^2 - s^2}} ds, \quad (2.2)$$

$\Delta\sigma(s)$ is the stress drop and μ is the rigidity. It should be noted that the elastic constant is not contained in the expression of $\varphi(x, a)$ and that l is a function of a . The function $\varphi(x, a)$ is determined if the stress drop is given in the range $|x| < l$. In Eq. (2.2) only the positive x is considered since $\Delta\sigma(x)$ is an even function of x .

Let the crack with a finite length of $2a_0$ be instantaneously formed at time $t=0$. Since the crack will in general extend for $t>0$, l and a are functions of t . For the displacement discontinuity at time t we obtain

$$[u_z(x)] = \frac{1}{\mu_0} \varphi(x, a(t)) + \int_0^t \dot{\Omega}(t-\tau) \varphi(x, a(\tau)) d\tau, \quad (2.3)$$

using the correspondence principle and Eq. (2.1). Here μ_0 is the instantaneous rigidity and $\dot{\Omega}(t)$ is the derivative of creep compliance function with respect to its argument. If the three-parameter solid, shown in Fig. 2, is assumed, then we have

$$\dot{\Omega}(t) = \exp(-\gamma\mu_0 t/\eta)/\eta, \quad (2.4)$$

where η is the viscosity.

The crack tip opening displacement criterion (COD criterion) is employed as the fracture criterion at the crack tips. In other words if the displacement discontinuity at the crack tip exceeds a certain value δ_c , the crack tip extends. The quantity δ_c is assumed to be a material constant. The fracture strength is defined as δ_c .

We find, from Eq. (2.3), for $t=0$

$$[u_z(x)]_{x=a_0} = \varphi(a_0, a_0)/\mu_0, \quad (2.5)$$

where a_0 is a half of the initial crack length, that is $a_0 = a(0)$. If $[u_z(x)]_{x=a_0}$ exceeds δ_c , the quasistatic solution does not exist and inertial term must be taken into consideration. Remaining within the framework of quasistatic problem, we may state that an instantaneous fracture occurs in such a case. The term, instantaneous fracture, denotes an ordinary fracture which propagates at a speed near shear-wave velocity. The condition of the occurrence of instantaneous fracture at $t=0$ is thus given by the inequality

$$\varphi(a_0, a_0)/\mu_0 > \delta_c. \quad (2.6)$$

If $[u_z(x)]_{x=a_0} < \delta_c$, the crack tip remains non-extending for some time. During this time interval, $a(t) = a_0$ and

$$[u_z(x)]_{x=a_0} = \varphi(a_0, a_0) \left\{ \frac{1}{\mu_0} + \int_0^t \dot{\Omega}(\tau) d\tau \right\}. \quad (2.7)$$

Since $\dot{\Omega}(\tau)$ is positive, the quantity in the brace increases with time. Therefore the crack tip begins to extend at time t_1 defined by the equation

$$\varphi(a_0, a_0) \left\{ \frac{1}{\mu_0} + \int_0^{t_1} \dot{\Omega}(\tau) d\tau \right\} = \delta_c. \quad (2.8)$$

If the inequality

$$\varphi(a_0, a_0) \left\{ \frac{1}{\mu_0} + \int_0^{\infty} \dot{\Omega}(\tau) d\tau \right\} < \delta_c \quad (2.9)$$

is satisfied, the crack tip remains non-extending indefinitely. The above analysis shows that the condition of the occurrence of quasistatic crack tip extension at $t=0$ is given by

$$\gamma \delta_c / (\gamma + 1) < \varphi(a_0, a_0) / \mu_0 < \delta_c. \quad (2.10)$$

When the crack tip extends quasistatically for $t > 0$, an equation

$$\delta_c = \varphi(a(t), a(t)) / \mu_0 + \int_{t_0}^t \dot{\Omega}(t-\tau) \varphi(a(t), a(\tau)) d\tau \quad (2.11)$$

must be satisfied (see Eq. (3.18) in paper I), where t_0 is the time at which $l(t)$ reaches the position where $a(t)$ is situated at time t , that is, $a(t) = l(t_0)$. In the derivation of Eq. (2.11) the assumption is made that the time required for the crack to advance the initial yielded zone $l_0 - a_0$ is far less than the total extending time. WNUK (1971) pointed out that if we confine our attention to the small-scale yielding and to a slowly moving crack, the second term on the righthand side of Eq. (2.11) is approximated as

$$\int_{t_0}^t \dot{\Omega}(t-\tau) \varphi(a(t), a(\tau)) d\tau \simeq \varphi(a(t), a(t)) \int_{t_0}^t \dot{\Omega}(t-\tau) d\tau. \quad (2.12)$$

If the three-parameter solid is assumed, we obtain

$$\int_{t_0}^t \dot{\Omega}(t-\tau) d\tau = \{1 - \exp(-\gamma \mu_0 \Delta / \eta \dot{l})\} / \gamma \mu_0, \quad (2.13)$$

where $\dot{l} = dl/dt$ and Δ is the length of yielded zone at time t , that is, $\Delta = l(t) - a(t)$. Then Eq. (2.11) is finally reduced to

$$\delta_c = \frac{\varphi(a(t), a(t))}{\mu} \left\{ 1 + \frac{1}{\gamma} - \frac{1}{\gamma} \exp(-\gamma \mu_0 \Delta / \eta \dot{l}) \right\}. \quad (2.14)$$

Since $\exp(-\gamma \mu_0 \Delta / \eta \dot{l})$ is always positive, the inequality

$$\gamma + 1 - \gamma \mu_0 \delta_c / \varphi(a(t), a(t)) > 0 \quad (2.15)$$

must be satisfied. Solving Eq. (2.14) for dl/dt , we get

$$\frac{dl}{dt} = - \frac{\gamma \mu_0}{\eta} \frac{\Delta}{\ln(\gamma + 1 - \gamma \mu_0 \delta_c / \varphi(a(t), a(t)))}. \quad (2.16)$$

Since a small-scale yielding is assumed, l is approximately equal to a . The rate of crack tip extension is, therefore, approximately described by Eq. (2.16). If the argument of the logarithm becomes equal to unity, then dl/dt becomes infinite.

This means the occurrence of instantaneous crack tip extension. Therefore the inequality

$$\gamma + 1 - \gamma \mu_0 \delta_c / \varphi(a(t), a(t)) < 1 \quad (2.17)$$

must also be satisfied for the occurrence of quasistatic crack tip extension. Combining the inequalities (2.15) and (2.17), we get

$$\gamma \delta_c / (\gamma + 1) < \delta < \delta_c \quad (2.18)$$

as the condition of quasistatic crack tip extension for $t > 0$, where

$$\delta = \varphi(a(t), a(t)) / \mu_0. \quad (2.19)$$

The inequalities (2.18) are equivalent to (2.10) if $a(t)$ is replaced by a_0 . This means that the inequalities (2.18) are valid not only for $t > 0$ but also for $t = 0$.

3. Summary of the Results Obtained in Paper I and Some Inferences

In paper I we specified the distribution of stress drop in a form

$$\Delta\sigma(x) = \sigma_0(1 - |x|/h) - \sigma^* \{H(|x| - a) - H(|x| - l)\} \quad (3.1.1)$$

or

$$\Delta\sigma(x) = \sigma_0 m^2 / (m^2 + x^2) - \sigma^* \{H(|x| - a) - H(|x| - l)\}, \quad (3.1.2)$$

on the segment of the x -axis, $-l < x < l$, where h , m and σ_0 are positive constants, σ^* is a parameter related to the yielding stress and $H(x)$ is a unit step function. Detailed analysis and discussion were made especially in the case of (3.1.1). The fracture strength δ_c was assumed to be homogeneously distributed or to have a step discontinuity.

When the distribution of stress drop is given by Eq. (3.1.1), $1/\sqrt{\delta}$ and $1/\sqrt{(1+1/\gamma)\delta}$ are expressed as functions of l such as illustrated in Fig. 3. The quantity δ is defined by Eq. (2.19). It was shown in the preceding section that the conditions of instantaneous and quasistatic fractures are $1/\sqrt{\delta_c} > 1/\sqrt{\delta}$ and $1/\sqrt{(1+1/\gamma)\delta} < 1/\sqrt{\delta_c} < 1/\sqrt{\delta}$, respectively: see Eq. (2.18). No fracture occurs in the range $1/\sqrt{\delta_c} < 1/\sqrt{(1+1/\gamma)\delta}$. Therefore when the point $(l(t), 1/\sqrt{\delta_c})$ is situated in regions I, II and III, instantaneous fracture, quasistatic fracture and no fracture occur, respectively, at this instant. The position $x = l(t)$ can be regarded as that of crack tip because of small-scale yielding, which implies $l \approx a$.

When the crack length increases with time, the abscissa can be regarded as a time-axis since $l(t)$ is in general proportional to t . The point $(l(t), 1/\sqrt{\delta_c})$ in Fig. 3 moves to the right as the crack length increases. The movement of this point is illustrated in the figure for three typical cases, in which δ_c is assumed to be homogeneously distributed. Here the largest and the smallest δ_c are assumed in the cases *F3* and *F1* respectively. Since δ_c is homogeneously distributed, the point $(l(t), 1/\sqrt{\delta_c})$ moves in parallel with the abscissa as time increases. The following rupture processes are expected from the inequalities (2.18). In the

Fig. 3. Typical three fracture processes, $F1$, $F2$ and $F3$, in the case when the stress drop distribution is given by Eq. (3.1.1). The solid and broken curves are proportional to $1/\sqrt{\delta}$ and $1/\sqrt{(1+1/\gamma)\delta}$, that is, $(\sigma_0/\mu_0)/\sqrt{(\pi/2)(\sigma^*/\mu_0)(\delta/h)}$ and $(\sigma_0/\mu_0)/\sqrt{(\pi/2)(\sigma^*/\mu_0)(1+1/\gamma)(\delta/h)}$, respectively. The value $\gamma=1$ is assumed.

cases of $F1$ and $F2$, the fractures begin to start a quasistatic extension at point P and grow like the arrows. The quasistatic extension is transformed into the instantaneous one at point $P1$. The instantaneous extension is reverted to the quasistatic one at point $Q1$, which finally stops at point Q . Only the quasistatic extension occurs in the case $F3$. It is seen that the ratio of quasistatic extension to instantaneous one is much higher in $F2$ than in $F1$. The rupture process $F2$ corresponds to a seismic focal process which consists of seismic faulting preceded and followed by aseismic faultings. Only the seismic faulting will be seismologically observed in the case $F1$. The rupture process $F3$ corresponds to fault slip which consists only of aseismic faultings.

As seen from (2.18), the boundaries between regions I, II and III are determined if the form of $\varphi(a(t), a(t))$ and the value γ 's are given. The form of $\varphi(a(t), a(t))$ is determined by the distribution of stress drop: see Eq. (2.2). Therefore it is sufficiently expected that the difference in the distribution of stress drop produces the diversity of rupture processes. Some important examples of the curves $1/\sqrt{\delta}$ and $1/\sqrt{(1+1/\gamma)\delta}$, which may be associated with certain stress drop distribution and γ , are schematically illustrated in Fig. 4. The rupture process illustrated in Fig. 4(a) will correspond to a mainshock-aftershocks sequence if the process time of each quasistatic fracture is somewhat large. This is because the small-scale instantaneous crack tip extensions occur intermittently following the large-scale instantaneous fracture. If the process time of each quasistatic fracture is very short, the rupture process will correspond to a multiple-rupture event since all instantaneous fractures occur in a very short time. The example illustrated in Fig. 4(b) will correspond to an earthquake swarm since there is no predominantly large segment in which the crack tip extends instantaneously. In

Fig. 4. Possible fracture diagrams to explain the following diverse earthquake rupture processes. (a) Mainshock-aftershock sequence or multiple-rupture event. (b) Earthquake swarm or multiple-rupture event. (c) Aseismic faulting accompanied by no seismic faulting. (d) Seismic faulting followed by a large amount of aseismic faulting.

this case also, the rupture process corresponds to a multiple-rupture event if the process time of each quasistatic fracture is very short. In contrast to the multiple-rupture event in Fig. 4(a), the multiple-rupture event in Fig. 4(b) has no predominantly large shock. The examples shown in Figs. 4(c) and 4(d) corresponds to aseismic faultings accompanied by no seismic faulting and preceded by seismic faulting respectively.

If the curve $1/\sqrt{\delta(t)}$ and the quantities γ , η and δ_0 are given, then the rupture mode is immediately determined in the framework of quasistatic theory. The assumption of the above curve and quantities is equivalent to that of rupture mode. The various combinations of $1/\sqrt{\delta(t)}$, γ , η , and δ_0 certainly produce the diversity of rupture processes as shown in Fig. 4. The quantities η , γ and δ_0 will be closely related to physical and chemical properties of the material. The curve $1/\sqrt{\delta(t)}$ is related to the stress drop distribution. In this paper we intend to determine the stress drop distribution, if it exists, corresponding to the

assumed curve of $1/\sqrt{\delta(l)}$. If the specific form is assumed as to the distribution of stress drop, which is the approach adopted in paper I, then the characteristics of rupture mode is unknown until the computation is over. Therefore the method of trial and error must be needed, in that approach, to reveal the relation between the distribution of stress drop and the rupture mode. Since the rupture mode is assumed in this paper, the present approach is more efficient. Since we have not much knowledge concerning δ_e , δ_e is assumed to be homogeneously distributed. IRÔ and SASAJIMA (1979) experimentally obtained the relations $\gamma \approx 3.0$ and $\eta = 10^{17} \sim 10^{18}$ poises with granite under the condition of room temperature and atmospheric pressure. It is suggested, in paper I, that the existence of water may lower the value γ .

4. General Expression of the Distribution of Stress Drop as the Function of $1/\sqrt{\delta(l)}$

We assume a series of adjacent box functions as shown in Fig. 5 to express the distribution of stress drop in a rather general form. Continuous variation of stress drop can be approximately expressed as a superposition of discontinuous phases. Since the stress drop $\Delta\sigma(x)$ is assumed to be an even function of x , its distribution is considered only in the positive part of x . The positive part of the x -axis is divided into the segments $L_0, L_1, \dots, L_i, \dots$, and in each segment the stress drop is assumed to be constant on the crack surface. Here $L_i = (a_i, a_{i+1})$ ($i=0, 1, \dots$) and a_0 coincides with the point $x=0$. We intend to obtain σ_i , assuming a_i and the curve $1/\sqrt{\delta(l)}$. Individual segment is thought to correspond to a tectonic block in which tectonic properties can be approximated to be constant. The yielded zone is assumed to be perfectly contained in the n -th segment, which is justified by the condition of small-scale yielding. Since the crack tip generally extends, n is a function of time t . If it is necessary to emphasize that n is a function of t , $n(t)$ is used instead of n . The function $n(t)$ increases stepwise with time, taking only integral values. The following stress drop is assumed in each segment:

Fig. 5. Stress drop distribution, given by Eq. (4.1), on the segment of x -axis, $0 < x < l$.

$$\begin{aligned}
\Delta\sigma(x) &= \sigma_0 && \text{in the segment } L_0, \\
&= \sigma_0 + \sigma_i && \text{in the segment } L_i \quad (1 \leq i \leq n-1), \\
&= \sigma_0 + \sigma_n - (\sigma_0 + \sigma_n + \sigma^*) \{H(x-a) - H(x-l)\} && \text{in the segment } L_n, \text{ but } x < l.
\end{aligned} \tag{4.1}$$

The sum of σ^* and the initial stress in the segment L_n is regarded as the yielding stress of the medium. If the initial stress is homogeneous, the yielding stress is spatially homogeneous.

The displacement discontinuity in the yielded zone ($a < x < l$) is, from Eqs. (2.1), (2.2) and (4.1),

$$[u_z(x)] = \frac{4}{\pi\mu} [(\sigma_0 + \sigma_n + \sigma^*) \int_x^l \frac{r \sin^{-1}(a/r)}{\sqrt{r^2 - a^2}} dr - \frac{\pi}{2} \sigma^* \sqrt{l^2 - x^2} + I_n], \tag{4.2}$$

where

$$\begin{aligned}
I_n &= \sum_{i=1}^{n-1} \sigma_i \left\{ \int_x^l \frac{r \sin^{-1}(a_{i+1}/r)}{\sqrt{r^2 - x^2}} dr - \int_x^l \frac{r \sin^{-1}(a_i/r)}{\sqrt{r^2 - x^2}} dr \right\} - \sigma_n \int_x^l \frac{r \sin^{-1}(a_n/r)}{\sqrt{r^2 - x^2}} dr \\
&\quad \text{for } n \geq 2, \\
&= -\sigma_n \int_x^l \frac{r \sin^{-1}(a_n/r)}{\sqrt{r^2 - x^2}} dr \quad \text{for } n = 1.
\end{aligned} \tag{4.3}$$

If $n=0$, we obtain $[u_z(x)]$ setting $\sigma_n=0$ and $I_n=0$ in Eq. (4.2).

The condition of finite stress around the margin of the yielded zone gives a relation between l and a , which is of the form

$$a = l \cos \beta_n, \tag{4.4}$$

where

$$\begin{aligned}
\beta_n &= -\frac{\pi}{2} \frac{\sigma_0 + \sigma_n}{\sigma_0 + \sigma_n + \sigma^*} + \frac{\sigma_n}{\sigma_0 + \sigma_n + \sigma^*} \sin^{-1} \frac{a_n}{l} \\
&\quad - \frac{1}{\sigma_0 + \sigma_n + \sigma^*} \sum_{i=1}^{n-1} \sigma_i \left\{ \sin^{-1} \frac{a_{i+1}}{l} - \sin^{-1} \frac{a_i}{l} \right\} \quad \text{for } n \geq 2, \\
&= -\frac{\pi}{2} \frac{\sigma_0 + \sigma_n}{\sigma_0 + \sigma_n + \sigma^*} + \frac{\sigma_n}{\sigma_0 + \sigma_n + \sigma^*} \sin^{-1} \frac{a_n}{l} \quad \text{for } n = 1, \\
&= -\frac{\pi}{2} \frac{\sigma_0}{\sigma_0 + \sigma^*} \quad \text{for } n = 0.
\end{aligned} \tag{4.5}$$

The subscript n in Eq. (4.4) stands for the existence of yielded zone in the segment L_n . If the small-scale yielding ($|\sigma_i|/\sigma^* \ll 1$) is satisfied, the displacement discontinuity at the crack tip is, from Eqs. (4.2), (4.3), (4.4), and (4.5), obtained as

$$[u_z(a)] = \frac{2\sigma_0}{\pi\mu} \frac{\sigma_0}{\sigma^*} \beta_n^{*2}(l) l, \tag{4.6}$$

where

$$\begin{aligned}
\beta_n^*(l) &= B_n^1 + B_n^2, \\
B_n^1 &= \frac{\pi}{2} \left(1 + \frac{\sigma_n}{\sigma_0} \right) - \frac{\sigma_n}{\sigma_0} \sin^{-1} \frac{a_n}{l} \quad \text{for } n \geq 1, \\
&= \frac{\pi}{2} \quad \text{for } n=0, \\
B_n^2 &= \sum_{i=1}^{n-1} \frac{\sigma_i}{\sigma_0} \left(\sin^{-1} \frac{a_{i+1}}{l} - \sin^{-1} \frac{a_i}{l} \right) \quad \text{for } n \geq 2, \\
&= 0 \quad \text{for } n=0, 1.
\end{aligned} \tag{4.7}$$

The quantities of the second order or higher in σ_i/σ^* ($i=0, 1, \dots, n$) are neglected in deriving Eq. (4.6). We finally obtain the equations, with the aid of (2.1), (2.18), (2.19) and (4.6), which describe the boundaries between regions I, II and III. The boundaries between regions I and II, and between regions II and III are described by the equations

$$\frac{2\sigma_0}{\pi\mu_0} \frac{\sigma_0}{\sigma^*} \beta_n^{*2}(l) l = \delta(l), \tag{4.8.1}$$

and

$$\frac{2\sigma_0}{\pi\mu_0} \frac{\sigma_0}{\sigma^*} \beta_n^{*2}(l) l = \frac{\gamma}{1+\gamma} \delta(l), \tag{4.8.2}$$

in the segment L_n , respectively. Equation (4.8.1) is rewritten, in a non-dimensional form, as

$$\frac{\sigma_0}{\mu_0} \sqrt{\frac{\pi}{2}} \frac{\sigma^*}{\mu_0} \frac{\delta}{a_1} = 1 / \left\{ \beta_n^*(l) \sqrt{\frac{l}{a_1}} \right\}. \tag{4.9}$$

The lefthand side of Eq. (4.9) is assumed to be given and to be described by a function $f(l)$, so that

$$f(l) = 1 / \{ \beta_n^*(l) \sqrt{l/a_1} \}. \tag{4.10}$$

The crack is assumed to be nucleated in the segment L_0 at time $t=0$. Now let us consider a situation at some time $t=t^*$. The crack tip exists in the segment $n(t^*)$ at this time, which is regarded as the result of crack tip extension. It is reasonable to assume $f(l)$ in the range $0 < l < a_{n(t^*)}$ because of this extension. We shall determine the stress drop distribution using Eq. (4.10). It should be noted that $f(l)$ is not arbitrarily given for all values of l since we have an additional condition that the stress drop be constant in each segment. The function $f(l)$ can be arbitrarily specified at only the discrete points, $a_1, a_2, \dots, a_{n(t^*)}$. If $f(l)$ is specified at these points, $\sigma_1/\sigma_0, \sigma_2/\sigma_0, \dots, \sigma_{n(t^*)-1}/\sigma_0$ are completely determined. The function $f(l)$ is determined in all the range after the determination of $\sigma_1/\sigma_0, \dots, \sigma_{n(t^*)-1}/\sigma_0$. It will be shown later that $f(l)$ varies rather smoothly in the range $a_i < x < a_{i+1}$ ($i=0, \dots, n(t^*)-1$). The specification of $f(l)$ at only the above points is, therefore, enough to see the general behavior of $f(l)$.

If $f(l)$ is specified at $l=a_k$ ($1 \leq k \leq n(t^*)$), Eq. (4.10) yields

$$f(a_k) = 1 / \left[\sqrt{\frac{a_k}{a_1}} \left\{ \frac{\pi}{2} + \sum_{i=1}^{k-1} \frac{\sigma_i}{\sigma_0} \left(\sin^{-1} \frac{a_{i+1}}{a_k} - \sin^{-1} \frac{a_i}{a_k} \right) \right\} \right] \quad \text{for } k \geq 2, \\ = 1 / \left\{ \frac{\pi}{2} \sqrt{\frac{a_k}{a_1}} \right\} \quad \text{for } k = 1. \quad (4.11)$$

In this derivation β_n in Eq. (4.10) is replaced by β_k . Solving Eq. (4.11) for σ_{k-1}/σ_0 , we obtain

$$\frac{\sigma_{k-1}}{\sigma_0} = \frac{1}{\pi/2 - \sin^{-1}(a_{k-1}/a_k)} \left[\sqrt{\frac{a_1}{a_k}} \frac{1}{f(a_k)} - \frac{\pi}{2} - \sum_{i=1}^{k-2} \frac{\sigma_i}{\sigma_0} \left\{ \sin^{-1} \frac{a_{i+1}}{a_k} - \sin^{-1} \frac{a_i}{a_k} \right\} \right] \\ \text{for } k \geq 3, \quad (4.12.1)$$

$$= \frac{1}{\pi/2 - \sin^{-1}(a_{k-1}/a_k)} \left[\sqrt{\frac{a_1}{a_k}} \frac{1}{f(a_k)} - \frac{\pi}{2} \right] \quad \text{for } k = 2. \quad (4.12.2)$$

Thus the non-dimensional stress drop σ_{k-1}/σ_0 is obtained if a_i ($i=1, \dots, k$), σ_i/σ_0 ($i=1, \dots, k-2$) and $f(a_k)$ are known. The value $f(a_1)$ is always equal to $2/\pi$. If $f(a_2)$ is given, σ_1/σ_0 is determined with the aid of Eq. (4.12.2). The stress drop σ_2/σ_0 is determined with the aid of $f(a_3)$, Eq. (4.12.1) and σ_1/σ_0 determined above. In this manner the non-dimensional stress drop distribution is determined in the range $0 < x < a_k$. If σ_i/σ_0 ($i=0, \dots, k-1$) are obtained like this, $f(l)$ is fully determined at points other than $a_1, a_2, \dots, a_{k-1}$ in the range $0 < l < a_k$, using Eq. (4.10). The rate of crack tip extension is approximated by Eq. (2.16). Using Eqs. (2.1) and (4.6) we obtain

$$\frac{dl}{dt} = - \frac{\gamma \mu_0}{2\eta} \left(\frac{\sigma_0}{\sigma^*} \right)^2 \frac{l \beta_k^{*2}}{\ln [\gamma + 1 - (\pi \gamma \mu_0 \delta_0 / 2 \sigma^* \beta_k^{*2} l (\sigma_0 / \sigma^*)^2)]} \quad (4.13)$$

in the segment $L_k (0 \leq k \leq n(t^*) - 1)$.

In the following sections we assume various kinds of $f(l)$, such as shown in Fig. 4, which seems to be important for the understanding of the diversity of seismic rupture process. The corresponding stress drop distributions are determined using the results obtained in this section.

5. Dependence of the Diversity of Rupture Process on the Stress Drop Distribution

In this section we will obtain the stress drop distributions for some qualitatively different rupture processes. It will be shown that the stress drop distribution is one of factors to cause the diversity of rupture processes.

5.1 Quasistatic crack tip extension

Let us suppose the curves of $1/\sqrt{\delta(l)}$ and $1/\sqrt{(1+1/\gamma)\delta(l)}$ illustrated in Fig. 6(a). These curves are, as stated in the preceding section, assumed to be specified only at the points where the stress drop varies discontinuously. The overall dependence on l is determined after the determination of the stress drop distribution. This procedure is hereafter followed in drawing the curves of $1/\sqrt{\delta(l)}$ and

Fig. 6. Quasistatic crack tip extension and stress drop distribution associated with its occurrence. (a) Assumed curves of $1/\sqrt{\delta}$ and $1/\sqrt{(1+1/\gamma)\delta}$. The uppermost solid curve is $1/\sqrt{\delta}$. The broken curve and the lowermost solid curves are $1/\sqrt{(1+1/\gamma)\delta}$ corresponding to the values $\gamma=3$ and $\gamma=1$, respectively. (b) Stress drop distribution corresponding to the assumed curve of $1/\sqrt{\delta}$. (c) Non-dimensional rate of crack tip extension $(\sigma^*/\sigma_0)^2 dl/d\bar{t}$ as a function of l/a_1 , where $l=l/a_1$ and $\bar{t}=\gamma\mu_0 t/\eta$. Here the values $(\sigma_0/\mu_0)/\sqrt{(\pi/2)(\sigma^*/\mu_0)(\delta_c/a_1)}=0.48$ and $\gamma=1$ are assumed.

$1/\sqrt{(1+1/\gamma)\delta(l)}$. In this and the following figures the non-dimensional forms are adopted for $1/\sqrt{\delta(l)}$ and $1/\sqrt{(1+1/\gamma)\delta(l)}$. It is seen that the crack tips continue to extend if $0.4 < (\sigma_0/\mu_0)/\sqrt{(\pi/2)(\sigma^*/\mu_0)(\delta_c/a_1)} < 0.6$ and $0.5 < (\sigma_0/\mu_0)/$

Fig. 7. Instantaneous fracture followed by a large amount of quasistatic fracture and stress drop distribution related to its occurrence. (a) Assumed curves of $1/\sqrt{\delta}$ and $1/\sqrt{1+1/\gamma}\delta$. Three pairs, ASA1, ASA2 and ASA3, are assumed for each value of γ . Curve parameters are the same as in Fig. 6(a). (b) Stress drop distributions corresponding to the assumed curves of $1/\sqrt{\delta}$. (c) Non-dimensional rate of crack tip extensions as functions of l/a_1 , where l and $\bar{l}$ are the same as in Fig. 6(c). Here $(\sigma_0/\mu_0)/\sqrt{(\pi/2)(\sigma^*/\mu_0)(\delta_0/a_1)}$ is assumed to take values 4.4, 2.9 and 1.4 in the cases ASA1, ASA2 and ASA3, respectively, and $\gamma=1$ is assumed in all cases.

$\sqrt{(\pi/2)(\sigma^*/\mu_0)(\delta_0/a_1)} < 0.6$ are satisfied in the cases of $\gamma=1$ and $\gamma=3$ respectively. The obtained stress drop distribution is illustrated in Fig. 6(b). The curve dI/dt is shown in Fig. 6(c) in a non-dimensional form, which indicates that the rate of

crack tip extension converges to a constant value with time.

5.2 Instantaneous fracture followed by a large amount of quasistatic fracture

Three examples are assumed as to a pair of the curves $(1/\sqrt{\delta}, 1/\sqrt{(1+1/\gamma)\delta})$ in Fig. 7(a). If $3.6 < (\sigma_0/\mu_0)/\sqrt{(\pi/2)(\sigma^*/\mu_0)(\delta_c/a_1)} < 5.2$, $2.4 < (\sigma_0/\mu_0)/\sqrt{(\pi/2)(\sigma^*/\mu_0)(\delta_c/a_1)} < 3.4$ and $1.2 < (\sigma_0/\mu_0)/\sqrt{(\pi/2)(\sigma^*/\mu_0)(\delta_c/a_1)} < 1.6$ are satisfied in the cases *ASA1*, *ASA2* and *ASA3* respectively, and if $\gamma=1$ is assumed, then the quasistatic extension follows the instantaneous fracture in each case. If γ becomes larger than unity, the permissible range of the value $(\sigma_0/\mu_0)/\sqrt{(\pi/2)(\sigma^*/\mu_0)(\delta_c/a_1)}$ becomes narrow. The stress drop distributions associated with the curves $1/\sqrt{\delta(l)}$ in Fig. 7(a) are shown in Fig. 7(b). It is noteworthy that the stress drops rapidly decrease beyond the point $x/a_1=1.0$ and they turn negative near the point $x/a_1=1.4$. The existence of the region of negative stress drop will be needed to arrest the instantaneous fracture. The absolute value of the stress drop gradually decreases beyond the point $x/a_1=1.5$ in each case. The change of $\Delta\sigma/\sigma_0$ in this region is the counterpart of that in Fig. 6(b), although the sign of $d\Delta\sigma(x)/dx$ is the reverse. If the curve $1/\sqrt{\delta(l)}$ is in parallel with the x -axis, and δ_c and γ take suitable values, then the quasistatic crack tip extension continues indefinitely. The curve $1/\sqrt{\delta(l)}$ of this kind will be associated with a gradual change in the stress drop distribution such as shown in Figs. 6(b) and 7(b). The rate of crack tip extension is illustrated in Fig. 7(c). The rate converges to a constant value after the instantaneous fracture.

5.3 Instantaneous fracture followed by a number of small-scale instantaneous and quasistatic fractures

Three examples are assumed for a pair of curves $(1/\sqrt{\delta}, 1/\sqrt{(1+1/\gamma)\delta})$ in Fig. 8(a). If the inequalities $4.0 < (\sigma_0/\mu_0)/\sqrt{(\pi/2)(\sigma^*/\mu_0)(\delta_c/a_1)} < 4.7$, $2.3 < (\sigma_0/\mu_0)/\sqrt{(\pi/2)(\sigma^*/\mu_0)(\delta_c/a_1)} < 2.6$ and $0.4 < (\sigma_0/\mu_0)/\sqrt{(\pi/2)(\sigma^*/\mu_0)(\delta_c/a_1)} < 0.7$ are satisfied in the cases *AF1*, *AF2* and *AF3* respectively, and if $\gamma=1$ is satisfied, a number of small instantaneous and quasistatic fractures then follow the large instantaneous fracture. The small instantaneous and quasistatic fractures occur alternately. The stress drop distributions corresponding to the examples *AF1*, *AF2* and *AF3* are illustrated in Fig. 8(b). The stress drop has comblike distribution beyond the point $x/a_1=1.0$. This kind of distribution will be needed to cause the small intermittent instantaneous fractures. The amplitude of the discontinuity in the distribution of $\Delta\sigma(x)/\sigma_0$ is smaller for larger $1/\sqrt{\delta}$. The rate of crack tip extension is shown in Fig. 8(c).

5.4 Swarm-type rupture

The rupture processes in Fig. 8(a) are regarded as swarm-type ruptures if the process time of each quasistatic fracture is somewhat large. It is seen that there is no predominantly large instantaneous fracture. The process time of quasistatic fracture increases as γ and/or η increases (see paper I). Both the rupture processes in Figs. 8(a) and 9(a) are regarded as multiple-rupture events if the process time

Fig. 8. Instantaneous fracture followed by a number of small-scale instantaneous and quasistatic fractures, and the stress drop distribution related to its occurrence. (a) Assumed curves for $1/\sqrt{\delta}$ and $1/\sqrt{(1+1/\gamma)\delta}$. Three pairs, AF1, AF2 and AF3, are assumed for each value of γ . Curve parameters are the same as in Fig. 6(a). (b) Stress drop distributions corresponding to the assumed curves for $1/\sqrt{\delta}$. (c) Non-dimensional rate of crack tip extensions as functions of l/a_1 , where l and $\bar{t}$ are the same as in Fig. 6(c). Here $(\sigma_0/\mu_0)/\sqrt{(\pi/2)(\sigma^*/\mu_0)(\delta_0/a_1)}$ is assumed to take the values 4.7, 2.7 and 1.1 in the cases AF1, AF2 and AF3, respectively, and $\gamma=1$ is assumed in all cases.

Fig. 9. Swarm-type fracture and the stress drop distribution related to its occurrence. (a) Assumed curves for $1/\sqrt{\delta}$ and $1/\sqrt{(1+1/\gamma)\delta}$. Three pairs, *SW1*, *SW2* and *SW3*, are assumed for each value of γ . Curve parameters are the same as in Fig. 6(a). (b) Stress drop distributions corresponding to the assumed curves of $1/\sqrt{\delta}$. (c) Non-dimensional rate of crack tip extensions as functions of l/a_1 , where l and $\bar{l}$ are the same as in Fig. 6(c). Here $(\sigma_0/\mu_0)/\sqrt{(\pi/2)(\sigma^*/\mu_0)(\delta_0/a_1)}$ is assumed to take the values 0.8, 0.96 and 1.1 in the cases *SW1*, *SW2* and *SW3*, respectively, and $\gamma=1$ is assumed in all cases.

of each quasistatic fracture is very short. The stress drop distributions associated with curves $1/\sqrt{\delta}(l)$ in Fig. 9(a) are shown in Fig. 9(b). The difference between in Fig. 8(a) and in Fig. 9(a) lies in the fact that there is no predominantly large segment in Fig. 9(b). The rate of crack tip extension is shown in Fig. 9(c).

The analyses in this section show the following facts. The crack tip extension is accelerated and instantaneous extension occurs in the segment of positive

high stress drop. When the crack tip extends into the segment of negative or small stress drop, the instantaneous fracture is sometimes transformed into the quasistatic fracture. The analyses also proved the existence of the stress drop distribution corresponding to the curves $1/\sqrt{\delta(l)}$ such as shown in Fig. 4.

6. Discussion and Conclusions

In our earthquake source model aftershocks occur only at the ends of fault plane (e.g., see Fig. 8(a)). This is closely related to the fact that the crack surface has a one-dimensional extent. If our source model is successfully applied to rupture propagation having a two-dimensional crack surface, it is easily inferred that aftershocks do not necessarily occur only at the ends of the fault plane of the main shock. This is because of the fact that the encounter with an unbreakable barrier does not necessarily arrest the propagation of rupture front, but the rupture front can detour the barrier. This barrier corresponds to a region of low or negative stress drop, in our case. The detouring is peculiar to the rupture propagation having a two-dimensional crack surface. If the crack surface has a two-dimensional extent, some regions will possibly be left unbroken in the main-shock crack surface. If no fracture or quasistatic fractures occur, after a time, in these unbroken regions, aftershocks will not be seismologically observed. If the high

Fig. 10. A possible pattern of rupture propagation in the case when the rupture area is two-dimensional. This is inferred from our source model, in which the rupture area is one-dimensional. Instantaneously and quasistatically fractured regions are expressed by the distributions of large and small dots, respectively. Unbroken regions are expressed by blank ones. The regions I_0 , I_1 , I_2 , I_4 , I_5 and I_7 are instantaneously fractured regions, and E is the nucleation point of the instantaneous fracture in region I_0 .

stress drop areas partly exist in the unbroken regions, however, the instantaneous fractures can be excited by the quasistatic crack tip extension into the unbroken regions. This kind of instantaneous fractures correspond to aftershocks occurring in the main-shock fault plane. This rupture process is schematically illustrated in Fig. 10.

The fracture initiates at the point E and instantaneous crack tip extension occurs over the region I_0 . Three regions I_1 , I_2 and I_3 are left unbroken. This is because the stress drop is considerably lower in I_1 , I_2 and I_3 than in I_0 . In regions I_1 , I_2 and I_3 , the quasistatic fractures extend toward the center of these regions. The generation of quasistatic fracture after the instantaneous fracture was shown to be possible, in our source mode, under suitable conditions. The high stress drop areas are assumed in the central parts of regions I_1 and I_2 , so that instantaneous fractures are excited there by the arrival of quasistatically extending rupture fronts. These instantaneous fractures are observed as aftershocks. The instantaneous fracture in region I_4 occurs near the end of main-shock fault plane. This fracture is caused by the arrival of quasistatically extending rupture front which initiated at the border of the main-shock fault plane. The same kind of fracture occurs in region I_5 . The regions I_6 and I_7 are left unbroken at the time of instantaneous fracture in I_5 . The instantaneous fracture in region I_7 will be seismologically regarded as a secondary aftershock.

If the instantaneous fractures in I_1 , I_2 , I_4 , I_5 and I_7 immediately follow the main shock in I_0 , the event may be seismologically observed as a multiple-rupture event. Whether the event is observed as multiple-rupture event or as mainshock-aftershocks sequence is, as stated previously, dependent on the process time of quasistatic rupture.

The source model developed in paper I and the present paper is based on the quasistatic theory. The fracture mode changes on the boundary between regions I, II and III in the framework of the quasistatic theory. A possibility was suggested in paper I that the instantaneous fracture may overshoot a point on the boundary if the inertia term is taken into account. In the present paper we will show another possibility that the instantaneous fracture makes no overshooting. This possibility is suggested from the analysis of semi-infinite crack in an elastic medium in which the energy balance fracture criterion is used. The analysis shows that the stopping positions of crack tip expected from the static and dynamic elasticity theories are the identical when the stress drop inhomogeneity is the cause to arrest the crack tip propagation. HUSSEINI *et al.* (1975) studied the unsteady high-speed propagation of antiplane strain semi-infinite crack in an elastic medium. They assumed the stress drop distribution in the form

$$\Delta\sigma(x) = T_0[H(x+a) - H(x-b)], \quad (6.1)$$

where T_0 , a and b are positive constants, and they assumed a constant fracture energy ν_0 . A semi-infinite crack was assumed to appear along the negative x -axis at $t=0$. The stopping position of crack tip is described as

$$x = \frac{(a+b)^2 T_0^2}{2\pi\mu\nu_0} + \frac{b-a}{2} + \frac{\mu\pi\nu_0}{8T_0^2} \quad (\geq b). \quad (6.2)$$

This is also obtainable from the static elasticity theory. If we use the theory of continuous dislocations (e.g., YOKOBORI, 1964), the gradient of displacement discontinuity is obtained in the form

$$\frac{d}{dx}[u_z(x)] = \frac{2}{\pi\mu} \frac{T_0}{\sqrt{q-x}} P \int_{-a}^b \frac{\sqrt{q-\xi}}{\xi-x} d\xi, \quad (6.3)$$

where the crack surface exists in the range $x \leq q (q \geq b)$ along the x -axis. The symbol P denotes the employment of the Cauchy principal value. The stress intensity factor is, therefore,

$$K = 2T_0 \sqrt{\frac{2}{\pi}} \{\sqrt{q+a} - \sqrt{q-b}\}. \quad (6.4)$$

If we consider the energy balance, the equality

$$K^2/2\mu = 2\nu_0 \quad (6.5)$$

must hold at the crack tip. Since K is a monotonically decreasing function of q , the stopping position is determined by Eq. (6.5). Solving Eq. (6.5) for q , we get Eq. (6.2). Thus it is proved that the same stopping position is obtained in the static and dynamic elasticity theories when the stress drop inhomogeneity is the cause to arrest the crack tip propagation and the energy balance fracture criterion is used. It would not be unreasonable to expect that the same conclusion is obtained in the case when the COD fracture criterion is used for the crack propagation in an elastic medium. It is also expected that the high-speed fracture gets much resistance to its propagation in a viscoelastic medium than in an elastic medium. If these expectations are true, the instantaneous fracture does not overshoot a point on the boundary between regions I and II since the cause to arrest the crack tip extension is the stress drop inhomogeneity in the present paper.

In this paper we did not treat the phenomenon of foreshocks. However this phenomenon can be explained in a manner the same as the aftershocks. If the stress drop has a comblike distribution near an earthquake nucleation point and if a high stress drop region, extending broadly, exists outside the nucleation region, then foreshocks can be observed. It is also easily inferred that the occurrence of aseismic slip prior to seismic faulting becomes easy when the stress drop make a gradual change, such as shown in Figs. 6(b) and 7(b), near an earthquake nucleation point.

The conclusion is obtained in this paper and paper I that the diversity of earthquake rupture process is closely related to the distributions of stress drop and fracture strength, and to the mechanical property of the medium. Since we confined our attention to the quasistatic crack extension in this paper and paper I, we did not investigate the diversity in the framework of the high-speed fracture

itself. However the same conclusion would be obtained in the study of high-speed fracture also. It is theoretically known that the rupture process of high-speed fracture has a diversity (e.g., MADARIAGA, 1977; YAMASHITA, 1980a). Madariaga showed that considerable high-frequency waves are radiated if the specific fracture energy is discontinuously distributed. Yamashita showed that slow earthquakes and multiple-rupture events can occur provided the stress drop and/or the specific fracture energy are inhomogeneously distributed. In their papers the specific fracture energy is a quantity corresponding to the fracture strength. Therefore the diversity is related to the distribution of stress drop and high-speed fracture.

Finally the meaning of the term "unified model" is touched upon. In each rupture phenomenon the occurrence of a series of ruptures is assumed to be restricted to a single fault plane. For example all aftershocks occur on the fault plane the same as that of the mainshock, and earthquake swarm consists of small ruptures occurring on a single fault plane. Our source model is a "unified model" for earthquake ruptures occurring on a single fault plane.

The author is grateful to Prof. Takuo Maruyama of Earthquake Research Institute, the University of Tokyo, and Dr. Tsutomu Sasatani of Hokkaido University for their valuable suggestions and advice.

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