STABILITY AND INSTABILITY OF INTERACTIVE SHEAR CRACKS—GENERATING MECHANISM FOR SLOW EARTHQUAKES AND FAULT CREEPS

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The stability and instability of two collinear shear cracks were investigated theoretically. When instability occurs in two pre-existing cracks, the cracks coalesce spontaneously and a large-scale crack is formed. Many seismological observations suggest the occurrence of this kind of rupture in the earth. Specific fracture energy or stress-drop is assumed to be inhomogeneously distributed. It is shown that the mode of coalescence has great diversity and is sensitive to the distribution of stress-drop or specific fracture energy. This diversity will be closely related to the diversity of earthquake rupture processes.

In this paper we specifically investigate the mechanism of generation of slow earthquakes and fault creeps along the San Andreas fault system. Our source model developed in the present paper satisfactorily explains observations for the above rupture phenomena. Our source model also explains, in a unified fashion, the existence of two distinct types of fault creeps, that is, stable continuous creep and propagating episodic creep, along the San Andreas fault system.

1. Introduction

In a series of papers, Nemat-Nasser, Keer, Sumi and their co-workers (NEMAT-NASSER, 1978; NEMAT-NASSER *et al.*, 1978, 1980; KEER *et al.*, 1979; SUMI *et al.*, 1980a, b) have studied the growth regime of a system of two-dimensional interactive cracks. They showed that cracks grow stably with an increase of stress when they are far apart and the interaction is weak. When the crack spacing and the crack length are of the same order of magnitude, the cracks become highly interactive and the stability of the system is lost: some cracks stop as others grow spontaneously.

SEKIYA (1976), EVISON (1977) and OHTAKE (1976) have found the precursory earthquake swarm whose epicenters are distributed widely in the eventual after-shock area and its surroundings. Precursory swarm is often followed by a period of abnormal quiescence which lasts until the onset of the major event. MOGI (1968), KELLEHER and SAVINO (1975), ISHIDA and KANAMORI (1980) and others showed that the seismic activity immediately before a large earthquake tends to cluster around the epicenter of the eventual main shock. WYSS *et al.* (1981)

indicated the existence of a preslipped large fault plane surrounding the epicenter of the eventual main shock.

These seismological observations suggest the existence of pre-slipped faults around the hypocenter of the main shock immediately before its occurrence. If the faults are so densely distributed as to be interactive, their interaction will play a role in the process of generation of main shock. It is inferred, on the basis of the analysis done by Nemat-Nasser and others, that high interaction makes the system of pre-slipped faults unstable and a large seismic event will be excited. The excited seismic event will grow dynamically, uniting pre-slipped faults, since the fracture resistance will be weak along the pre-slipped faults.

Nemat-Nasser and others specifically studied thermally induced interactive tension cracks in a homogeneous elastic solid. They treated the problem as a strain-controlled problem, in which no external load is applied and the accumulation of strain energy is associated with thermal processes in the medium. We also treat the problem as a strain-controlled problem. Usual earthquake ruptures in the earth can also be treated as a strain-controlled problem since the accumulation of tectonic stress will be related to some kind of thermal process in the earth. The tectonic stress is regarded as internal stress in a strain-controlled problem. SAVAGE (1969) showed the importance of the concept of internal stress for the energy balance of fault motions in a medium surrounded by a free surface, such as the earth, and also that the initial stress associated with earthquake occurrence should be regarded as an internal stress. The tectonic initial stress, regarded as internal stress, will be very inhomogeneous since there will exist many internal-stress origins in the shallow part of the earth. The sliding frictional stress and the specific fracture energy will also be inhomogeneously distributed here. It is, therefore, sufficiently reasonable to assume inhomogeneous stress-drop on a fault surface. YAMASHITA (1980b, 1981) has pointed out the importance of inhomogeneous stress-drop for the diversity of the earthquake rupture process.

In this paper we investigate the instability of interactive shear cracks in inhomogeneous elastic solids, in which stress-drop on crack surfaces or critical value of stress-intensity factor is inhomogeneously distributed. The critical value of the stress-intensity factor and the stress-drop on the crack surface are the most fundamental parameters in static fracture mechanics if Griffith's fracture criterion is employed (e.g., RUDNICKI, 1980).

To simplify the problem we consider the instability of two collinear plane strain shear cracks. When instability occurs, the two cracks coalesce spontaneously. It will be shown that the mode of the instability has great diversity in the inhomogeneous media. To be more concrete, the extent of the spontaneously fractured region between the two cracks, the coalescence velocity of the two cracks and so on are highly dependent on the distributions of stress-drop and critical value of the stress-intensity factor. This dependence will be closely related to the diversity of the earthquake rupture processes.

The mechanisms of generation of the fault creep along the San Andreas fault

system and the slow earthquake are specifically studied in detail in terms of the source model developed in this paper. It will be shown that our source model satisfactorily accounts for observations of these ruptures. Our source model explains, in a unified manner, the existence of two distinct types of fault creeps, stable continuous creep and propagating episodic creep, along the San Andreas fault.

2. *Stability and Instability of Isolated Crack*

Assuming individual cracks are not yet sufficiently grown and are far apart, each crack can be treated as being independent. Initially, it is therefore meaningful to study the stability and instability of isolated cracks. The stability analysis of isolated cracks is very simple and is found in some literature (e.g., RICE, 1968). Let us assume a straight crack of length $2a$ in the infinite xy -plane (Fig. 1). A plane strain shear crack is assumed here. For simplicity the distributions of stress-drop, $\Delta\sigma(x)$, and critical value of stress-intensity factors, $K^c(x)$, are assumed to be even functions of x . The fracture criterion at the crack tip is described in terms of the stress-intensity factor as follows (e.g., RUDNICKI, 1980):

$$K(a) = K^c(a), \quad (2.1)$$

where $K(a)$ is the stress-intensity factor for the crack tips at $|x|=a$.

The necessary and sufficient conditions for stability and instability of the equilibrium state, (2.1), are

$$dK(a)/da < dK^c(x)/dx|_{x=a}, \quad (2.2)$$

and

$$dK(a)/da > dK^c(x)/dx|_{x=a}, \quad (2.3)$$

respectively. Although Nemat-Nasser and others assumed a constant K^c , the same method of analysis can be employed for the analysis of stability and instability whether K^c is constant or inhomogeneous. Equation (2.1) is obtained by setting as equal to zero the first variation of the total potential energy of the system. The stability and instability conditions, (2.2) and (2.3), are obtained by examining the sign of the second variation of the total potential energy. Refer to the paper by NEMAT-NASSER (1978) for details.

First the effect of the inhomogeneity in the distribution of $\Delta\sigma(x)$ is investigated, and $K^c(x)$ is assumed to be constant, i.e., $K^c(x) = K^c(\text{const.})$. Assume the stress-drop in the form

$$\Delta\sigma(x) = \sigma_0 - \mu x^2/r^2 \quad (2.4)$$

on the crack surface, $|x| < a$, where μ is the rigidity, and σ_0 and r are constants having the dimensions of stress and length, respectively. In Eq. (2.4) the fracture-resistance is assumed to be low near $x=0$ so that the crack is expected to be nucleated near $x=0$. The stress-drop (2.4) represents the difference between

tectonic initial stress and final stress on the crack surface. Since the final stress on the crack surface will not vary much with time, the stress-drop will increase with the accumulation of tectonic stress. Hence it will be reasonable to assume that σ_0 increases with the accumulation of tectonic stress. Nemat-Nasser and others call the quantity such as σ_0 *load parameter*.

The stress-intensity factor for the crack tips at $|x|=a$ is

$$K(a) = \sqrt{\pi a} [2\sigma_0 - \mu(a/r)^2]/2. \quad (2.5)$$

This is obtained upon following the method outlined in Appendix 1. The equation

$$\sqrt{\pi a} [2\sigma_0 - \mu(a/r)^2]/2 = K^c \quad (2.6)$$

and the inequality

$$\sigma_0 - 5\mu(a/r)^2/2 < 0 \quad (2.7)$$

must be satisfied during stable crack growth (see Eqs. (2.1), (2.2) and (2.5)). If the left-hand side of (2.7) becomes positive, instability occurs and the crack grows spontaneously. The state where the left-hand side of (2.7) vanishes is called the *critical state* (NEMAT-NASSER, 1978). The crack length corresponding to the critical state is $2a_c = 2r\sqrt{2\sigma_0/5\mu}$, which is hereafter referred to as *critical crack length*.

The curves described by Eq. (2.6) are drawn as the dependence of σ_0 on a in Fig. 1. Two typical cases, high and low K^c , are considered. Unstable and stable branches are drawn by broken and solid curves, respectively. Let us consider a pre-existing crack with length $2a_0$, where the stress-intensity factor is assumed, at first, to be lower than its critical value. First we assume the high- K^c curve in Fig. 1. As σ_0 increases, the stress-intensity factor becomes higher and

Fig. 1. Isolated crack and curves along which the fracture condition (2.6) is satisfied.

Eq. (2.6) is first satisfied at point A . This point is located on the unstable branch so that the crack tip extends spontaneously along the segment AB . The crack tip extends as far as point B in the framework of the static theory. Further stress increase makes the crack tip extend stably along the stable branch. In the above case the length of the pre-existing crack is assumed to be shorter than the critical crack length, i.e., $a_0 < a_c$. If a_0 is larger than a_c , Eq. (2.6) is first satisfied on the stable branch so that only stable growth occurs. In fact this situation occurs in the case of the low- K^c curve in Fig. 1. The fracture condition is first satisfied at point A' on the stable branch. It is also seen in Fig. 1 that the fracture condition is satisfied at the lowest value of σ_0 for each curve when the length of the pre-existing crack is equal to the critical crack length, i.e., $a_0 = a_c$. It is concluded from the above considerations that the probability of the occurrence of spontaneous extension decreases as K^c decreases. This is because the projected length of the unstable branch on the a -axis decreases as K^c decreases. If K^c is zero, there is no unstable branch.

Next let us briefly consider the effect of inhomogeneity in the distribution of K^c . The stress-drop is assumed to be homogeneously distributed, i.e., $\Delta\sigma(x) = \sigma_0$ (const.). We obtain the stress-intensity factor, setting $r \rightarrow \infty$ in Eq. (2.5), in the form

$$K(a) = \sigma_0 \sqrt{\pi a}, \quad (2.8)$$

so that

$$dK(a)/da = \sigma_0 \sqrt{\pi}/2\sqrt{a}. \quad (2.9)$$

If the equality $K(a) = K^c(a)$ is satisfied and $dK^c/dx|_{x=a}$ is greater than the righthand side of Eq. (2.9), then the crack grows stably. It should be noted that there is no stable branch in the case when both K^c and $\Delta\sigma(x)$ are homogeneously distributed.

As shown above the values of K and K^c and their derivatives are required for the analysis of stability. Therefore both the assumed distribution and its derivative immediately influence the problem when $K^c(x)$ is assumed. On the other hand when $\Delta\sigma(x)$ is assumed, its derivative will not immediately influence the problem since the stress-intensity factor is expressed as an integral of $\Delta\sigma(x)$ (see Appendix 1). The model-dependence of analysed results is, therefore, much stronger in the case of K^c -assumption than in the case of $\Delta\sigma$ -assumption. A too-detailed consideration of analysed results is, then, meaningless in the former case.

3. Stability and Instability of Two Collinear Cracks

As stated in the introduction a number of faults will be in the state of interaction around the hypocenter of a main shock before its occurrence. It will therefore be unavoidable to consider many interactive cracks in the physical

Fig. 2. Two collinear cracks.

consideration of the mechanism of generation of main shock. However, the analysis becomes extremely complicated if a great number of cracks are assumed. In this paper we study the problem of two collinear cracks, which may be the most elementary problem in crack interaction.

The cracks are assumed to exist on the segments of the x -axis, $a \leq |x| \leq b$, in the infinite xy -plane (Fig. 2). These are plane strain shear cracks. For simplicity $\Delta\sigma(x)$ and $K^c(x)$ are assumed to be even functions of x so that the analysis of the half-plane $x \geq 0$ is sufficient. The fracture criteria at the crack tips are written as

$$K_a = K^c(a) \quad \text{and} \quad K_b = K^c(b), \quad (3.1)$$

where K_a and K_b are stress-intensity factors for the inner and outer crack tips at $x=a$ and b , respectively. When Eqs. (3.1) are satisfied, the necessary and sufficient conditions for stability are written in the forms,

$$\begin{aligned} \partial K_a / \partial a > dK^c(x)/dx|_{x=a} \quad \text{and} \quad \partial K_b / \partial b < dK^c(x)/dx|_{x=b} \\ \text{if} \quad \partial K_a / \partial b = -\partial K_b / \partial a < 0, \end{aligned} \quad (3.2)$$

and

$$\begin{aligned} \partial K_a / \partial a > dK^c(x)/dx|_{x=a}, \quad \partial K_b / \partial b < dK^c(x)/dx|_{x=b} \quad \text{and} \\ (\partial K_a / \partial b)^2 + (\partial K_a / \partial a - dK^c(x)/dx|_{x=a})(\partial K_b / \partial b - dK^c(x)/dx|_{x=b}) < 0 \\ \text{if} \quad \partial K_a / \partial b = -\partial K_b / \partial a > 0. \end{aligned} \quad (3.3)$$

As stated in the preceding section the stability conditions (3.2) and (3.3) are obtained upon examining the sign of the second variation of the total potential energy. Reference to the paper by NEMAT-NASSER (1978) are recommendable for details.

If the two collinear cracks grow stably for a while, a high fracture-resistant zone must exist between the two cracks. The high fracture-resistant region is mathematically described as zones having high- K^c and/or low- $\Delta\sigma$ values. A low stress-drop makes for a low stress-intensity factor (e.g., see Eq. (2.8)) so that a low $\Delta\sigma$ makes fracture-resistance high. It is essential to assume the inhomogeneity in the distribution of $\Delta\sigma$ or K^c if a stable phase exists in the growth of the cracks.

3.1 $\Delta\sigma$ -inhomogeneity

In this subsection we consider the inhomogeneity in the distribution of $\Delta\sigma$, and K^c is assumed to be constant. This model is hereafter referred to as the $\Delta\sigma$ -inhomogeneity model. We assume the following stress-drop distribution

$$\Delta\sigma(x) = -(\sigma_0 - c)[(x/x_a)^2 - 1]^2 + \sigma_0, \quad \sigma_0 - c > 0, \quad x_a > 0 \quad (3.4)$$

on the crack surfaces, $a < |x| < b$. This stress-drop stands for the lowering of stress measured from the initial reference state in which no crack exists. The function $\Delta\sigma(x)$ takes maximum and minimum values at $|x| = x_a$ and $x = 0$, respectively. A high fracture-resistant region is supposed near $x = 0$ in Eq. (3.4). Since the fracture-resistance is assumed to be low near $|x| = x_a$ in Eq. (3.4), two cracks are expected to be nucleated near $x = x_a$ and $x = -x_a$. The quantity $(\sigma_0 - c)$ denotes the degree of inhomogeneity in the distribution of $\Delta\sigma$, and x_a the degree of extent of $\Delta\sigma$. The quantity σ_0 is assumed to increase with the accumulation of tectonic stress. The quantity $(\sigma_0 - c)$ is assumed to remain constant during the accumulation of tectonic stress.

Stress-intensity factors are obtained, in the non-dimensional forms, as (see Appendix 1)

$$\tilde{K}_a \equiv K_a/(\sigma_0 - c)\sqrt{2\pi x_a} = (\xi(\tilde{a}) + \lambda)/\sqrt{2\tilde{a}(\tilde{b}^2 - \tilde{a}^2)} \quad (3.5.1)$$

and

$$\tilde{K}_b \equiv K_b/(\sigma_0 - c)\sqrt{2\pi x_a} = (\xi(\tilde{b}) + \lambda)/\sqrt{2\tilde{b}(\tilde{b}^2 - \tilde{a}^2)}, \quad (3.5.2)$$

where

$$\xi(x) = -x^6 + (\tilde{a}^2 + \tilde{b}^2 + 4)x^4/2 + [(\tilde{a}^2 - \tilde{b}^2)^2/8 + c/(\sigma_0 - c) - (\tilde{a}^2 + \tilde{b}^2)]x^2 + (\tilde{a}^2 + \tilde{b}^2)(\tilde{a}^2 - \tilde{b}^2)^2/16 - (\tilde{a}^2 - \tilde{b}^2)^2/4 - c(\tilde{a}^2 + \tilde{b}^2)/2(\sigma_0 - c), \quad (3.6.1)$$

$$\lambda = -b[C_0 K(q) + C_1 E(q)]/K(q), \quad (3.6.2)$$

$$C_0 = \{\tilde{a}^2 \tilde{b}^2 [(\tilde{a}^2 + \tilde{b}^2)/10 - 2/3] + (\tilde{a}^2 + \tilde{b}^2 - 4)(\tilde{a}^2 - \tilde{b}^2)^2 - c(\tilde{a}^2 + \tilde{b}^2)/2(\sigma_0 - c)\}/\tilde{b}, \quad (3.6.3)$$

$$C_1 = \tilde{b}[-(\tilde{a}^2 + \tilde{b}^2)^2/5 + (\tilde{a}^2 - \tilde{b}^2)^2/8 + (\tilde{a}^2 + \tilde{b}^2)/3 + 3\tilde{a}^2 \tilde{b}^2/5 + c/(\sigma_0 - c)], \quad (3.6.4)$$

$$q = \sqrt{\tilde{b}^2 - \tilde{a}^2}/\tilde{b}, \quad \tilde{b} = b/x_a, \quad \tilde{a} = a/x_a \quad (3.6.5)$$

and $K(q)$ and $E(q)$ are the complete elliptic integrals of the first and second kinds, respectively, given by

$$K(q) = \int_0^{\pi/2} \frac{d\varphi}{\sqrt{1 - q^2 \sin^2 \varphi}} \quad \text{and} \quad E(q) = \int_0^{\pi/2} \sqrt{1 - q^2 \sin^2 \varphi} d\varphi. \quad (3.6.6)$$

The non-dimensional form of K^c is defined in the same manner as in Eqs. (3.5), i.e.,

$$\tilde{K}^c \equiv K^c/(\sigma_0 - c)\sqrt{2\pi x_a}. \quad (3.7)$$

Equations (3.1) are thus reduced to

$$[\xi(\bar{a}) + \lambda] / \sqrt{2\bar{a}(\bar{b}^2 - \bar{a}^2)} = \tilde{K}^c(\bar{a}) \quad \text{and} \quad [\xi(\bar{b}) + \lambda] / \sqrt{2\bar{b}(\bar{b}^2 - \bar{a}^2)} = \tilde{K}^c(\bar{b}). \quad (3.8)$$

We obtain the locations of the crack tips by solving Eqs. (3.8) with respect to $\bar{a}$ and $\bar{b}$. The solutions are numerically obtained. The stability is examined by use of the partial derivatives of $\tilde{K}_a$ and $\tilde{K}_b$ at $\bar{a} = \bar{a}(\tilde{K}^c)$ and $\bar{b} = \bar{b}(\tilde{K}^c)$ expressed in the finite difference scheme. Although NEMAT-NASSER *et al.* (1978) confirmed $\partial K_a / \partial b = -\partial K_b / \partial a < 0$ in the analysis of stable tension cracks, these quantities are always positive in our case.

The growth path of the inner crack tip is illustrated in Fig. 3 for various values of $\tilde{K}^c$. There is no stable solution beyond the rightmost point of each curve. If σ_0 exceeds its value corresponding to the rightmost point, the two cracks coalesce spontaneously. The rightmost point of the curve in Fig. 3 is hereafter referred to as the *instability point*.

If the stable solution begins to exist in the cases $\tilde{K}^c = 0.01, 0.1, 0.2, 0.3$ and 0.32 as a result of σ_0 -increase, then the crack is assumed to be immediately formed. This means that the length of the pre-existing crack is equal to the critical crack length. Note that individual cracks are approximately regarded as independent if they are far apart. It was shown in the preceding section that the fracture condition is satisfied by the lowest stress-increase when the length of the pre-existing crack is equal to the critical crack length. Since the stress-drop given by Eq. (3.4) in the half-plane $x > 0$ or $x < 0$ is distributed similarly to that given by

Fig. 3. Stable growth curve of inner crack tip for the $\Delta\sigma$ -inhomogeneity model. Numerical parameters denote the values of $\tilde{K}^c$. Closed circles represent stable crack tip locations obtained by numerical computation. There is no stable solution beyond the rightmost closed circle on each curve. Only the instability point is computed in the cases $\tilde{K}^c = 0.025, 0.05, 0.15, 0.26, 0.28$ and 0.29 . In the other cases the stable solution begins to exist at the leftmost solid circle on each curve.

Eq. (2.4), the result obtained in the preceding section will be applicable. If the length of the pre-existing crack is smaller or larger than the critical crack length, the stable solution will begin to exist at some point on the growth curve in Fig. 3.

The two cracks coalesce spontaneously after the occurrence of instability and a large-scale single crack is formed. The zone between the instability points of the two cracks is hereafter referred to as the *unstable asperity zone*. It is seen in Fig. 3 that the larger unstable asperity zone is formed for higher $\tilde{K}^c$. It is confirmed in the computation that the stable growth regime still exists even for $\tilde{a}=0.03$ when $\tilde{K}^c=0.01$. Practically, the two cracks stably coalesce in this case. An accelerated phase is more clearly seen in the stable growth rate for a lower value of $\tilde{K}^c$ near the instability point.

When $\tilde{K}^c \geq 0.33$, the stable growth regime of two collinear cracks does not exist. A large-scale single crack is immediately formed in this case. Large $\tilde{K}^c$ is associated with large K^c , small $(\sigma_0 - c)$ and/or small x_a . Two initial cracks are expected to be formed in the neighborhoods of $x = x_a$ and $x = -x_a$ since the stress-drop is highest there. If x_a is small, the two cracks formed are highly interactive from the beginning. Then two stable cracks cannot exist for an extremely small

Fig. 4. Growth curve of outer crack tip for the $\Delta\sigma$ -inhomogeneity model. Only typical numerical examples are illustrated. Numerals are the values of $\tilde{K}^c$. Closed circles represent stable crack tip locations obtained by numerical computation. Broken curves denote stable growth paths. Solid lines denote paths along which the outer crack tips extend spontaneously at the time of instability. Open circles represent equilibrium locations of the tips of single cracks formed by instability. We obtain these locations, assuming isolated cracks in the medium where the stress-drop is described by Eq. (3.4), and assuming the fracture criterion (2.1).

Fig. 5. Stress-drops associated with the spontaneous coalescence of two collinear cracks. The stress-drops averaged over the unstable asperity zone and over the entire crack surface, $|x| < p$, are denoted as $\langle \Delta \sigma \rangle_{a.z.}$ and $\langle \Delta \sigma \rangle$, respectively, where $|x| = p$ are the equilibrium locations of the tips of single cracks, represented by open circles in Fig. 4, formed by instability. The closed and open circles represent $\langle \Delta \sigma \rangle_{a.z.}$ and $\langle \Delta \sigma \rangle$, respectively.

x_a , which means a large $\bar{K}^c$. As $(\sigma_0 - c)$ decreases, the degree of inhomogeneity in $\Delta \sigma(x)$ decreases and the fracture-resistance lowers near $x=0$. The stability of the two cracks thus lowers as $(\sigma_0 - c)$ decreases. It is also seen in Fig. 3 that there is no stable growth for $\sigma_0/(\sigma_0 - c) > 1$, i.e., $c > 0$. This may be because of the lowering of fracture-resistance near $x=0$. The stress-drop is positive there if $c > 0$.

The growth curve of the outer crack tip is illustrated in Fig. 4. The broken curves are stable growth paths. At the time of instability the outer crack tip extends spontaneously, in the framework of the static theory, as far as the point plotted by open circles along the solid line. The stress-drops associated with the spontaneous coalescence of the two cracks are illustrated in Fig. 5. We show the stress-drops averaged over the entire crack surface and over the unstable asperity zone. Refer to Appendix 2 for the solution. It should be noted here that the stress-drops in Fig. 5 are the lowering of stresses measured from the state immediately before the instability. There is a difference in the initial reference state between the stress-drops in Eq. (3.4) and in Fig. 5. Both stress-drops in Fig. 5 are generally lower for lower $\bar{K}^c$, but this tendency is reversed for $\bar{K}^c > 0.29$ in the case of the stress-drop $\langle \Delta \sigma \rangle_{a.z.}$. This reversal may be associated with the phenomenon seen in the growth curve of the inner crack tip (Fig. 3). The instability occurs invariably at the point $\sigma_0/(\sigma_0 - c) = 1$ when $\bar{K}^c > 0.29$.

3.2 $\bar{K}^c$ -inhomogeneity

In this subsection we consider the inhomogeneity in the distribution of

K^c , and $\Delta\sigma$ is assumed to be homogeneous, i.e., $\Delta\sigma = \sigma_0$ (const.). This model is hereafter referred to as the K^c -inhomogeneity model. Since the model-dependence of analysed results is much stronger in the K^c -inhomogeneity model than in the $\Delta\sigma$ -inhomogeneity model, only short consideration is given here.

We can get the stress-intensity factors, setting $c \rightarrow \sigma_0$ in Eqs. (3.5.1) and (3.5.2), in the forms

$$K_a = \sigma_0 \sqrt{\pi b^2/a} [E(q)/K(q) - 1 + q^2]/q \quad \text{and} \quad K_b = \sigma_0 \sqrt{\pi b} [1 - E(q)/K(q)]/q, \quad (3.9)$$

where $q = \sqrt{b^2 - a^2}/b$, and $K(q)$ and $E(q)$ are given by Eq. (3.6.6). These are rewritten in the non-dimensional forms

$$\begin{aligned} \tilde{K}_a &\equiv K_a / \mu \sqrt{b} = \frac{\sigma_0}{\mu} \sqrt{\frac{\pi}{c}} \frac{E(q')/K(q') - 1 + q'^2}{q'}, \quad \text{and} \\ \tilde{K}_b &\equiv K_b / \mu \sqrt{b} = \frac{\sigma_0}{\mu} \sqrt{\frac{\pi}{c}} \frac{1 - E(q')/K(q')}{q'}, \end{aligned} \quad (3.10)$$

where $c = a/b$ and $q' = \sqrt{1 - c^2}$. The outer crack tip, at $x = b$, is assumed to be kept fixed. This will be justified by the assumption of unbreakable barriers for $|x| > b$. Since the outer crack tip is fixed, the stability conditions are written as

$$dK_a/da > dK^c(x)/dx|_{x=a_0} \quad \text{and} \quad K_a = K^c(a). \quad (3.11)$$

First let us assume the K^c -distribution shown in Fig. 6a. The two collinear cracks are assumed to be formed at some time along the segments $a_0 < |x| < b_0$. The distribution of K^c is assumed to be discontinuous at $|x| = a_0$. It is assumed that the stress-intensity factor at $x = a_0$ is, at that time, below the critical value $K^c(a_0 - \epsilon)$, where ϵ is an infinitesimally small positive number. The stress-intensity factor will attain the critical value $K^c(a_0 - \epsilon)$ after a while by the increase of tectonic stress. Then the inner crack tip begins to extend. Since $\partial K_a / \partial a$ is negative when $\Delta\sigma = \text{const.}$ (e.g., see Fig. 5 in the paper by RUDNICKI and KANAMORI (1981)), the crack tip extends spontaneously in the range $0 < x < a_0$. Note that

Fig. 6. Assumed K^c -distributions for the K^c -inhomogeneity model.

the stability conditions, (3.11), cannot be satisfied when both $\partial K_a/\partial a < 0$ and $dK^c(x)/dx=0$ are satisfied. When the inner crack tip extends stably over some portion in the range $0 < x < a_0$, a negative $dK^c(x)/dx$ is required along that portion.

The K^c -distribution such as shown in Fig. 6b is next assumed. In the range $0 < x < a_0$, K^c is written as

$$K^c = -x \cdot \Delta K^c / a_0 + K_1^c, \quad \Delta K^c = K_1^c - K_0^c. \quad (3.12)$$

The growth curve of the inner crack tip is illustrated in Fig. 7. There is a tendency, although it is weak, that the extent of unstable asperity zone decreases with the increases of K_0^c and/or ΔK^c .

Fig. 7. Stable growth curve of inner crack tip for the K^c -inhomogeneity model. Numerical parameters are the values of $\Delta \tilde{K}^c \equiv \Delta K^c / \mu \sqrt{b_0}$. $\tilde{K}_0^c$ denotes $K_0^c / \mu \sqrt{b_0}$. Refer to the right-hand scale for σ_0/μ only in the case $\Delta \tilde{K}^c = 1.0$. The left-hand scale is referred to in the other cases.

4. Interpretation of Slow Fault-Slip Phenomena as Ruptures Excited by the Instability of Pre-existing Faults

KANAMORI (1972) investigated the mechanisms of the Aleutian earthquake of 1946 and the Sanriku earthquake of 1896. These earthquakes generated, despite their relatively small earthquake magnitude, two of the largest and most widespread tsunamis in history. Kanamori attributed this phenomenon to the abnormally slow fault slip and called this type of earthquake 'tsunami earthquake.' Some examples of tsunami earthquakes were thereafter found through observation of long-period seismic waves and tsunamis (FUKAO and FURUMOTO, 1975; PFLUKE, 1978; TAKEMURA *et al.*, 1977). KANAMORI and STEWART (1979) used the term 'slow earthquake,' which expresses general slow fault-slip phenomena, including tsunami earthquakes.

Aseismic fault slip which emits no seismic wave is also known to exist. The occurrence of aseismic fault slip is well known along the San Andreas fault system. Detailed observations have been carried out there in many aspects, and the mechanism of this type of fault slip has been revealed. It is known that there

are two seemingly distinct mechanisms by which aseismic slip occurs along the San Andreas fault system (BURFORD *et al.*, 1978; EVANS *et al.*, 1981). The creep records demonstrate that stable continuous creep (or stable creep) within the narrow surface expression of the fault is the most dominant form of shallow aseismic slip along the central section of the San Andreas fault. There are, however, several active sections where apparent steady creep is replaced by, or has superimposed upon it, episodes of accelerated creep. It has been revealed that this event, called 'propagating episodic creep' or 'episodic creep,' has a form of propagating failure at velocities of the order of kilometers per hour. The episodic creep is detected along the ends of the section of the San Andreas fault where the stable creep is the most dominant form of aseismic slip. No creep is detected beyond the segments where the episodic creep is observed. See Fig. 1 in the paper by EVANS *et al.* (1981) for details. Source processes of episodic creep have been kinematically analysed with the aid of the elastostatic theory of dislocations (KING *et al.*, 1973; MORTENSEN *et al.*, 1977; MCHUGH and JOHNSTON, 1978; EVANS *et al.*, 1981).

We now obtain the general fracture-characteristics of slow earthquakes and episodic fault creeps. The general fracture-characteristics will be described by the stress-drop $\langle \Delta\sigma \rangle$ and the specific fracture energy $\langle \nu_f \rangle$, both averaged over the final fault surface. It will be shown later that the general fracture-characteristics of these events are successfully accounted for by our source model, and that the generating mechanisms are revealed.

In Fig. 8 we plot the stress-drop $\langle \Delta\sigma \rangle$ for episodic creeps and for seismic events which include slow earthquakes. We employ the table of earthquake source

Fig. 8. Stress-drops for seismic events and episodic creeps. Symbols are defined as follows: ●, interplate events; ○, intraplate events; ⊙, interplate slow earthquakes; ⊗, intraplate slow earthquakes. Numbers are attached to the slow earthquakes: 1, Mid-Atlantic Ridge earthquake of 1967; 2, Mid-Atlantic Ridge earthquake of 1974; 3, Spain earthquake of 1964; 4, Nemuro-oki earthquake of 1975; 5, Shakotan earthquake of 1940; 6, Aleutian earthquake of 1946; 7, Sanriku earthquake of 1896.

Table 1. Seismic events (A) and episodic creeps (B).

A)						
No.	Event	Date	M_0 (dyn·cm)	S (km ²)	$\bar{D}$ (cm)	$\langle\Delta\sigma\rangle$ (bars)
1	Sanriku	15 Jun. 1896	6.3×10^{28}	300×160	190	<u>14.6</u>
2	South Izu	21 Mar. 1934	9.5×10^{28}	7×4	10	11.0
3	Shakotan	1 Aug. 1940	4.2×10^{27}	170×50	110	17.0
4	Aleutian	1 Apr. 1946	3.7×10^{28}	100×100	170	<u>90.1</u>
5	Mid-Atlantic	13 Feb. 1967	3.4×10^{28}	60×10	160	<u>56.4</u>
6	Amagi	9 Jul. 1974	3.2×10^{28}	3.5×3	9	13.0
7	Mid-Altantic	16 Oct. 1974	4.5×10^{28}	70×10	180	<u>59.2</u>
8	Nemuro-oki	10 Jun. 1975	3.0×10^{27}	85×75	—	<u>14.4</u>
9	Kawazu	17 Aug. 1976	2.1×10^{24}	9×3.5	20	25.0

B)						
No.	Date	M_0 (dyn·cm)	S (km ²)	$\bar{D}$ (mm)	$\langle\Delta\sigma\rangle$ (bars)	
10	17 Jul. 1971	<u>1.69×10^{22}</u>	14.4	6.0	<u>0.75</u>	
11	17 Sep. 1975	<u>3.84×10^{21}</u>	4.0×2.0	2.4	<u>0.41</u>	
12	8 Aug. 1976	<u>1.40×10^{21}</u>	2.0×1.0	3.5	<u>0.75</u>	
13	20 Aug. 1976	<u>4.04×10^{21}</u>	3.06×3.0	2.2	<u>0.35</u>	
14	26 Jul. 1977	<u>4.22×10^{21}</u>	6.59×0.8	4.0	<u>0.85</u>	

Seismic events Nos. 1, 3, 4, 5, 7 and 8 are considered to be slow earthquakes. The other seismic events are tabulated here since GELLER's (1976) table does not contain such small events. Values with underlines are determined by the present author on the assumptions $M_0 = \mu \bar{D} S$ and $\langle\Delta\sigma\rangle = 7M_0/16R^3$ (ESHELBY, 1957). M_0 , seismic moment; S , fault area; $\bar{D}$, average dislocation; $\Delta\sigma$, stress drop; R , characteristic radius of fault plane.

1) KANAMORI (1972), 2) ABE (1978), 3) FUKAO and FURUMOTO (1975), 4) KANAMORI (1972), 5) KANAMORI and STEWART (1976), 6) ABE (1978), 7) KANAMORI and STEWART (1976), 8) moment from TAKEMURA *et al.* (1977), fault area from SUZUKI and SUGIMOTO (1975), 9) ABE (1978), 10) KING *et al.* (1973), 11) MORTENSEN *et al.* (1977), 12) McHUGH and JOHNSTON (1978), 13) McHUGH and JOHNSTON (1978), 14) EVANS *et al.* (1981).

parameters compiled by GELLER (1976) and Table 1 in the present paper. If the stress-drop is not determined in the literature in Table 1, it is now determined with the aid of the formula $\langle\Delta\sigma\rangle = 7\pi\bar{D}\mu/16R$ (ESHELBY, 1957), where $\bar{D}$ is the average dislocation and R is the characteristic radius of the fault plane. The value 2×10^{11} dyn/cm² is assumed for the rigidity, μ , for the episodic creeps, which is considered as representative by some authors (RICE and SIMONS, 1976; RUDNICKI, 1980). The stress-drops for the episodic creep are seen to be extremely low compared with those for moderate- or large-size earthquakes. The stress-drop for the episodic creep is even lower than the average value of stress-drops for small earthquakes (HANKS, 1977).

It is also seen that the stress-drops for slow earthquakes are generally lower in a corresponding range of seismic moment than for the usual type of seismic event except for three events, that is, the Aleutian earthquake of 1946 and the Mid-

Atlantic Ridge earthquakes of 1967 and 1974. Hence it can be stated that the slow fault-slip events are generally characterized by low stress-drop. The reason for the three exceptional earthquakes are considered later. We denote both slow earthquake and episodic creep by a single term *slow fault-slip event*. Since the 1964 Spanish earthquake has an unusually low rupture velocity of 1.4 km/sec in the table of GELLER (1976), this earthquake is placed in the category of slow earthquakes.

Not only the values of stress-drop but also of specific fracture energy (or the critical value of stress-intensity factor) must be revealed in order to understand the mechanics of fracture if the usual Griffith's fracture theory (e.g., see RICE, 1968) is applied. Let us estimate the specific fracture energy $\langle \nu_f \rangle$, averaged over each fault surface, for the events plotted in Fig. 8 with the aid of the formula

$$\langle \nu_f \rangle = R \langle \Delta \sigma \rangle^2 / 2 \mu \pi \quad (4.1)$$

derived by HUSSEINI *et al.* (1975). Since the stress-drop is sometimes estimated through the value of seismic moment M_0 , Eq. (4.1) is rewritten in the form

$$M_0 / S \sqrt{R} = 16 \sqrt{2 \mu \langle \nu_f \rangle} / 7 \sqrt{\pi} \cong 1.82 \sqrt{\mu \langle \nu_f \rangle}, \quad (4.2)$$

where S is the fault area, i.e., $S = \pi R^2$. The formula $M_0 = 16 R^3 \langle \Delta \sigma \rangle / 7$, applicable to a circular fault with radius R (ESHELBY, 1957), is employed in the derivation of Eq. (4.2). The quantity $M_0 / S \sqrt{R}$ is therefore a measure of average specific fracture energy. The expression (4.2) has an advantage in that $M_0 / S \sqrt{R}$ is approximately equal to the average value of K^c , i.e., $\langle K^c \rangle$. When the plane strain shear is assumed, ν_f is related to K^c in the form (e.g., RUDNICKI, 1980)

$$\mu \nu_f = (1 - \nu) (K^c)^2 / 2, \quad (4.3)$$

where ν is Poisson's ratio. Substituting this equation into (4.2) and assuming $\nu = 0.1-0.3$, we get

$$M_0 / S \sqrt{R} \cong (1.1-1.2) \langle K^c \rangle. \quad (4.4)$$

The estimated values of $M_0 / S \sqrt{R}$ for the events in Fig. 8 are plotted in Fig. 9.

Fig. 9. Values of $M_0 / S \sqrt{R}$ for seismic events and episodic creeps. Symbols and numbers are explained in the caption of Fig. 8.

Since the seismic moment for the episodic creeps is not estimated in the literature, we determine it using the formula $M_0 = \mu \bar{D} S$. It is seen that the values of $M_0/S\sqrt{R}$ are smaller for the slow earthquakes than for the other seismic events except for the three above-mentioned exceptions. The episodic creeps have values of $M_0/S\sqrt{R}$ far lower than those for the seismic events.

The three exceptional earthquakes are briefly considered here. Since there is an arbitrariness in the determination of fault width for the Mid-Atlantic Ridge earthquakes, the stress-drop and $M_0/S\sqrt{R}$ for these earthquakes are doubtful. Note the dependence of $M_0/S\sqrt{R}$ and $\langle \Delta\sigma \rangle$ on source dimension, i.e., $M_0/S\sqrt{R} \propto R^{-2.5}$ and $\langle \Delta\sigma \rangle \propto R^{-3}$ when M_0 are given. The fault plane will be comparatively well determined for the 1946 Aleutian earthquake, and it is found to be relatively small for its corresponding earthquake magnitude. The mechanism of generation of the 1946 Aleutian earthquake may be essentially different from that of the other slow fault slip events. The above three earthquakes are therefore excluded in the following investigation of the mechanism of generation of slow fault-slip events.

General fracture-characteristics of the slow fault-slip events are thus obtained; low average stress-drop and low $M_0/S\sqrt{R}$ are distinctive features. The slow earthquake and the episodic creep will be therefore regarded as similar rupture phenomena from the viewpoint of fracture mechanics. Hereafter we intend to account for these characteristics of the slow fault-slip events with the aid of the source model developed in the present paper.

Since detailed observations have been made in many aspects on the creep along the San Andreas fault system, we pay particularly attention to episodic creep. EVANS *et al.* (1981) showed in a figure that episodic creep coexists with stable creep. This observation suggests the coexistence of two distinct fracture mechanisms in the same fault section. This may be a key to the understanding of the mechanism of generation of slow fault-slip events. This observation is satisfactorily explained by our source model developed in the preceding sections. It is assumed in the interpretation of the observations that K° is homogeneously distributed and that some inhomogeneity exists in the distribution of $\Delta\sigma$. The validity of this assumption will be shown later.

The mechanism of the coexistence of episodic creep and stable creep is now examined. The fracture mechanism of stable creep will be fully understood from the results obtained in the analysis of the isolated crack. It was shown that the stable growth regime is dominant in the case of extremely low K° . The stable creep is thus explained from the condition of very low K° . This is in harmony with the observation that the episodic creep is associated with very low $\langle K^\circ \rangle$. General fracture-characteristics observed for both types of creep events—steady creep and episodic creep should be similar in the aspect of fracture mechanics because of the coexistence in the same fault section. The stable creeps will be nucleated in places where the fracture-resistance is locally weak. The cracks thus nucleated will grow stably with the increase of tectonic stress and the interac-

tion gradually becomes strong. If instability due to interaction occurs, stably grown cracks coalesce spontaneously such as shown in Fig. 3. A slower coalescence velocity is expected for lower K^e since the extent of unstable asperity zone decreases and the stress-drop in the unstable asperity zone lowers as K^e lowers (see Figs. 3 and 5).

Our source model shows that coalescence velocity is very low and that the stress-drop averaged over the final crack surface is also low in the fault section where K^e is low. This will account for the observed characteristics of episodic creeps. The episodic creep is understood as a rupture excited by the instability of interacting cracks associated with the stable creep. The coexistence of stable creep and episodic creep is fully understood from this interpretation.

The specific fracture energy will be further lower along the central section of the San Andreas fault than along the sections where the episodic creep is detected. It is known that the stable creep is dominant along the central section of the San Andreas fault. If K^e is close to zero, the interactive cracks coalesce nearly stably: see the example $\bar{K}^e=0.01$ in Fig. 3. Practically, instability does not occur in such a case, and accelerated ruptures like episodic creep are not observed. It is therefore concluded that the stable crack growth is dominant in the case $K^e \simeq 0$ even if many interactive cracks exist.

It is inferred from the above considerations that the specific fracture energy increases with the distance, along the San Andreas fault, from the central section. The episodic creep is considered to occur along the end portions of the central section because of the somewhat high specific fracture energy there. It is considered that no creep is detected beyond the portions where the episodic creep is observed because of considerably high specific fracture energy there.

Although no more-detailed observations have been carried out for slow earthquakes than for creep events along the San Andreas fault system, the fracture mechanism will be similar. It is revealed in this paper that general fracture-characteristics are the same for slow earthquakes and episodic creeps from the viewpoint of the fracture mechanics: low stress-drop and low specific fracture energy are the common features.

5. Discussion

It was shown in the preceding section that the slow fault-slip events can be successfully explained by the $\Delta\sigma$ -inhomogeneity model. It is shown here that explanation by use of the K^e -inhomogeneity model is difficult to make. It is shown in Fig. 7 on the assumption of Eq. (3.11) holding that the extent of the unstable asperity zone decreases as $\bar{K}_0^e$ and/or $\Delta\bar{K}^e$ increase. The average stress-drop associated with spontaneous coalescence decreases with the decrease of the extent of the unstable asperity zone. A lower rupture extension velocity is expected (see Eq. (4.5)) as the stress-drop decreases, and $\bar{K}_0^e$ and $\Delta\bar{K}^e$ increase. Hence slower coalescence velocity is expected for higher $\bar{K}_0^e$ and/or $\Delta\bar{K}^e$ for the

examples in Fig. 7. This is contradictory to the observations: the slow fault-slip events are characterized by low specific fracture energy. However, we do not completely rule out the role of inhomogeneous K^c -distribution since the model-sensitivity is stronger when K^c is assumed than when $\Delta\sigma$ is assumed.

Since the specific fracture energy is nearly zero on the preslipped crack surface, K^c for the extension of pre-existing cracks is generally higher than the average value of K^c , i.e., $\langle K^c \rangle$, for the rupture associated with the spontaneous coalescence of the pre-existing cracks. The former is now referred to as K_{pr}^c for comparative purposes. It is associated with a rupture of fresh structure. While the quantity K^c assumed in Sec. 3 naturally stands for K_{pr}^c , $M_0/S\sqrt{R}$ approximately equals $\langle K^c \rangle$. However, the proportional relation $K_{pr}^c \propto \langle K^c \rangle$ is known to exist if the $\Delta\sigma$ -inhomogeneity model is assumed. With the decrease of K_{pr}^c , there is an increase in the ratio of the size of the pre-existing crack immediately before instability to that of the crack formed by the instability (see Figs. 3 and 4). Since $\langle K^c \rangle$ will decrease with the increase of this ratio and/or the decrease of K_{pr}^c , we have the proportional relation in the $\Delta\sigma$ -inhomogeneity model. Therefore higher and lower $\langle K^c \rangle$ certainly reflect higher and lower K_{pr}^c , respectively. Then Fig. 9 will be able to be used to estimate the relative magnitude of K_{pr}^c . This procedure was taken in section 4 to estimate K_{pr}^c .

It was shown previously that the 1946 Aleutian earthquake was accompanied by exceptionally high stress-drop and high specific fracture energy in the group of slow earthquakes. These values are high even for the group of interplate earthquakes. This event may not be related to the failure of the unstable asperity zone. A large-scale single fault might be formed from the beginning. It is shown in Fig. 3 that a single large-scale crack is immediately formed for considerably high $\bar{K}^c$, i.e., $\bar{K}^c \geq 0.33$. This kind of rupture will show higher stress-drop and higher specific fracture energy compared with a rupture related to the spontaneous coalescence of pre-existing cracks: the stress-drop and the specific fracture energy are zero, for the latter rupture, on the pre-slipped crack surface.

YAMASHITA (1980a) investigated theoretically the causes leading to the slow earthquakes on the assumption of inhomogeneously distributed stress-drop and specific fracture energy. He analysed an isolated crack propagating dynamically and concluded that high specific fracture energy and low and inhomogeneously distributed stress-drop may account for the slow earthquakes. However it is shown in Fig. 9 in the present paper that the specific fracture energies for the slow earthquakes are not necessarily high. He considered that the model developed in his paper can account for the fact that some slow earthquakes are accompanied by multiple events. This phenomenon was observed by KANAMORI and STEWART (1979) and KASAHARA and SASATANI (1979). However it is explained by the present source model, also. The slow earthquake is understood, in the present paper, as a rupture related to the spontaneous coalescence of the pre-existing faults. Although only the two pre-existing cracks are assumed to coalesce in the present paper, many such cracks will coalesce one after another in

the actual source process. In such a case somewhat strong seismic waves will be emitted on each occasion of coalescence and the rupture will be observed as a multiple event.

RUDNICKI and KANAMORI (1981) defined asperity as the high fracture-resistant zone between the pre-existing cracks. They may imagine only the spontaneous rupture as the failure of asperity. It is shown in the present paper that the asperity ruptures not only spontaneously but also stably.

WYSS *et al.* (1981) found the existence of an asperity zone for the 1975 Hawaii earthquake through detailed geodetic and seismological measurements. They also obtained evidence that the size of asperity zone decreases, before the occurrence of the earthquake, with the aseismic growth of the pre-existing fault. This observation seems to be in harmony with our $\Delta\sigma$ -inhomogeneity model.

It appears to be physically unreasonable in Eqs. (2.4) and (3.4) that the stress-drop on the crack surface, $\Delta\sigma(x)$, tends towards negative infinity as $|x| \rightarrow \infty$. However this will be justified if the region far from the origin is regarded as an unbreakable zone. The forms of the stress-drop such as in Eqs. (2.4) and (3.4) are adopted mainly to simplify the mathematical treatment. When the distribution of stress-drop is given by a polynomial of x , the mathematical treatment becomes extremely simple. The stress-drop, $\Delta\sigma(x)$, can be regarded as the expression of the function $\Delta\sigma(z)$ on the real-axis of the z -plane, where $z = x + iy$ and $\Delta\sigma(z)$ is obtained upon replacing x in $\Delta\sigma(x)$ by z . Since a polynomial of z is regular everywhere on the z -plane except at infinity, the integral on the z -plane is easily carried out as shown in Appendix 1 if $\Delta\sigma(x)$ is expressed as a polynomial of x . The assumptions of stress-drop distribution in Eqs. (2.4) and (3.4) seem to be mathematically the simplest ones under which a single crack and two stable cracks are expected to be nucleated, respectively, in the initial state.

6. Conclusions

It is shown that the mode of spontaneous coalescence of pre-existing cracks depends on the values and the distributions of stress-drop and specific fracture energy. For example if the specific fracture energy is sufficiently low and the inhomogeneous stress-drop is assumed, then the pre-existing cracks coalesce slowly. On the other hand if the specific fracture energy is high enough, the pre-existing cracks coalesce highly dynamically.

The slow earthquake and episodic creep can be understood as ruptures associated with the instability of interacting pre-existing cracks. The existence of distinct types of fault creeps along the San Andreas fault may be due to the gradual increase of specific fracture energy from the central section toward the end portions along the fault.

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Appendix 1. Determination of Stress-Intensity Factors

Suppose the crack or cracks lie on the x -axis of the infinite xy -plane, and that there are n finite cracks with left and right ends at $a_1, a_2, \dots, a_n$ and $b_1, b_2, \dots, b_n$, respectively. Denote by L the crack line or union of all crack lines. The j -th crack line is denoted as $L_j = (a_j, b_j)$ so that $L = L_1 + L_2 + \dots + L_n$.

Upon following the methods of MUSKHELISHVILI (1953a, b) the stress-intensity factors for the crack tips at $x=a_j$ and $x=b_j$ are written as

$$K_{a_j}/\sqrt{2\pi} = -2 \lim_{t \rightarrow a_j} \sqrt{a_j - t} \operatorname{Im} \varphi(t) \quad (\text{A.1})$$

and

$$K_{b_j}/\sqrt{2\pi} = -2 \lim_{t \rightarrow b_j} \sqrt{t - b_j} \operatorname{Im} \varphi(t), \quad (\text{A.2})$$

respectively, where

$$\varphi(z) = \frac{\chi(z)}{2\pi} \int_L \frac{\Delta\sigma(t)}{[\chi(t)]^+(t-z)} dt + P(z)\chi(z), \quad (\text{A.3.1})$$

$$\chi(z) = \prod_{j=1}^n [(z-a_j)(z-b_j)]^{-1/2}, \quad (\text{A.3.2})$$

$$P(z) = C_0 + C_1 z + \dots + C_{n-1} z^{n-1}, \quad (\text{A.3.3})$$

$z = x + iy$, $\Delta\sigma(t)$ is the stress-drop, C_j ($j=1, 2, \dots, n$) are complex constants and points on the crack lines are denoted by $z=t$, t being real. In the limiting procedure, t approaches a_j from the left and b_j from the right in (A.1) and (A.2), respectively. In (A.3.1) and in the following, the superscripts (+) and (−) denote the limits of the analytic function as the crack line is approached from the regions $y>0$ and $y<0$, respectively. Choose the branch cut along L such that $\chi(z)$ behaves as z^{-n} for large z .

The complex constants C_k ($k=0, 1, \dots, n-1$) are determined from the conditions of the single-valuedness of displacement, that is,

$$\int_{a_j}^{b_j} \varphi^+(t) dt - \int_{a_j}^{b_j} \varphi^-(t) dt = 0 \quad (j=1, 2, \dots, n-1). \quad (\text{A.4})$$

As shown above, the mathematical method for analysing an isolated crack is essentially the same as that for analysing collinear cracks, so that only the problem of two collinear cracks are concretely analysed.

Assume the stress-drop distribution (3.4), and $n=2$, $a_1=-b$, $b_1=-a$, $a_2=a$ and $b_2=b$ in the above formulae. It can be easily shown, for large $|z|$, that

$$\begin{aligned} \Delta\sigma(z)\chi^{-1}(z) &= [z^2 - (a^2 + b^2)/2] \{ -(\sigma_0 - c)[(z/x_a)^2 - 1]^2 + \sigma_0 \} \\ &\quad + (\sigma_0 - c)(a^2 - b^2)^2 [(z/x_a)^2 - 2]/8x_a^2 \\ &\quad + (\sigma_0 - c)(a^2 + b^2)(a^2 - b^2)^2/16x_a^4 + O(z^{-1}) \\ &= \eta(z) + O(z^{-1}). \end{aligned} \quad (\text{A.5})$$

Let us consider the following integral to evaluate the integral in Eq. (A.3.1).

$$I = \int_{\lambda_1 + \lambda_2} \frac{\chi^{-1}(s) \Delta\sigma(s) - \eta(s)}{s - z} ds, \quad (\text{A.6})$$

where λ_1 and λ_2 are contours surrounding the cracks L_1 and L_2 , respectively, in a clockwise direction, and $\eta(s)$ is given in Eq. (A.5). Upon applying Cauchy's formula and noting Eq. (A.5), the right-hand side of Eq. (A.6) is reduced to

$$I = \int_{A_z} \frac{\chi^{-1}(s) \Delta\sigma(s) - \eta(s)}{s - z} ds, \quad (\text{A.7})$$

where A_z is a contour surrounding the point z in a counter-clockwise direction. Hence

$$I = 2\pi i [\chi^{-1}(z) \Delta\sigma(z) - \eta(z)]. \quad (\text{A.8})$$

It is also known that

$$I = \int_{A_1 + A_2} \frac{\chi^{-1}(s) \Delta\sigma(s) - \eta(s)}{s - z} ds = 2 \int_L \frac{\Delta\sigma(t)}{[\chi(t)]^+(t - z)} dt \quad (\text{A.9})$$

since $[\chi^{-1}(s)]^+ = -[\chi^{-1}(s)]^-$, and $\Delta\sigma(s)$ and $\eta(s)$ are regular on the x -axis. We get, from Eqs. (A.8) and (A.9),

$$\int_L \frac{\Delta\sigma(t)}{[\chi(t)]^+(t - z)} dt = \pi i [\chi^{-1}(z) \Delta\sigma(z) - \eta(z)]. \quad (\text{A.10})$$

The integral in Eq. (A.3.1) is thus evaluated.

We obtain, from Eqs. (A.3.1) and (A.10),

$$\varphi^+(t) - \varphi^-(t) = -i[\eta(t) + C_0' + C_1't] \chi^+(t), \quad (\text{A.11})$$

where

$$C_0' = 2iC_0 \quad \text{and} \quad C_1' = 2iC_1.$$

The conditions of (A.4) are concretely written, with the aid of Eq. (A.11), in the form

$$\int_a^b \frac{\eta(t) + C_0' + C_1't}{\sqrt{(b^2 - t^2)(t^2 - a^2)}} dt = \int_{-b}^{-a} \frac{\eta(t) + C_0' + C_1't}{\sqrt{(b^2 - t^2)(t^2 - a^2)}} dt = 0. \quad (\text{A.12})$$

Solving Eqs. (A.12) with respect to C_0' and C_1' and expressing the solutions with the aid of the complete elliptic integrals, we obtain $C_1' = 0$ and $C_0' / (\sigma_0 - c) x_a^2 = \lambda$, where λ is defined by Eq. (3.6.2) in the text. The stress-intensity factors for the crack tips at $x = a$ and b are thus completely determined. They are explicitly expressed in Eqs. (3.5.1) and (3.5.2) in the text.

Appendix 2. Determination of Stress-Drop

According to the method of MUSKHELISHVILI (1953a, b), the change of stress

component p_{xy} due to the collinear cracks in Fig. 2 is written, with the aid of $\varphi(z)$ defined by Eq. (A.3.1), as

$$p_{xy} = -2 \operatorname{Im} \varphi(t) \quad (\text{A.13})$$

on the x -axis. This is easily calculated upon using the expression of $\varphi(z)$ obtained in Appendix 1. Thus we get

$$p_{xy}(t) = -\Delta\sigma(t) + \frac{\eta(t) + C_1't}{\sqrt{(t^2 - a^2)(t^2 - b^2)}} \quad \text{for } |t| > b$$

(A.14)

and

$$p_{xy}(t) = -\Delta\sigma(t) - \frac{\eta(t) + C_1't}{\sqrt{(a^2 - t^2)(b^2 - t^2)}} \quad \text{for } 0 \leq |t| < a,$$

where $\eta(t)$ is defined by Eq. (A.5) and $\Delta\sigma(t)$ is given by Eq. (3.4).

Assume that a large-scale crack, with left and right crack tips at $x = -p$ and p , is formed by the instability of the two interacting cracks. Then the stress drops in an amount equal to $\Delta\sigma(t) + p_{xy}(t)$ in the regions $b < |t| < p$ and $0 \leq |t| < a$ at the time of the coalescence. The stress-drop is zero in the regions $a < |t| < b$. The stress-drop associated with spontaneous coalescence, $\Delta\sigma(t)_{s.c.}$, is, therefore, written as

$$\Delta\sigma(t)_{s.c.} = -\frac{\eta(t) + C_1't}{\sqrt{(a^2 - t^2)(b^2 - t^2)}} \quad \text{for } 0 \leq |t| < a,$$

(A.15)

and

$$\Delta\sigma(t)_{s.c.} = \frac{\eta(t) + C_1't}{\sqrt{(t^2 - a^2)(t^2 - b^2)}} \quad \text{for } b < |t| < p.$$

If $\Delta\sigma(t)_{s.c.}$ is averaged over the unstable asperity zone and over the entire crack surface of the large-scale crack, we have

$$\langle \Delta\sigma \rangle_{a.z.} = -\frac{1}{a} \int_0^a \frac{\eta(t) + C_1't}{\sqrt{(a^2 - t^2)(b^2 - t^2)}} dt,$$

(A.16)

and

$$\langle \Delta\sigma \rangle = \frac{1}{p} \int_0^a \frac{\eta(t) + C_1't}{\sqrt{(a^2 - t^2)(b^2 - t^2)}} dt + \frac{1}{p} \int_b^p \frac{\eta(t) + C_1't}{\sqrt{(t^2 - a^2)(t^2 - b^2)}} dt,$$

respectively. These integrals are numerically evaluated in this paper.

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