The field of accelerations of a kinematic fault-model are evaluated in terms of fundamental interference-integrals that depend upon the source-observer geometry and the various spatial and temporal source elements.

The dependence of the accelerations at various distances $r$ from the fault’s center, on the fault’s major dimension $L$ and the radiation’s wavelength $\lambda$ are scaled to the three dimensionless ‘regionalization-indices’: $2\pi r/\lambda$, $2L^2/\lambda r$ and $0.62 \times (L/r) \sqrt{L/\lambda}$. These determine the limits of the far-field, the outer Fresnel-zone and the inner Fresnel-zone respectively.

The interference integrals are then evaluated through the stationary-phase approximation and the Fresnel approximation both in the time and frequency domains. In the Fresnel zone they are expressed in terms of the Lommel functions of two variables. The ensuing acceleration field is shown to depend strongly on the shear Mach-number. In subshear-rupture, the acceleration in the near-fault and Fresnel zones decreases exponentially in a direction normal to the fault. In supershear-rupture the acceleration is a Mach-wave that propagates without attenuation along the Mach-lines.

The theoretical scaling law for the acceleration in each region is determined. We assert: (1) peak accelerations in the near-fault zone are essentially independent of the earthquake’s magnitude; (2) peak accelerations in the near-fault and Fresnel zones are proportional to the particle velocity on the fault; (3) accelerations in the near-fault and Fresnel zones are determined by the radiation from the nearest fault-segment. This explains the elliptical shape of isoseismals, which are locii of equidistant points from the fault.

1. Introduction

The damage caused by earthquakes to human habitation and life-lines in a given zone can be assessed through the deterministic model which takes into account the characteristics of the causative fault (dimensions, time-history, offset, etc.), specified local site conditions (dynamic soil properties, surface effects, topography, layering characteristics, etc.) and the specific source-site transmission path. Clearly, a deterministic calculation of ground acceleration, prerrequisites the
knowledge of parameters, most of which are only partially known and some of which are totally unknown. Fortunately, details of the acceleration time-series are not always needed, especially in problems of engineering design where the knowledge of peak acceleration over a given time-window is sufficient. Thus, a prerequisite for the estimation of seismic hazard in a given region is the determination of certain relations involving the peak horizontal acceleration in the period range of 0.1–1.0 s, the basic source parameters and the epicentral distance.

The literature on the subject is rather abundant and its survey will require a special review which is outside the scope of the present paper. We may, however, mention a number of contributions which represent three avenues of approach to the problem: (1) Exact solutions of some canonical problems which involve the kinematic source model alone (Niazy, 1975; Boatwright and Boore, 1975). (2) Derivation of empirical formulas based on observed data (Shebalin, 1973; Hanks and Johnson, 1976; Maley and Cloud, 1971; Joyner et al., 1981). (3) A phenomenological approach through which observed data is interpreted by the ad-hoc theory (Murray, 1973).

In the present paper we have used a different approach: Starting from a finite dislocation model in a three dimensional elastic continuum, we derive the acceleration field at an arbitrary distance from the fault in terms of interference-integrals. These integrals are then evaluated by different approximations, depending on the observer’s distance from the fault and the shear Mach-number. The peak acceleration in each region is evaluated. It is then shown that the theory yields the known empirical dependence of the peak acceleration upon the magnitude and epicentral distance. Finally we discuss the relationship between isoseisms and curves of equal horizontal peak acceleration.

2. The Fundamental Interference-Integrals

The explicit expression for the displacement field $u(r, t)$ in an unbounded medium due to a point dislocation with area $dS$, slip vector $\vec{U} = U (\cos \lambda, \sin \lambda \cos \delta, \sin \lambda \sin \delta)$ and normal $\vec{n} = (0, -\sin \delta, \cos \delta)$ is given by the well-known formula (Ben-Menahem and Singh, 1981)

$$u(r, t) = \frac{U dS}{4\pi} \left[ \frac{1}{R} D_1 + \frac{1}{R^2} D_2 \right]. \quad (2.1)$$

The vectors $D_1$ and $D_2$ are given explicitly in Table 3 for a general source time-function $g(t)$ and in Table 4 for the special function $H(t)$. Spectral displacements are shown in Table 5. These tables also include the corresponding velocities and accelerations. $\alpha$ and $\beta$ are the respective velocities of the compressional and shear waves. From (2.1) we obtain the accelerations by a two-fold differentiation w.r.t. time [(A.6) and (A.7)]. We may generalize the foregoing expressions for a finite fault, the strike of which coincides with the $x_1$ axis (Fig. 2). To obtain the total contribution to the acceleration at $P$, we must integrate over the entire fault area,
taking into account the dependence of $R$, $i_h$, and $\phi_h$ on the source coordinates. Assuming that $\bar{U}$ is constant over the fault and denoting by $\alpha$ and $\beta$ the respective intrinsic compressional and shear wave velocities, Eqs. (2.1), (A.6)-(A.7) and Table 3 yield,

$$u_x^{(p)} = \frac{\bar{U} \beta^2}{12 \pi \alpha^3} \int_S \frac{dS}{R} \left( A_x F_1 \cos i_h + B_x F_2 \sin i_h \right)$$

$$u_x^{(S)} = -\frac{\bar{U}}{24 \pi \beta} \int_S \frac{dS}{R} \left( C_{\beta} F_2 \sin i_h + 6 D_{\beta} F_1 \cos i_h \right)$$

$$u_x = \frac{\bar{U} \beta^2}{12 \pi \alpha^3} \int_S \frac{dS}{R} \left[ (A_x F_1 \cos i_h - B_x F_2 \cos i_h) \cos \phi_h + B_x F_3 \sin \phi_h \right]$$

$$u_y^{(p)} = \frac{\bar{U} \beta^2}{12 \pi \alpha^3} \int_S \frac{dS}{R} \left[ (A_y F_1 \sin i_h - B_y F_2 \cos i_h) \cos \phi_h - C_y F_3 \cos \phi_h \right]$$

$$u_y^{(S)} = \frac{\bar{U}}{24 \pi \beta} \int_S \frac{dS}{R} \left[ (C_y F_2 \cos i_h + 6 D_y F_1 \sin i_h) \sin \phi_h - C_y F_3 \sin \phi_h \right]$$

$$u_y = \frac{\bar{U} \beta^2}{12 \pi \alpha^3} \int_S \frac{dS}{R} \left[ (A_y F_1 \cos i_h - B_y F_2 \cos i_h) \sin \phi_h - B_y F_3 \sin \phi_h \right]$$

with

$$A_x(t) = \bar{g}_x + \frac{4 \alpha}{R} \frac{\bar{g}_x}{R^2} \frac{9 \alpha^2}{R^3} \frac{\bar{g}_x}{R^4} + \frac{9 \alpha^3}{R^5} g_x, \quad g_x = g \left( t - \frac{R}{\alpha} \right)$$

$$B_x(t) = \frac{\alpha}{R} \frac{\bar{g}_x}{R^2} \frac{3 \alpha^2}{R^3} \frac{\bar{g}_x}{R^4} + \frac{3 \alpha^3}{R^5} g_x$$

$$C_{\beta}(t) = \bar{g}_{\beta} + \frac{3 \beta}{R} \frac{\bar{g}_{\beta}}{R^2} \frac{6 \beta^2}{R^3} \frac{\bar{g}_{\beta}}{R^4} + \frac{6 \beta^3}{R^5} g_{\beta}$$

$$D_{\beta}(t) = \frac{\beta}{R} \frac{\bar{g}_{\beta}}{R^2} \frac{3 \beta^2}{R^3} \frac{\bar{g}_{\beta}}{R^4} + \frac{3 \beta^3}{R^5} g_{\beta}$$

Fig. 1. Spherical coordinates of a field-point relative to the point of rupture initiation.
Fig. 2. Elements of a strike-slip fault.

Fig. 3. Geometry of a circular fault and its field.

The entities $F_1, F_2, F_3$ are defined in (A.5) and the angles $i_h, \phi_h$ are shown in Fig. 1. Since $g(\omega)\exp(-ik_cR)$ is the Fourier-transform of $g(t-R/c)$, the corresponding spectral accelerations are obtained with

\[
\begin{align*}
A_\alpha(\omega) &= (i\omega)^3 g(\omega) \left[ 1 - \frac{4i}{k_\alpha R} - \frac{9}{k_\alpha^2 R^2} + \frac{9i}{k_\alpha^3 R^3} \right] \exp(-ik_\alpha R) \\
B_\alpha(\omega) &= (i\omega)^3 g(\omega) \left[ -\frac{i}{k_\beta R} - \frac{3}{k_\beta^2 R^2} + \frac{3i}{k_\beta^3 R^3} \right] \exp(-ik_\beta R) \\
C_\beta(\omega) &= (i\omega)^3 g(\omega) \left[ 1 - \frac{3i}{k_\beta R} - \frac{6}{k_\beta^2 R^2} + \frac{6i}{k_\beta^3 R^3} \right] \exp(-ik_\beta R) \\
D_\beta(\omega) &= (i\omega)^3 g(\omega) \left[ -\frac{i}{k_\beta R} - \frac{3}{k_\beta^2 R^2} + \frac{3i}{k_\beta^3 R^3} \right] \exp(-ik_\beta R)
\end{align*}
\]

(2.4)

\[
k_\alpha = \frac{\omega}{\alpha}; \quad k_\beta = \frac{\omega}{\beta}.
\]

In those regions of the seismic field where we may assume $k_c R \gg 1$ in the amplitude
terms, Eqs. (2.2) lends itself to the approximations

\[ A_\alpha(\omega) \approx (i\omega)^3 g(\omega) \exp(-ik_\alpha R); \quad B_\alpha(\omega) \approx 0 \]

\[ C_\beta \approx (i\omega)^3 g(\omega) \exp(-ik_\beta R); \quad D_\beta \approx 0. \]

Since the variation of the spectral acceleration across the fault is dominated by the behavior of the propagator \( \{\exp(-ik_\alpha R)/R \} \) in the integrand (first-order effect), we can write

\[ \tilde{u}_x^{(p)}(\omega) = \frac{\bar{U} \beta^2}{12\pi^2} (i\omega)^3 g(\omega) [F_1 \cos i_\alpha]_{(0)} \int_S \frac{\exp(-ik_\alpha R)}{R} dS \]

\[ \tilde{u}_x^{(S)}(\omega) = -\frac{\bar{U}}{24\pi \beta} (i\omega)^3 g(\omega) [F_2 \sin i_\alpha]_{(0)} \int_S \frac{\exp(-ik_\beta R)}{R} dS \]

\[ \tilde{u}_y^{(p)}(\omega) = \frac{\bar{U} \beta^2}{12\pi^2} (i\omega)^3 g(\omega) [F_1 \sin i_\alpha \cos \phi_\alpha - F_3 \sin \phi_\alpha]_{(0)} \int_S \frac{\exp(-ik_\alpha R)}{R} dS \]

\[ \tilde{u}_y^{(S)}(\omega) = \frac{\bar{U}}{24\pi \beta} (i\omega)^3 g(\omega) [F_2 \cos i_\alpha \sin \phi_\alpha + F_3 \cos \phi_\alpha]_{(0)} \int_S \frac{\exp(-ik_\beta R)}{R} dS \]

where the subscript \((0)\) indicates a value at the source. The functions

\[ \int_S \frac{\exp(-ik_\alpha R)}{R} dS, \quad \int_S \frac{\exp(-ik_\beta R)}{R} dS \]

constitute the fundamental interference integrals. One must evaluate it in order to obtain a quantitative assessment of the acceleration field in those ranges where \( k_\alpha R \gg 1 \).

Next, we represent the source time-function by the expression

\[ g(t) = H(t-\tau) + Y(t)[H(t) - H(t-\tau)], \quad \tau > 0, \quad Y(0) = 0, \quad 1 - Y(\tau) = 0, \quad (2.6) \]

where \( H(t) \) is the Heavyside unit step-function and \( \tau \) is the rise-time. We choose \( Y(t) \) in such a way that near \( t=0 \) it behaves like \( t^N \) where \( N \geq 3 \), while near \( t=\tau \), \( 1 - Y(t) \) behaves like \( (t-\tau)^N \). It is then found that

\[ \frac{\partial^m}{\partial t^m} g(t) = [H(t) - H(t-\tau)] \frac{\partial^m}{\partial t^m} Y(t), \quad m = 1, 2, 3. \quad (2.7) \]

The specific choice of \( Y(t) \) is problematic: BEN-MENAHEM and TOKSOZ (1963) deduced \( Y(t) = 1 - \exp(-t/\tau) \) from observations of long-period surface-waves. PEKERIS et al. (1963) introduced a step-function with rounded shoulders \( Y(t) = 2(t/\tau)^2, \ 0 < t < \tau/2, \ Y(t) = 1 - 2(t/\tau - 1)^2, \ \tau/2 < t < \tau \). The corresponding \( g(t) \) is then
composed of four segments welded together at three joints $\tau = 0, \tau/2, \tau$, such that the values of the function and its first derivative are continuous at the joints. Haskell (1964), selected

$$ Y(t) = \frac{t}{\tau} - \frac{1}{\tau} \sin \frac{\omega_0 t}{\omega_0}, \quad \omega_0 = \frac{2\pi}{\tau^*} \tag{2.8} $$

where $\tau^*$ is the period of the high-frequency particle-motion that is superimposed on the ramp motion.

To date, however, there is some observational evidence that at wavelength scales smaller than the rupture extent, the source time-function may be irregular. Nevertheless, one can always define an effective differentiable time-function with the characteristics of (2.8), provided that the source time-function embodies two different time scales: (1) a rise-time $\tau$; (2) a high-frequency perturbation with $\tau^* \ll \tau$. This latter time scale is in effect a representation of the irregular source motion that accompanies the slower build-up of the displacement from zero to maximal offset. In this sense, the choice of (2.8) keeps the essential features of a realistic time-function without being too complicated.

The explicit expression for the first three time derivatives of $g(t)$ are

$$ \dot{g}(t) = \frac{1}{\tau} (1 - \cos \omega_0 t)[H(t) - H(t - \tau)] $$

$$ \ddot{g}(t) = \frac{2\pi}{\tau \tau^*} \sin \omega_0 t \left[ H(t) - H(t - \tau) \right] \tag{2.9} $$

$$ \dddot{g}(t) = \frac{4\pi^2}{\tau^2 \tau^*^2} \cos \omega_0 t \left[ H(t) - H(t - \tau) \right]. $$

If we denote the Fourier transform of $g(t)$ by $g(\omega)$ and write $g(t) \rightarrow g(\omega)$, then $\dot{g}(t) \rightarrow i\omega g(\omega)$, $\ddot{g}(\omega) \rightarrow (i\omega)^2 g(\omega)$, etc.

The equivalents of Eqs. (2.5) in the time-domain are obtained directly with the aid of Eqs. (2.7) and (2.8)

$$ \dddot{u}_2^{(p)}(t) = \frac{\pi \bar{U} \beta^2}{3\alpha^2 \pi (\tau^*)^2} \left( F_1 \cos i_i_0 \right) \int_S \frac{\cos \omega_0 \left( \frac{t - R}{\alpha} \right)}{R} \times \left[ H \left( t - \frac{R}{\alpha} \right) - H \left( t - \frac{R}{\alpha} - \tau \right) \right] dS \tag{2.10} $$

$$ \dddot{u}_2^{(s)}(t) = -\frac{\pi \bar{U}}{6\beta \pi (\tau^*)^2} \left( F_2 \sin i_i_0 \right) \int_S \frac{\cos \omega_0 \left( \frac{t - R}{\beta} \right)}{R} \times \left[ H \left( t - \frac{R}{\beta} \right) - H \left( t - \frac{R}{\beta} - \tau \right) \right] dS, $$
and similar expressions for the remaining components.

Exact evaluation of the interference integrals is possible only for special source geometries. For example, let us take the acceleration field of a circular fault. Here we must evaluate the integral

$$ I(a, \Delta_0, y; k_c) = \int_0^{2\pi} \int_0^a \frac{\exp(-ik_c R)}{R} d\phi d\Delta $$(2.11)

with a geometrical set-up like that in Fig. 3. We begin with the known expansion

$$ \frac{\exp(-ik_c R)}{R} = \sum_{m=-\infty}^{\infty} \{\exp[im(\phi - \phi_0)]\} \int_0^{\infty} J_m(k\Delta)J_m(k\Delta_0) \{\exp(-\nu_c y)\} \frac{k dk}{\nu_c}, $$ (2.12)

where

$$ R^2 = y^2 + \rho^2, \quad \rho^2 = \Delta^2 + \Delta_0^2 - 2\Delta\Delta_0 \cos(\phi - \phi_0), \quad \text{and} \quad \nu_c = [k^2 - k_c^2]^{1/2}. $$

Applying the known results

$$ \int_0^{2\pi} \exp[im(\phi - \phi_0)]d\phi = 2\pi \delta_{m0}, \quad \int_0^a J_m(k\Delta)\Delta d\Delta = \frac{a}{k} J_1(ka), $$ (2.13)

Eqs. (2.10) becomes

$$ I = 2\pi a \int_0^{\infty} J_0(k\Delta_0)J_1(ka) \{\exp[-y(k^2 - k_c^2)^{1/2}]\} \frac{dk}{[k^2 - k_c^2]^{1/2}}. $$ (2.14)

The value of the interference integral on the axis $\Delta_0 = 0$ is found to be

$$ I = -\frac{8\pi}{k_c} \sin \frac{k_c}{2} \sqrt{y^2 + a^2 - y} \exp \left[ -\frac{ik_c}{2} \sqrt{y^2 + a^2 + y} \right] $$ (2.15)

$$ \approx -\frac{2S}{y} \exp(-ik_c y) \quad \text{for} \quad y \gg a, \quad S = \pi a^2. $$

3. Regionalization of the Far Acceleration Field

The behavior of the acceleration at various distances from the fault depends on the relative sizes of the wavelength $\lambda$, the distance $R$ and the fault dimensions $L$ and $W$. The key element is the distance $R$ which we shall next expand in terms of auxiliary geometrical entities. Let us focus our attention on Fig. 4. A binomial expansion of $R$, according to

$$ (1+\varepsilon)^{1/2} = 1 + \frac{1}{2} \varepsilon - \frac{1}{8} \varepsilon^2 + \frac{1}{16} \varepsilon^3 + \cdots, \quad |\varepsilon| \ll 1 $$ (3.1)

yields

$$ R = (r^2 + r_1^2 - 2rr_1 \cos \Theta)^{1/2}. $$
We shall divide the far-field of the source into two regions:

3.1 The true far-field zone \([k_c r \gg 1, (L/r)^2 \ll 1, (W/r)^2 \ll 1]\)

In the expansion of \(R\), we neglect terms of order \(1/r\) and beyond, such that (Fig. 4)

\[
R \approx r - r_1 \cos \Theta + \frac{r_1^2}{2r} \sin^2 \Theta + \frac{r_1^3}{2r^2} \cos \Theta \sin^2 \Theta + O\left(\frac{r_1^4}{r^3}\right),
\]

provided

\[
\left|\frac{r_1^2}{r^2} - \frac{2r_1}{r} \cos \Theta\right| \ll 1.
\]

Since the extreme value of \(r_1^2\) (Fig. 4) is \(L^2 + W^2\), the validity of Eq. (3.4) is governed by the condition

\[
\frac{L^2 + W^2}{2r^2} \ll 1.
\]

With this approximation, the source appears only as a point radiator. The source dimensions are significant only in the phase term \(\exp(-ik_r R)\), where the first two terms of the expansion in Eq. (3.2) are kept. Geometrically, this means that the rays along \(R\) and \(r\) (Fig. 4) are parallel. This is strictly true for point \(P\) at infinity only, but the region in which this is a satisfactory approximation defines the far-field...
region. Quantitatively, the neglect of quadratic terms in the phase leads to a maximal error of \((k_c/2r)\left[(L/2)^2 + (W/2)^2\right] \text{ rad. This error will not exceed, say, } \pi/8 \text{ radians (or } \lambda/16) \text{ if for } L > W\)

\[
r > r_c = \frac{2(L^2 + W^2)}{\lambda} \approx \frac{2L}{\lambda}.
\]

For any given wavelength, we may consider \(r_c\) as a limiting distance from the fault, beyond which the 'far-field' begins. Since \(k_c r >> 1\) is a prerequisite in the far-field, the range of 'permissible' wavelengths for a given distance \(r\) and fault dimension \(L, W\) are

\[
\frac{2(L^2 + W^2)}{r} < \lambda \ll 2\pi r.
\]

The physical significance of \(r_c\) is that a point sensor located at the distance \(8(L^2 + W^2)/\lambda\) or more from the fault would record a plane wave whose phase does not deviate more than \(\pi/8\) from a spherical wave.

### 3.2 The Fresnel zone \([L^2 + W^2]^{3/4}/\lambda^{1/2} < r < 2(L^2 + W^2)/\lambda, k_c r \gg 1]\)

In this region, the second order term in the binomial expansion of \(R\) is included in the phase term. However, it is still sufficiently accurate to drop the second term in \(R\) in the amplitude factor. Again, all terms with radial variation other than \(1/R\) are neglected as are those in the far-field. Unlike the far-field, the azimuthal and colatitudinal variation of the field in the Fresnel zone do change with \(R\), and the wave-forms are distorted.

The point where one stops using the Fresnel approximation depends on how accurately one needs to know the field at that point. A practical lower limit for the Fresnel region may be taken at distance \(r\) for which the fourth term in the expansion for \(R\) produces a phase error of not more than \(\pi/8\) radians.

Since the extreme value of \(\{\sin^2 \Theta \cos \Theta\}\) is \(2/3\sqrt{3} = (0.6204)^2\), the above condition reads

\[
r_t \geq 0.62 \frac{(L^2 + W^2)^{3/4}}{\lambda^{1/2}} \approx 0.62 L \sqrt{\frac{L}{\lambda}}.
\]

The limits of the Fresnel regions are then

\[
r_t < r < r_c.
\]

Clearly, this zone does not have rigid boundaries and the transition to the far-field is gradual.

### 4. Approximations of the Interference Integrals

#### 4.1 The stationary phase approximation

Under certain conditions, one may evaluate the double Fourier integral over a
finite domain $S$ by the two-dimensional stationary phase approximation. For a single stationary point $(x_m, y_m)$, the integral is approximated as follows

$$\int_S A(\xi, \eta) \exp[-ik_c s(\xi, \eta)] d\xi d\eta = \frac{2\pi A(\xi_m, \eta_m)}{k_c \sqrt{s_{\xi m} s_{\eta m} - (s_{\eta m})^2}} \exp\left[-\frac{i}{2} \left(-ik_c s(\xi_m, \eta_m) - \frac{\pi i}{2}\right)\right], \quad c = \alpha, \beta. \quad (4.1)$$

Applying this method to the integrals in Eqs. (2.10) over a rectangular fault $0 \leq \xi \leq L$, $0 \leq \eta \leq W$ with $A(\xi, \eta) = f(\xi, \eta)/R^n$, $s(\xi, \eta) = R$, we find

$$Q(x, y, 0) = \int_0^L \int_0^W d\xi d\eta f(\xi, \eta) \exp\left(-ik_c R\right) R^n$$

$$= \frac{2\pi}{k_c y^{n-1}} \left\{ \exp\left(-ik_c y - \frac{\pi i}{2}\right) \right\} f(\xi_m, \eta_m) \quad (4.2)$$

where (Fig. 4)

$$R^2 = (x - x_1)^2 + y^2 + z^2, \quad k_c y \gg 1,$$

$$x_1 = \xi, \quad z = r \cos i_h - \eta.$$

If we incorporate the factor

$$\{\exp(i\omega_n t)\} \left[ H\left(t - \frac{R}{c}\right) - H\left(t - \frac{R}{c} - \tau\right) \right]$$

in Eq. (4.2) and take the real part of each side, we obtain

$$\int_0^L \int_0^W d\xi d\eta f(\xi, \eta) \frac{\cos \omega_0 \left(t - \frac{R}{c}\right)}{R^n} \left[ H\left(t - \frac{R}{c}\right) - H\left(t - \frac{R}{c} - \tau\right) \right]$$

$$\approx \frac{2\pi c}{\omega_0 y^{n-1}} f(\xi_m, \eta_m) \sin \omega_0 \left(t - \frac{y}{c}\right) \left[ H\left(t - \frac{y}{c}\right) - H\left(t - \frac{y}{c} - \tau\right) \right]. \quad (4.3)$$

Note that, within the bounds of the limitation imposed on this approximation, the main contribution to the integrals in Eqs. (2.10) is independent of the fault dimensions $L$ and $W$. This has already been demonstrated in one dimension by Aki (1968).

A typical acceleration component, such as in Eqs. (2.10), will now have the leading term

$$\ddot{u}_{sl}(t) \approx \frac{\bar{U}}{\tau^*} \left[ \frac{\pi}{6} (F_2 \sin i_h) \right] \sin \omega_0 \left(t - \frac{y}{\beta}\right) \left[ H\left(t - \frac{y}{\beta}\right) - H\left(t - \frac{y}{\beta} - \tau\right) \right], \quad (4.4)$$

along with similar expressions for the other components.
4.2 Fresnel approximation

We evaluate each integral in Eqs. (2.10) with \(y_1=0\). Using the binomial expansion (3.1) we have

\[
R = \left[ (x-x_1)^2 + y^2 + z_1^2 \right]^{1/2} \\
= y \left[ 1 + \frac{(x-x_1)^2}{y} + \left( \frac{z_1}{y} \right)^2 \right]^{1/2} = y[1 + \varepsilon]^{1/2} \\
= y \left[ 1 + \frac{1}{2} \left( \frac{x-x_1}{y} \right)^2 + \left( \frac{z_1}{y} \right)^2 \right] + \frac{y}{8} \left[ \left( \frac{x-x_1}{y} \right)^2 + \left( \frac{z_1}{y} \right)^2 \right]^2 + \cdots \\
= y + \frac{(x-x_1)^2}{2y} + \frac{z_1^2}{2y} + \frac{1}{8y^3} [(x-x_1)^2 + z_1^2]^2 + \cdots.
\]

Then, we replace \(1/R\) by \(1/y\) in the amplitude of the integrand while we put

\[
k_e R = k_e y + \frac{k_s}{2y} (x-x_1)^2 + \frac{k_s}{2y} z_1^2
\]

in the phase of the integrand, assuming there that

\[
y^3 \gg \frac{\pi}{4\lambda} [(x-x_1)^2 + z_1^2]_{\text{max}}.
\]

Apart from factor \(f(\xi_m, \eta_m)\), the integral in (4.2) becomes

\[
Q(x, y) = \frac{2\pi}{k_e} \left\{ \exp(-ik_e y) \right\} \left[ \Phi(\eta_2) - \Phi(\xi_1) \right] \Phi(\eta_2),
\]

where

\[
\Phi(\alpha) = \int_0^\alpha \exp \left( -\frac{i\pi}{2} \tau^2 \right) d\tau = C(\alpha) - iS(\alpha).
\]

\[
\xi_1 = \frac{L+2x}{\sqrt{2y\lambda}}; \quad \xi_2 = \frac{L-2x}{\sqrt{2y\lambda}}; \quad \eta_2 = \frac{W}{\sqrt{2y\lambda}} = -\eta.
\]

An alternative derivation of (4.9) is given in Appendix A.

If \((y\lambda)\) is sufficiently small, we can approximate \(\Phi(\alpha)\) by \(\Phi(\infty)\). Under these conditions

\[
\Phi(\xi_2), \Phi(\eta_2) \to \frac{1-i}{2}, \Phi(\xi_1), \Phi(\eta_1) \to \left( \frac{1-i}{2} \right) \quad \text{and} \quad Q \to \frac{2\pi}{ik_e} \exp(-ik_e y).
\]

This agrees with the stationary-phase approximation of (4.2) for \(n=1\) and \(f=1\). The second equation in (2.10) then assumes the form

\[
\tilde{u}_z^{(S)}(t) = \left[ \frac{US}{\tau(\pi)^2} \right] \left[ \frac{\pi}{12y\beta} \right] \left\{ F_2 \sin i_0(pq) \sin \left\{ \omega_0 \left( t - \frac{y}{\beta} \right) + (\theta_1 - \theta_2) \right\} \right\}
\]
Here are the Fresnel integrals. We have noted that the integrated acceleration field from a rectangular fault has similar characteristics to the diffraction pattern of a rectangular aperture illuminated by a plane wave.

As mentioned above, for sufficiently small values of \( y \), \( k_c/\pi y \) is an exceedingly large number and the following approximations are accurate

\[
\begin{align*}
\int_{\xi_1}^{\xi_2} \cos \left( \frac{\pi \tau^2}{2} \right) d\tau & = \int_{\eta_1}^{\eta_2} \sin \left( \frac{\pi \tau^2}{2} \right) d\tau = \begin{cases} 1 & \frac{L}{2} < x < \frac{L}{2} \\ 0 & x > \frac{L}{2} \text{ or } x < -\frac{L}{2} \end{cases} \\
\int_{\eta_1}^{\eta_2} \cos \left( \frac{\pi \tau^2}{2} \right) d\tau & = \int_{\eta_1}^{\eta_2} \sin \left( \frac{\pi \tau^2}{2} \right) d\tau = \begin{cases} 1 & \frac{W}{2} < z < \frac{W}{2} \\ 0 & z > \frac{W}{2} \text{ or } z < -\frac{W}{2} \end{cases}
\end{align*}
\]

It means that deep within the Fresnel region, the acceleration field is the geometrical projection of the particle velocity distribution, \( \vec{U}/\tau \), on the fault itself (\( pq = 2 \), \( \theta_1 - \theta_2 = 0 \)).

The acceleration on a line normal to the fault, \( (x = L/2) \), is given by (4.12) with

\[
\begin{align*}
p & = \frac{1}{v_L} \left[ C^2(v_L) + S^2(v_L) \right]^{1/2} ; \\
v_L & = \frac{L}{\sqrt{2y\lambda}} ;
\end{align*}
\]

The values of \( v_L \) vs. \( f(v_L) = [C^2(v_L) + S^2(v_L)]^{1/2} \) are shown in Table 1. It is observed that beyond \( v_L = 1/2 \) we may accept \( f(v_L) \approx 1/\sqrt{2} \) without a serious error. The corresponding relation between the various length-parameters is \( y = 2L^2/\lambda \). We may thus use the \( v_L \)-value as a regionalization-index:
Table 1. Modulus of the Fresnel integral as a function of the regionalization-index.

<table>
<thead>
<tr>
<th>$v_r$ = $\frac{L}{\sqrt{2y\lambda}}$</th>
<th>$[C^2(v_r) + S^2(v_r)]^{\frac{1}{2}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>0.5</td>
<td>0.496</td>
</tr>
<tr>
<td>0.7</td>
<td>0.682</td>
</tr>
<tr>
<td>0.9</td>
<td>0.837</td>
</tr>
<tr>
<td>1.0</td>
<td>0.895</td>
</tr>
<tr>
<td>1.5</td>
<td>0.828</td>
</tr>
<tr>
<td>1.8</td>
<td>0.561</td>
</tr>
<tr>
<td>2.0</td>
<td>0.597</td>
</tr>
<tr>
<td>3.5</td>
<td>0.675</td>
</tr>
<tr>
<td>5.0</td>
<td>0.753</td>
</tr>
<tr>
<td>8.5</td>
<td>0.696</td>
</tr>
<tr>
<td>$\infty$</td>
<td>$0.7071=\frac{1}{\sqrt{2}}$</td>
</tr>
</tbody>
</table>

Note that the regionalization in Chapter 3 is made in terms of the distance $r$ from the rupture initiation to the observer, whereas in Chapter 4 we use the normal distance $y$ from the faults strike to the observer, as the regionalization-scale. Simple geometrical relations exist between $r$ and $y$, which are obvious from Fig. 4. In the far-field one can use either of them since both are of the same order of magnitude. However, in the Fresnel and near-fault zones, $y$ is physically more meaningful since it arises naturally by means of the stationary-phase approximation.

5. The Effect of Propagating Rupture

5.1 The-far-field

So far we have considered the acceleration field $\ddot{u}(r, t)$ excited by a finite and fully coherent shear fault with dislocation $\ddot{U}$, area $S$ and a source time function $g(t)$ with rise-time $\tau$ and modulation-time $\tau^*$. We shall make this source model more realistic by introducing a rupture velocity $v_r$. Figure 5 shows a model of a propagating rupture in an unbounded elastic solid. $\alpha$ is the angle that the point of fracture initiation makes with the direction of the rupture. A rupturing source introduces three new parameters into the physical setup: The scalars $\alpha/v_r$ and $\beta/v_r$.
Fig. 5. Propagating rupture in an unbounded medium.

known as the Mach-numbers and the angle on the fault plane between the direction of the displacement and the direction of rupture.

Let us begin with the simplest case of a rupture with a fixed velocity $v_f$ in the direction of the strike. The radial component of the compressional acceleration from a point dislocation is given explicitly in Table 1 (Ben-Menahem and Singh, 1981, pp. 299–231)

$g_{\beta} = g(t - R/\beta)$ etc.

The evaluation of the time-domain displacement from a propagating rupture in the far-field ($R \gg L, W$) is obtained from Eq. (5.1) via a number of steps:

1. We replace $R$ by $r$ (Fig. 5) in the amplitude factors of Eq. (5.1) and $(t - R/\beta)$ in the argument of the source time function by \{\(t - r/\beta - \xi/v_f(1 - \xi/v_f)\cos \Theta\)\} where $\cos \Theta = \sin \phi_a \cos \phi_h$ (Fig. 5).

2. We integrate the r.h.s. of Eq. (5.1) with $dS = d\xi d\eta$ over the rectangle \{(0, L), (0, W)\} while keeping $U$ fixed. The values of $i_h$ and $\phi_h$ in $F_1$ and $\cos \Theta$ are taken as those at the initiation of rupture.

3. With $g(t)$ defined in Eq. (2.8), we keep in Eq. (5.1) the term with $\delta_a$ only. This approximation implies that
which is certainly true everywhere except in the close neighborhood of the fault. The integration over $\xi$ yields

\[
\int_0^L \frac{\partial}{\partial t} \frac{\partial}{\partial \xi} \left[ t - \frac{r}{\alpha} - \frac{\xi}{v_r} \left( \frac{1 - \frac{v_r}{\alpha} \cos \Theta}{1 - \frac{v_r}{\alpha} \cos \Theta} \right) \right] d\xi
= \frac{v_r}{1 - \frac{v_r}{\alpha} \cos \Theta} \int_0^L \frac{\partial}{\partial \xi} \left[ t - \frac{r}{\alpha} - \frac{\xi}{v_r} \left( \frac{1 - \frac{v_r}{\alpha} \cos \Theta}{1 - \frac{v_r}{\alpha} \cos \Theta} \right) \right] d\xi
= \frac{v_r}{1 - \frac{v_r}{\alpha} \cos \Theta} \left[ g \left( t - \frac{r}{\alpha} \right) - g \left( t - \frac{r}{\alpha} - \frac{L}{v_r} \left( 1 - \frac{v_r}{\alpha} \cos \Theta \right) \right) \right],
\]

while the integration over $W$ simply multiplies the entire expression by $W$. The result is

\[
\ddot{u}_R^{(p)} = \frac{\bar{U} W}{12\pi r} \left[ \frac{\beta^2}{\alpha^2} \right] \left[ \frac{M_a F_1(l_h, \phi_h)}{1 - M_a \cos \Theta} \right] \frac{\partial^2}{\partial \xi^2} \left[ g \left( t - \frac{r}{\alpha} - \frac{L}{t_d a} \right) \right].
\]

Carrying out the integration with the aid of Table 1 as indicated in Eq. (5.3), we arrive at the explicit expressions for the entire acceleration field:

Compressional acceleration

\[
\ddot{u}_R^{(p)}(t) = \frac{\bar{U} W}{12\pi r} \left[ \frac{\beta^2}{\alpha^2} \right] \left[ \frac{M_a F_1(l_h, \phi_h)}{1 - M_a \cos \Theta} \right] \frac{\partial^2}{\partial \xi^2} \left[ g \left( t - \frac{r}{\alpha} - \frac{L}{t_d a} \right) \right]
\times \left[ 4a \frac{G_a}{r} + \frac{9a^2}{r^2} G_a + \frac{9a^3}{r^3} \right] \int_0^L g \left( t - \frac{r}{\alpha} - \frac{\xi}{L} \right) d\xi
\]

\[
\ddot{u}_t^{(p)}(t) = -\frac{\bar{U} W}{12\pi r^2} \left[ \frac{\beta^2}{\alpha^2} \right] \left[ \frac{M_a F_2(l_h, \phi_h)}{1 - M_a \cos \Theta} \right] \frac{\partial^2}{\partial \xi^2} \left[ g \left( t - \frac{r}{\alpha} - \frac{L}{t_d a} \right) \right]
\times \left[ aG_a + 3\frac{a^2}{r} G_a + \frac{3a^2}{r^2} \int_0^L g \left( t - \frac{r}{\alpha} - \frac{\xi}{L} \right) d\xi \right]
\]

\[
\ddot{u}_g^{(p)}(t) = -\frac{\bar{U} W}{12\pi r^2} \left[ \frac{\beta^2}{\alpha^2} \right] \left[ \frac{M_a F_3(l_h, \phi_h)}{1 - M_a \cos \Theta} \right] \frac{\partial^2}{\partial \xi^2} \left[ g \left( t - \frac{r}{\alpha} - \frac{L}{t_d a} \right) \right]
\times \left[ aG_a + 3\frac{a^2}{r} G_a + \frac{3a^2}{r^2} \int_0^L g \left( t - \frac{r}{\alpha} - \frac{\xi}{L} \right) d\xi \right].
\]

Shear acceleration

\[
\ddot{u}_R^{(p)}(t) = -\frac{\bar{U} W}{4\pi r^2} \left[ \frac{M_b F_1(l_h, \phi_h)}{1 - M_b \cos \Theta} \right].
\]
In the following we shall neglect all terms in Eqs. (5.5)-(5.7) which fall off with distances faster than \( r^{-1} \).

We specify next that the source time-function is the one given by Eq. (2.8). The temporal behavior of \( \tilde{u}_{\phi}^{(S)}(t) \) depends on whether \( \tau > t_{ds} \) or \( \tau < t_{ds} \). In both cases we write

\[
\tilde{u}_{\phi}^{(S)}(t) = \frac{\tilde{U} W}{24\pi r} \left[ \frac{M_x F_3(i_{ph}, \phi_h)}{1 - M_x \cos \Theta} \right] \times \left[ \frac{G_x(t)}{\beta} \hat{G}_x + \frac{6\beta^2}{r^2} G_x + \frac{6\beta^2}{r^3} \right] \int_0^L g \left( t - \frac{r}{\beta} \frac{\xi}{L} t_{ds} \right) d\xi
\]

(5.6)

where

\[
M_x = \frac{v_r}{\alpha} < 1, \quad t_{ds} = \frac{L}{v_r} (1 - M_x \cos \Theta)
\]

\[
M_y = \frac{v_r}{\beta} < 1, \quad t_{dy} = \frac{L}{v_r} (1 - M_y \cos \Theta)
\]

(5.7)

\[
G_x(t) = g \left( t - \frac{r}{\alpha} \right) - g \left( t - \frac{r}{\alpha} - t_{ds} \right)
\]

\[
G_y(t) = g \left( t - \frac{r}{\beta} \right) - g \left( t - \frac{r}{\beta} - t_{dy} \right)
\]

In the following we shall neglect all terms in Eqs. (5.5)-(5.7) which fall off with distances faster than \( r^{-1} \).

We specify next that the source time-function is the one given by Eq. (2.8). The temporal behavior of \( \tilde{u}_{\phi}^{(P)}(t) \) depends on whether \( \tau > t_{ds} \) or \( \tau < t_{ds} \). In both cases we write

\[
\tilde{u}_{\phi}^{(P)}(t) = \frac{\tilde{U} W}{6\pi \tau^2 r} \left[ \frac{M_x F_1(i_{ph})}{1 - M_x \cos \Theta} \right] \Gamma(t),
\]

(5.8)

where

\[
\Gamma(\tau < t_{ds}) = \begin{cases} 
0 & \quad t - \frac{r}{\alpha} < 0 \\
\sin \omega_0 \left( t - \frac{r}{\alpha} \right) & \quad 0 < t - \frac{r}{\alpha} < \tau \\
0 & \quad \tau < t - \frac{r}{\alpha} < t_{ds} \\
-\sin \omega_0 \left( t - \frac{r}{\alpha} - t_{ds} \right) & \quad t_{ds} < t - \frac{r}{\alpha} < t_{ds} + \tau \\
0 & \quad t - \frac{r}{\alpha} > t_{ds} + \tau
\end{cases}
\]

(5.9)
Thus, the acceleration signal for the case \( t_{da} > \tau \) consists of two distinct arrivals: The first, of duration \( \tau \), peaks at \( t_1 = r/\alpha + \tau^*/4 \) and the second, with reverse polarity and duration \( \tau \), peaks at \( t_2 = t_1 + t_{da} \). In the case where \( \tau > t_{da} \), there is one continuous signal of duration \( (\tau + t_{da}) \).

Introducing our source time-function as indicated in Eqs. (5.9)-(5.10), and deleting the phase function \( \Gamma(t) \), we obtain explicit formulae for the peak accelerations of the various field components. Results depend on whether:

1. \( M_\beta < 1 \) (subshear rupture)
2. \( M_\beta > 1 \) (supershear rupture)
3. \( M_\beta = 1 \) (transhear rupture);

The cases of interest are:

1. \( M_\beta \leq 1 \)
2. \( M_\beta > 1 \)

\[
\Gamma(\tau > t_{da}) = \begin{cases} 
0 & \frac{r}{\alpha} - \tau < 0 \\
\sin \omega_n \left( \frac{t - \frac{r}{\alpha}}{\tau} \right) & 0 < \frac{r}{\alpha} - \tau < t_{da} \\
2 \sin \left( \frac{\omega_n t_{da}}{2} \right) \cos \omega_n \left( \frac{t - r}{\alpha} - \frac{1}{2} t_{da} \right) & t_{da} < \frac{r}{\alpha} - \tau < \tau \\
-\sin \omega_n \left( \frac{t - r}{\alpha} - t_{da} \right) & \tau < \frac{r}{\alpha} - t_{da} + \tau \\
0 & \frac{r}{\alpha} > t_{da} + \tau.
\end{cases}
\] (5.10)

\[
\{ \tilde{u}_R \}_{\text{peak}}^{(P)} = \frac{\bar{U} W}{6 \tau^* r} \left[ \frac{\beta^2}{\alpha^2} \right] \frac{M_\beta F_1}{1 - M_\beta \cos \Theta} \\
\{ \tilde{u}_{ih} \}_{\text{peak}}^{(S)} = \frac{\bar{U} W}{12 \tau^* r} \left[ \frac{M_\beta F_2}{1 - M_\beta \cos \Theta} \right] \\
\{ \tilde{u}_{\phi h} \}_{\text{peak}}^{(S)} = \frac{\bar{U} W}{12 \tau^* r} \left[ \frac{M_\beta F_3}{1 - M_\beta \cos \Theta} \right]
\] (5.11) (5.12) (5.13)

\[
t_{db} < t - r/\beta < \tau < t_{da};
\]

\[
\{ \tilde{u}_{ih} \}_{\text{peak}}^{(S)} = \frac{\pi U S}{6 \beta r \tau^*} \left[ \sin \frac{X}{X} \right] F_2, \quad X = \frac{\pi L}{C \tau^*} (1 - M_\beta \cos \Theta) \\
\{ \tilde{u}_{\phi h} \}_{\text{peak}}^{(S)} = \frac{\pi U S}{6 \beta r \tau^*} \left[ \sin \frac{X}{X} \right] F_3.
\] (5.14)
Let us denote $\frac{\beta}{\nu_f} = \cos \Theta_0$. Then for arbitrary $g(t)$, we have according to Eq. (2.8)

$$\ddot{u}_b^{(S)} = \left[ \frac{\bar{U} W}{24\pi r} \right] F_2 \frac{\partial^2}{\partial t^2} \left\{ \frac{g \left( \frac{t-r}{\beta} \right) - g \left( \frac{t-r-L}{\beta} \cos \Theta_0 - \cos \Theta \right)}{\cos \Theta_0 - \cos \Theta} \right\}. \quad (5.15)$$

Define $h = (L/\beta)(\cos \Theta_0 - \cos \Theta), \ t^* = t - r/\beta$ and write

$$\ddot{u}_b^{(S)} = \left[ \frac{\bar{U} L}{24\pi r \beta} \right] F_2 \frac{\partial^2}{\partial t^2} \left\{ \frac{g(t^*) - g(t^* - h)}{h} \right\}$$

$$= \frac{\bar{U} L}{24\pi r \beta} F_2 \frac{\partial^3}{\partial t^3} g \left( \frac{t-r}{\beta} \right). \quad (5.16)$$

Specifying $g(t)$ as in Eq. (2.8), we obtain

$$\left\{ \ddot{u}_{b, \text{peak}}^{(S)} \right\} = \left[ \frac{\pi \bar{U} S}{6\beta r \tau(t^*)^2} \right] F_2,$$

$$\left\{ \ddot{u}_{b, \text{peak}}^{(S)} \right\} = \left[ \frac{\pi \bar{U} S}{6\beta r \tau(t^*)^2} \right] F_3. \quad (5.17)$$

Note that these are the same as Eqs. (5.14) in the limit $X \to 0$.

### 5.2 The near fault zone

Our former analysis can account for the kinematic effect of a propagating rupture with velocity $\nu_f$, say in the x direction. In the frequency domain it is done by adding a phase delay $\{ -i\omega(\xi/\nu_f) \}$ to $\{ -ik_c R \}$ in the interference integrals. In the

![Fig. 6. Geometrical interpretation of the stationary-phase approximation. (a) Side-view; (b) Top-view. For $\xi_m = 0$ we have $x/y = \cot \Lambda$ where $\Lambda$ indicates the apparent direction of an 'equivalent source.'](image)
time domain, we add the time-delay \(-\omega_n(\xi/v_f)\) to the argument of the cosine function in the integrands.

A quick look at the results of this operation is afforded by again using the stationary-phase approximation. We put (Fig. 6)

\[ s(\xi, \eta) = \frac{c}{v_f} \xi + R(\xi, \eta), \quad R^2 = (x - \xi)^2 + y^2 + \eta^2. \]

Differentiation yields

\[
\frac{\partial s}{\partial \eta} = \frac{\partial R}{\partial \eta} = \frac{\eta}{R} = 0; \quad \eta_m = 0
\]

\[
\frac{\partial^2 s}{\partial \eta \partial \xi_m} = 0; \quad \frac{\partial^2 s}{\partial \eta^2} = \frac{1}{R_m}; \quad \frac{\partial^2 s}{\partial \xi^2} = \frac{1}{R_m} \left(1 - \frac{c^2}{v_f^2}\right)
\]

Therefore, with no restrictions on \(R\)

\[
\int_0^L \int_0^W f(\xi, \eta) \frac{1}{R^m} \exp \left[ -i \omega \left( \frac{\xi}{v_f} + \frac{R}{c} \right) \right] d\xi d\eta 
\]

\[
\approx \frac{2\pi f(\xi_m 0)}{ikc R_m^{n-1}} \left(1 - \frac{c^2}{v_f^2}\right)^{-1/2} \exp \left[ -ikc \left( \frac{c}{v_f} x + y \left(1 - \frac{c^2}{v_f^2}\right)^{1/2} \right) \right].
\]

Note that the condition \(\partial s/\partial \xi = 0\) has the geometrical interpretation (Fig. 6)

\[
\frac{c}{v_f} = \cot \theta = \sin \phi_1 \cos \phi_1.
\]

Consequently, there will be extreme radiation in the rupture direction (\(i_1 = \pi/2, \phi_1 = 0\)) if \(v_f = c\). If \(v_f < c\), there is an exponential attenuation perpendicular to the fault governed by the factor \(\exp(-((\omega I)/v_f)\gamma(1 - v_f^2/c^2)^{1/2}))\).

The shape of the arriving signal is obtained by the application of the foregoing result to (2.10) in which a time-delay term was added to the phase, namely

\[
\ddot{u}_{z}^{(S)}(t) = -\frac{\pi \bar{U}}{6 \beta \tau(\tau^*)^2} \left( F_2 \sin i_a \right) \int_{S} \frac{dS}{R} \cos \omega_0 \left( t - \frac{R}{\beta} - \frac{\xi}{v_f} \right) \times \left[ H \left( t - \frac{R}{\beta} - \frac{\xi}{v_f} \right) - H \left( t - \frac{R}{\beta} - \frac{\xi}{v_f} - \tau \right) \right].
\]

Hence, for \(v_f > \beta\)
6. Applications: Scaling Laws and the Shape of Isoseismals

Our former theory can yield an approximation to the observed dependence of the peak horizontal acceleration on the source's magnitude and epicentral distance. A summary of our theoretical results is shown in Table 2.

We have restricted our discussion to regions of the acceleration field where \( K_c R > 1 \). In the frequency domain this restriction will imply \( R > R_c = \lambda / 2\pi \). For shear accelerations with periods around 0.5 s, \( R_c \) is of the order of a few hundred meters.

Table 2. Peak horizontal accelerations as predicted by the kinematic source model (\( r^* \) is assumed to be independent of magnitude).

| Case | Universal scaling-low | Eq. | Values of shear Mach-number (\( M_s \)) rupture time and rise-time (\( \tau \)) Mag. dependence |
|------|-----------------------|-----|-------------------------------------------------|---------------------------------|
| I    | \( \bar{U} S / (\tau r^*) \) | \( R^{-1} e^{-\gamma R} \) | (5.17) \( M_s > 1 \) or \( e^{3.42M_L} \) |
| II   | \( \bar{U} W / (\tau r^*) \) | \( R^{-1} e^{-\gamma R} \) | (5.12) \( M_s \leq 1 \); \( \tau \leq (1 - M_s \cos \theta) \) \( e^{1.21M_L} \) |
| III  | \( \bar{U} / \sqrt{1 - M_s^2} e^{-\gamma \lambda} \) | (5.23) \( M_s < 1 \) independent |
| IV   | \( \bar{U} / (\tau r^*) \) | \( 1 / \sqrt{1 - M_s^2} \) \( \sin (\omega_0 T) \) | (5.22) \( M_s > 1 \) |

* The coefficients 2.42 and 1.4 refer to Israel and vicinity only (Table 6, Appendix C; BEN-MENAHEM et al., 1977). In other regions it may assume different values (e.g., JOYNER et al., 1981).
Beyond this distance, up to the boundary of the Fresnel zone \([r_f = (0.62)\sqrt{L/\lambda} \text{ (Eq. (3.7))}]\), peak accelerations can be fairly approximated by the stationary phase approximation \([\text{Eqs. (3.4), (5.21)-(5.23)}]\). The peak shear acceleration in this region is independent of the coordinates of the field-point relative to the fault provided that

\[
y \leq \frac{\sqrt{\beta \tau}}{2\pi} < \frac{M_\rho}{\sqrt{1 - M_\rho}}.
\]  

Indeed, observations of GUTENBERG and RICHTER (1956) and HANKS and JOHNSON (1976) have confirmed this phenomenon for the magnitude range \(4.5 \leq M \leq 7.1\). Since \(\beta \tau/2\pi\) is of the order of 300 m and \(M_\rho(1 - M_\rho^2)^{-1/2} \approx 10\) for most earthquakes, the range of validity for \(y\) is up to 10 km from the fault. This value is close to that of \(r_f = (0.62)\sqrt{L/\lambda}\) for \(\lambda = \beta \tau = 2\) km and \(L = 10\) km.

Note also from Table 2 that the acceleration in the near fault region is proportional to the particle velocity on the fault. This dependence also holds in the Fresnel zones. At the far-field, however, the dependence of the accelerations on the magnitude is derived as follows: BEN-MENAHEM (1977a) has shown that \(\bar{U}, \bar{W}, L,\) and \(\tau\) each scale like \((\bar{US})^{1/3}\). It implies that \(\bar{US}/\tau, \bar{U}W/\tau,\) and \(\bar{U}/\tau\) scale like \((\bar{US})^{2/3}, (\bar{US})^{13}, (\bar{US})^{13}\) and 1 respectively. We also know that in general (e.g., JOYNER et al., 1981)

\[
[S(cm)(cm^2)] = \exp(AM_L + B) \tag{6.2}
\]

where \(A\) and \(B\) depend on the region under consideration. Thus, our theory predicts the well known scaling law for the peak horizontal acceleration at the far-field (e.g., MURPHY and O'BRIEN, 1977)

\[
a_p = a_0(\exp(bM_L))R^{-1}\exp(-\gamma R). \tag{6.3}
\]

In a more realistic earth model, such as a multilayered half-space, the decay of \(a_p\) with epicentral distance is stronger then \(R^{-1}\) due to the interference of the direct pulse with the surface reflections ("Lloyd-mirror effect"). Indeed, MURPHY and
LAHOUD (1969) found $a_0 \propto R^{-1.43}$ for nuclear explosions.

Since (6.3) is singular at $R=0$, its use for the near-fault zone is extended by the replacement of $R$ with $(R + R_0)$, where $R_0$ varies from one zone to another. BEN-MENAHEM et al. (1982) (see Table 6) adopted the value of $R_0 = 25$ km for Israel and vicinity.

A second application of our theory concerns the relationship between the peak acceleration and the shape of isoseismals. Isoseismals, by definition, are curves of equal intensity and the intensity scales linearly with the logarithm of the peak acceleration.

It is clear from the results of Section 1 that the radiation pattern of the acceleration field is the same as that of the displacements. Yet isoseismals, which are curves on which the horizontal acceleration is fixed, are shaped like confocal ellipses and do not show strong dependence either upon the orientation of the displacement vector or on the dip of the fault. It seems as if the kinematic seismic source model fails to explain the shape of isoseismals.

This paradox is resolved in the following way: both in the near-fault zone and the Fresnel zone, the acceleration at a given point is most strongly affected by the radiation from the nearest fault segment and not from the average radiation of the entire fault as is the case in the far-field. This indeed is the physical interpretation of the stationary-phase approximations, as given in Eq. (4.3). Thus, a quantitative assessment of the acceleration field near a finite fault is realized by taking $\phi_h = \pi/2$ for each small segment of the fault. The equi-peak acceleration contours thus constitute a family of closed curves where each such curve is the loci off all points which are equi-distant from the fault. SHEBALIN (1973) explained the ellipticity of near isoseismals by similar arguments and claimed therefore that the first isoseismal reflects the horizontal extension of the focal zone such that $L \approx d_1 - d_2$ where $d_1$ and $d_2$ are the major and minor axes of the isoseim (Fig. 7).

The effect of the frequency of the radiated spectrum is latent in the factor $a_0$ in Eq. (6.3). Although we chose in our model a number of representative spectral source functions [see Eqs. (2.8), (5.14), and (6.3)], the choice of $g(\omega)$ was left open. In an earlier paper (BEN-MENAHEM, 1977b), I have recommended a displacement source function of the form $[1 + (\omega/\omega_1)^2]^{-1/2}[1 + (\omega/\omega_2)^2]^{-1/2}$, where $\omega_1 \equiv (2/T_r)$ ($T_r$ is the rupture time on the causative fault) and $\omega_2 = 1/\tau$ where $\tau$ is the rise-time. This yields an acceleration-spectrum that is proportional to

$$\frac{\omega^2}{1 + \left(\frac{\omega}{\omega_1}\right)^2} \frac{1}{1 + \left(\frac{\omega}{\omega_2}\right)^2}$$

(6.4)

It is similar to the spectral dependence of the rms value for acceleration suggested by recent data analysis of strong ground motion (e.g., LUCCO, 1985), except that the expression in (6.4) has two corner frequencies.
7. Discussion and Conclusions

We have studied the theoretical relations between the acceleration of an earthquake source model and the kinematic source parameters. In spite of the simplicity of the model we were able to give a quantitative explanation of the shape of isoseisms and obtain some scaling laws which connect the ground acceleration, the rupture-time, the rise-time and the shear Mach-number.

In recent years some authors (e.g., Madariaga, 1983), have shown that the radiation of high-frequency acceleration-waves depends on the motion of the rupture front and that abrupt changes in rupture velocity radiate high-frequency acceleration-waves. Our results, which are summarized in Table 2, are not opposed to, but rather complement the above findings. We claim that peak accelerations in the near-fault zone are caused on the most part by supershear acceleration-waves. It means that abrupt changes that lead to supershear source velocities, even over short time intervals, constitute the main acceleration signal in the near-field. Thus, the information obtained from both source-kinematics and fracture-mechanics are complementary.

However, we recognize that our results cannot be used to interpret details of the complex acceleration field around real earthquake faults. These may depend on non-linear soil response, various resonance phenomena and other local factors which may amplify or diminish the average peak accelerations by factors as high as ten. Nevertheless, in order to recognize these abnormalities one must have a good knowledge of the expected average acceleration field and this is precisely what we have endeavored to achieve in the present manuscript.

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Sarah Fliegelmann of our department kindly typed the manuscript via a computer Tex program.

APPENDIX

A. Displacement, Velocity and Acceleration Fields of a Point-Dislocation

The horizontal radiation-pattern functions \( F_1, F_2, F_3 \) (Table 5), depend on the four angles: \( \lambda \) (slip), \( \delta \) (dip), \( \phi_h \) (azimuth), \( i_h \) (take-off) and are given explicitly in the form

\[
F_m(\phi_h) = a_0^{(m)} + a_1^{(m)} \cos \phi_h + b_1^{(m)} \sin \phi_h + a_2^{(m)} \cos 2\phi_h + b_2^{(m)} \sin 2\phi_h \quad (A.1)
\]

\( m = 1, 2, 3; \) \( (1) = \text{P waves}; \) \( (2) = \text{SV waves}; \) \( (3) = \text{SH waves}. \)

Here
Table 3. Radiation of seismic waves from a point-dislocation in an unbounded medium with an arbitrary time-function \( g(t) \).

\[
\begin{align*}
\textbf{Displacement} & & \textbf{Velocity} \\
D_1^R & = & \frac{F_1}{3\beta^3} \tilde{\varphi}_x \\
D_1^\varphi & = & \frac{F_2}{6\beta} \tilde{\varphi}_\varphi \\
D_1^{\varphi\varphi} & = & \frac{F_3}{6\beta} \tilde{\varphi}_\varphi
\end{align*}
\]

\[
\begin{align*}
D_2^R & = & \frac{F_1}{3\beta^3} \left[ \frac{4}{3} \frac{\beta^2}{\alpha^2} \varphi_x - \frac{3}{R} \frac{\beta^2}{\alpha^2} \varphi_x \right] \\
+ & 3 \frac{1}{R} \int_{\varphi_{\varphi}}^{1} g(t-s) \frac{R}{\beta} \frac{R}{\beta} ds \\
D_2^{\varphi} & = & \frac{F_2}{6\beta^2} \left[ -2 \frac{\beta^2}{\alpha^2} \varphi_x + 3 \varphi_\varphi \right] \\
- & 6 \frac{1}{R} \int_{\varphi_{\varphi}}^{1} g(t-s) \frac{R}{\beta} \frac{R}{\beta} ds \\
D_2^{\varphi\varphi} & = & \frac{F_3}{6\beta^3} \left[ -2 \frac{\beta^2}{\alpha^2} \varphi_x + 6 \frac{\beta^2}{\alpha^2} \varphi_x + 6 \varphi_\varphi \right] \\
- & 6 \frac{1}{R} \int_{\varphi_{\varphi}}^{1} g(t-s) \frac{R}{\beta} \frac{R}{\beta} ds
\end{align*}
\]

\[
\begin{align*}
\textbf{Acceleration} \\
D_1^R & = & \frac{F_1}{3\beta^3} \tilde{\varphi}_x \\
D_1^\varphi & = & \frac{F_2}{6\beta} \tilde{\varphi}_\varphi \\
D_1^{\varphi\varphi} & = & \frac{F_3}{6\beta} \tilde{\varphi}_\varphi
\end{align*}
\]

\[
\begin{align*}
D_2^R & = & \frac{F_1}{3\beta^3} \left[ 4 \alpha^2 \tilde{\varphi}_x + \frac{9\alpha^2}{R} \tilde{\varphi}_x + \frac{9\alpha^2}{R^2} \varphi_x \right] \\
D_2^{\varphi} & = & \frac{F_2}{6\beta^2} \left[ \frac{3\alpha}{R} \tilde{\varphi}_x + \frac{3\alpha}{R^2} \varphi_x \right] \\
D_2^{\varphi\varphi} & = & \frac{F_3}{6\beta^3} \left[ \frac{3\alpha}{R} \tilde{\varphi}_x + \frac{3\alpha}{R^2} \varphi_x \right]
\end{align*}
\]
<table>
<thead>
<tr>
<th>Displacement</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_i^x$</td>
<td>$D_i^y$</td>
</tr>
<tr>
<td>$D_i^z$</td>
<td>$D_i^\phi$</td>
</tr>
<tr>
<td>$D_i^n$</td>
<td>$D_i^\theta$</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
 u(r, t) &= \frac{U_0 dS}{4\pi \left[ \frac{1}{R} D_1 + \frac{1}{R^2} D_2 \right]}, \quad H_x = H\left(t - \frac{R}{\alpha} \right), \quad H_y = H\left(t - \frac{R}{\beta} \right), \quad H(0) = \frac{1}{2} \\
 D_i^x &= \frac{F_1}{3\beta} \left( \frac{\beta}{\alpha} \right)^3 \delta\left(t - \frac{R}{\alpha} \right) \\
 D_i^y &= \frac{F_2}{6\beta} \delta\left(t - \frac{R}{\beta} \right) \\
 D_i^z &= \frac{F_3}{6\beta} \delta\left(t - \frac{R}{\beta} \right) \\
 D_i^\phi &= \frac{F_3}{6\beta} \delta\left(t - \frac{R}{\beta} \right) \\
 D_i^n &= \frac{4\beta^2 F_1}{3\alpha^2} \delta\left(t - \frac{R}{\alpha} \right) - \frac{3\beta^2 F_1}{R^2} t(H_x - H_y) + \frac{3\beta^2 F_1}{R^2} t(H_a - H_y) \\
 D_i^\theta &= \frac{-\beta^2 F_2}{3\alpha^2} \delta\left(t - \frac{R}{\alpha} \right) + \frac{1}{2} F_2 \delta\left(t - \frac{R}{\beta} \right) - \frac{\beta^2 F_2}{R^2} t(H_a - H_y) \\
 D_i^\psi &= \frac{-\beta^2 F_3}{3\alpha^2} \delta\left(t - \frac{R}{\alpha} \right) + \frac{1}{2} F_3 \delta\left(t - \frac{R}{\beta} \right) - \frac{\beta^2 F_3}{R^2} t(H_a - H_y) \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>Acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{D}_i^x$</td>
</tr>
<tr>
<td>$\dot{D}_i^y$</td>
</tr>
<tr>
<td>$\dot{D}_i^z$</td>
</tr>
<tr>
<td>$\dot{D}_i^\phi$</td>
</tr>
<tr>
<td>$\dot{D}_i^n$</td>
</tr>
<tr>
<td>$\dot{D}_i^\theta$</td>
</tr>
<tr>
<td>$\dot{D}_i^\psi$</td>
</tr>
</tbody>
</table>
Table 5. Field-spectrums generated by a point-dislocation in an unbounded medium with an arbitrary time-function $g(t)$.

\[ u(r, \omega) = \frac{U_0}{4\pi} \text{Log}(\omega) \left[ \frac{1}{R} D_1(r, \omega) + \frac{1}{R^2} D_2(r, \omega) \right], \]

\[ D_1 = S_{1\beta} \exp(-ik_\omega R) + S_{2\beta} \exp(-ik_\omega R), \]

\[ D_2 = S_{2\alpha} \exp(-ik_\omega R) + S_{2\beta} \exp(-ik_\omega R), \quad g(\omega) = \int_{-\infty}^{\infty} g(t) \exp(-i\omega t) dt \]

<table>
<thead>
<tr>
<th>Displacement</th>
<th>Compressional</th>
<th>Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{1\alpha}^h/F_1$</td>
<td>$\frac{1}{3\beta} \left( \frac{1}{\alpha} \right)^3$</td>
<td>$-$</td>
</tr>
<tr>
<td>$S_{1\beta}^h/F_2$</td>
<td>$-$</td>
<td>$\frac{1}{6\beta}$</td>
</tr>
<tr>
<td>$S_{2\alpha}^h/F_3$</td>
<td>$-$</td>
<td>$\frac{1}{6\beta}$</td>
</tr>
<tr>
<td>$S_{2\alpha}^h/F_1$</td>
<td>$\frac{4\beta^2}{3} \left( \frac{1}{\alpha} \right)^3 + \frac{3\beta^2}{\alpha R} \left( \frac{1}{\alpha \omega} \right)^2$</td>
<td>$-\frac{3\beta\left(1/\alpha \omega\right)^2}{R^2} + \frac{3\beta^2 \left(1/\alpha \omega\right)^3}{R^2}$</td>
</tr>
<tr>
<td>$S_{2\beta}^h/F_2$</td>
<td>$-\frac{1}{3} \frac{\beta^2}{\alpha^2} \left( \frac{1}{\omega} \right)$</td>
<td>$\frac{1}{2} \left( \frac{1}{\alpha \omega} \right)^2$</td>
</tr>
<tr>
<td>$S_{2\beta}^h/F_3$</td>
<td>$-\frac{1}{3} \frac{\beta^2}{\alpha^2} \left( \frac{1}{\omega} \right)$</td>
<td>$\frac{1}{2} \left( \frac{1}{\alpha \omega} \right)^2$</td>
</tr>
<tr>
<td>$S_{\alpha}^h/F_4$</td>
<td>$-\frac{\beta^2}{\alpha R} \left( \frac{1}{\alpha \omega} \right)^2$</td>
<td>$\frac{\beta}{R} \left( \frac{1}{\alpha \omega} \right)^2$</td>
</tr>
<tr>
<td>$S_{\beta}^h/F_5$</td>
<td>$-\frac{\beta^2}{\alpha R} \left( \frac{1}{\alpha \omega} \right)^2$</td>
<td>$\frac{\beta}{R} \left( \frac{1}{\alpha \omega} \right)^2$</td>
</tr>
<tr>
<td>$S_{\alpha}^|/F_6$</td>
<td>$-\frac{\beta^2}{\alpha R} \left( \frac{1}{\alpha \omega} \right)^2$</td>
<td>$\frac{\beta}{R} \left( \frac{1}{\alpha \omega} \right)^2$</td>
</tr>
<tr>
<td>$S_{\beta}^|/F_7$</td>
<td>$-\frac{\beta^2}{\alpha R} \left( \frac{1}{\alpha \omega} \right)^2$</td>
<td>$\frac{\beta}{R} \left( \frac{1}{\alpha \omega} \right)^2$</td>
</tr>
</tbody>
</table>
Table 5. (continued)

<table>
<thead>
<tr>
<th>Velocity</th>
<th>Compressional</th>
<th>Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{11}^p/F_1$</td>
<td>$\frac{1}{3\beta} \left( \frac{\beta}{\alpha} \right)^3 (i\omega)$</td>
<td>$-$</td>
</tr>
<tr>
<td>$S_{11}^s/F_2$</td>
<td>$-$</td>
<td>$\frac{1}{6\beta} (i\omega)$</td>
</tr>
<tr>
<td>$S_{22}^s/F_3$</td>
<td>$-$</td>
<td>$\frac{1}{6\beta} (i\omega)$</td>
</tr>
<tr>
<td>$S_{22}^p/F_1$</td>
<td>$\frac{4\beta^2}{3\alpha^2} + \frac{3\beta^2}{\alpha R} \left( \frac{1}{i\omega} \right)$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$S_{22}^s/F_2$</td>
<td>$-\frac{\beta^2}{\alpha R} \left( \frac{1}{i\omega} \right)$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$S_{22}^p/F_3$</td>
<td>$-\frac{\beta^2}{R^2} \left( \frac{1}{i\omega} \right)^2$</td>
<td>$+ \frac{\beta}{R} \left( \frac{1}{i\omega} \right)$</td>
</tr>
<tr>
<td>$S_{22}^s/F_4$</td>
<td>$-\frac{\beta^2}{R^2} \left( \frac{1}{i\omega} \right)^2$</td>
<td>$+ \frac{\beta}{R} \left( \frac{1}{i\omega} \right)$</td>
</tr>
</tbody>
</table>
Table 5. (continued)

<table>
<thead>
<tr>
<th>Acceleration</th>
<th>Compressional</th>
<th>Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S^{k_1}_{1}/F_1$</td>
<td>$\frac{1}{3\beta} \left( \frac{\beta}{\alpha} \right)^3 (i\omega)^2$</td>
<td>—</td>
</tr>
<tr>
<td>$S^{k_2}_{2}/F_2$</td>
<td>—</td>
<td>$\frac{1}{6\beta} (i\omega)^2$</td>
</tr>
<tr>
<td>$S^{k_3}_{3}/F_3$</td>
<td>—</td>
<td>$\frac{1}{6\beta} (i\omega)^2$</td>
</tr>
</tbody>
</table>

$$\frac{4\beta^2}{3\alpha^2} (i\omega)$$

$$\frac{3\beta^2}{\alpha R} + \frac{3\beta^2}{R^2} \left( \frac{1}{i\omega} \right)$$

$$\frac{3\beta}{R} - \frac{3\beta^2}{R^2} \left( \frac{1}{i\omega} \right)$$

$$\frac{1}{2} (i\omega)$$

$$\frac{\beta^2}{\alpha R} + \frac{\beta}{R}$$

$$\frac{\beta^2}{R^2} \left( \frac{1}{i\omega} \right)$$

$$\frac{1}{2} (i\omega)$$

$$\frac{\beta^2}{\alpha R} + \frac{\beta}{R}$$

$$\frac{\beta^2}{R^2} \left( \frac{1}{i\omega} \right)$$
Thus, for a vertical left lateral strike-slip fault

\[
\begin{align*}
F_1 &= -3 \sin^2 \phi h \sin 2 \phi h, \\
F_2 &= -6 \sin \phi h \cos \phi h \sin 2 \phi h, \\
F_3 &= -6 \sin \phi h \cos 2 \phi h.
\end{align*}
\]

In these equations and in Tables 1–3, \((R, \phi h, \phi h)\) are spherical coordinates of the sensor w.r.t., a fixed cartesian coordinate system with origin at the point of rupture initiation, such that the strike and rupture are in the direction of the \(x_1\) axis, dip direction along the \((-x_2)\) axis and the \(x_3\) axis is directed upwards (Fig. 1).

The acceleration field, \(\ddot{u}(r, t)\), at the observation-point \(P(R)\), due to a source time-function \(g(t)\), is obtained from the explicit expressions of the displacements by the two-fold differentiation w.r.t. to time:

\[
\ddot{u}(r, t) = \ddot{u}^{(P)}(r, t) + \ddot{u}^{(S)}(r, t)
\]

\[
\ddot{u}^{(P)}(r, t) = \ddot{u}_R^{(P)} e_R + \ddot{u}_\phi h^{(P)} e_\phi h + \ddot{u}_\phi h^{(P)} e_\phi h.
\]

\[
\ddot{u}^{(S)}(r, t) = \ddot{u}_R^{(S)} e_R + \ddot{u}_\phi h^{(S)} e_\phi h + \ddot{u}_\phi h^{(S)} e_\phi h.
\]

In Eqs. (A.6), \((e_R, e_\phi h, e_\phi h)\) is an orthogonal vector-base at \(P\). The superscripts \((P)\),
(S) indicate compressional and shear waves respectively.

The cartesian components of the compressional acceleration at Q are,
\[ \ddot{u}_x^{(p)} = \ddot{u}_x^{(P)} \cos i_h - \ddot{u}_h^{(p)} \sin i_h \]
\[ \ddot{u}_y^{(p)} = [\ddot{u}_x^{(P)} \sin i_h + \ddot{u}_h^{(p)} \cos i_h] \cos \phi_h - \ddot{u}_h^{(p)} \sin \phi_h \]
\[ \ddot{u}_z^{(p)} = [\ddot{u}_x^{(P)} \sin i_h + \ddot{u}_h^{(p)} \cos i_h] \sin i_h + \ddot{u}_h^{(p)} \cos \phi_h \]  
(A.7)

with similar expressions for the shear components.

B. Lommel Diffraction Integrals

The functions
\[ U_\nu(u, v) = \sum_{m=0}^{\infty} \left(-\frac{v}{u}\right)^{2m+\nu} J_{2m+\nu}(v) \]  
(B.1)

\[ V_\nu(u, v) = \sum_{m=0}^{\infty} \left(-\frac{v}{u}\right)^{2m+\nu} J_{2m+\nu}(v) \]  
(B.2)

are known as the Lommel functions of two variables of order \( \nu \) (BORN and WOLF, 1964; WATSON, 1966; BORESMA, 1962; DEKANOSIDZE, 1960). It can be shown that the Lommel functions are representable by means of the integrals
\[ v^{1-\nu} u^\nu \left\{ \exp \left( -\frac{i}{2} u \right) \right\} \int_{0}^{1} J_{\nu-1}(v \tau) \left\{ \exp \left( -\frac{i}{2} u \tau^2 \right) \right\} \tau \, d\tau = U_\nu(u, v) + i U_{\nu+1}(u, v) \]  
(B.3)

\[ v^{1-\nu} u^\nu \left\{ \exp \left( -\frac{i}{2} u \right) \right\} \int_{1}^{\infty} J_{\nu-1}(v \tau) \left\{ \exp \left( -\frac{i}{2} u \tau^2 \right) \right\} \tau \, d\tau = V_{\nu}(u, v) + i V_{\nu+1}(u, v) \]  
(B.4)

We shall now show how the above functions arise naturally from our interference integrals.

Consider first the one-dimensional case of a line-source radiator of finite length. The interference integral then reduces to
\[ I(W, z_0; k) = \int_{0}^{W} \exp(-ik_0 \sqrt{\eta^2 + z_0^2}) \frac{dk}{\sqrt{\eta^2 + z_0^2}} \]  
(B.5)

We apply the Sommerfeld integral
\[ \frac{\exp(-ik_0 \sqrt{\eta^2 + z_0^2})}{\sqrt{\eta^2 + z_0^2}} = \int_{0}^{\infty} J_0(k\eta) \left\{ \exp(-z_0 \sqrt{k^2 - k_0^2}) \right\} \frac{kd\eta}{\sqrt{k^2 - k_0^2}}, \]

and obtain at once
\[ I = \int_{0}^{\infty} \left\{ \exp(-z_0 \sqrt{k^2 - k_0^2}) \right\} \frac{kd\eta}{\sqrt{k^2 - k_0^2}} \int_{0}^{W} J_0(k\eta) \, d\eta \]
Peak Accelerations from Subshear and Supershear Radiation

\[ \begin{align*}
\text{(B.6)} & \quad = 2 \sum_{m=0}^{\infty} \int_{0}^{\infty} J_{2m+1}(kW) \left\{ \exp \left( -z_{0}\sqrt{k^{2} - k_{c}^{2}} \right) \right\} \frac{dk}{\sqrt{k^{2} - k_{c}^{2}}} \\
\text{(B.7)} & \quad = -\pi i \sum_{m=0}^{\infty} J_{m+1/2}(N) H_{m+1/2}^{(2)}(M),
\end{align*} \]

with

\[ N = \frac{k_{c}}{2} \left[ \sqrt{W^{2} + z_{0}^{2}} - z_{0} \right]; \quad M = \frac{k_{c}}{2} \left[ \sqrt{W^{2} + z_{0}^{2}} + z_{0} \right]. \]

If we let \( k_{c}z_{0} \gg 1 \), we may use the approximation

\[ H_{m+1/2}^{(2)}(M) = i \sqrt{\frac{2}{\pi M}} \exp \left( -iM + \frac{\pi mi}{2} \right). \]

Equation (B.5) then assumes the form

\[ I = \sqrt{\frac{2\pi}{M}} \left\{ \exp(-iM) \right\} \left[ U_{1/2} + iU_{3/2} \right]; \quad u = v = N \quad \text{(B.7)} \]

where, according to (A.1)

\[ U_{1/2} = \sum_{m=1}^{\infty} (-)^{m} J_{2m+1/2}(N); \quad U_{3/2} = \sum_{m=0}^{\infty} (-)^{m} J_{2m+3/2}(N). \quad \text{(B.8)} \]

Furthermore, since (WATSON, 1966, pp. 544–545)

\[ U_{1/2} + iU_{3/2} = \frac{1}{\sqrt{2}} \left\{ \exp(iN) \right\} \int_{0}^{\sqrt{2N/\pi}} \exp \left( -\frac{\pi t^{2}}{2} \right) dt, \]

(B.7) becomes

\[ I = \sqrt{\frac{2}{M}} \left\{ \exp(-ik_{c}z_{0}) \right\} \int_{0}^{\sqrt{2N}} \exp(-i\tau^{2}) d\tau. \quad \text{(B.9)} \]

The condition \( k_{c}z_{0} \ll (1/2)(k_{c}W)^{2} \) secures that \( \sqrt{2N} \gg 1 \). We may then replace the upper limit of the integral in (B.9) by infinity. Since

\[ \int_{0}^{\infty} \exp(-i\tau^{2}) d\tau = \frac{1}{2} \sqrt{\frac{\pi}{2} (1 - i)}, \]

it transpires that

\[ I \approx \sqrt{\frac{\pi}{2k_{c}z_{0}}} \exp \left( -ik_{c}z - \frac{\pi}{4} \right). \quad \text{(B.10)} \]

Our second case is that of the interference integral in (2.10) (Fig. 3). We make the Fresnel approximation

\[ D = \left[ y^{2} + A^{2} + A_{0}^{2} - 2A_{0}A \cos(\phi - \phi_{0}) \right]^{1/2}. \]
The integral in Eqs. (2.10) will thus become
\[
I = \frac{\exp(-ikz)}{y} \int_{\phi_0}^{\phi_0+2\pi} d\phi \int_{0}^{\phi} \left\{ \exp \left( -ik \frac{\Delta^2 + \Delta_0^2}{2y} + ik \Delta \cos(\phi - \phi_0) \right) \right\} \Delta d\Delta.
\] (B.11)

Since
\[
\int_{\phi_0}^{\phi_0+2\pi} \exp[i\beta \Delta \cos(\phi - \phi_0)] d\phi = 2\pi J_0(\beta \Delta), \quad \beta = \frac{k \Delta_0}{y},
\] (B.12)
the integral becomes
\[
I = \frac{2\pi}{k_c} \left\{ \exp \left[ -ik_c \left( \frac{\Delta_0^2 + \alpha^2}{2y} \right) \right] \right\} \left\{ u \left( \frac{iu}{2} \right) \right\} J_0(\nu \tau) \left\{ \exp \left( -\frac{i}{2} \nu \tau^2 \right) \right\} \tau d\tau,
\] (B.13)

where
\[
u = \frac{k_c \Delta_0}{y}, \quad v = \frac{k_c \Delta_0}{y}, \quad \frac{u}{v} = \frac{a}{\Delta_0}.
\]

Therefore

Fig. 8. Geometry of a circular fault in the presence of a free surface at the x-y plane. The angle \(\phi\) is between \(x\) and \(\Delta\) in the \(xz\) plane. The angle \(\Theta\) is between \(r\) and \(\Delta\) in the \((r,R,\Delta)\) plane.
\[ I = \frac{2\pi}{k_c} \left\{ \exp \left[ -ik_c \left( y + \frac{\Delta^2 + a^2}{2y} \right) \right] \right\} \left[ U_0 \left( \frac{k_c a^2}{y}, \frac{k_c a \Delta_0}{y} \right) + i U_1 \left( \frac{k_c a^2}{y}, \frac{k_c a \Delta_0}{y} \right) \right]. \]

\[(B.14)\]

The geometrical set-up of Fig. 5 is not convenient in the case where a free surface is involved. To this end we use the geometry of Fig. 8, where the free surface is in the \(xy\) plane and the disc-center is at depth \(h\). The various distances and angles are related here by the equations

\[ R^2 = r^2 + \Delta^2 - 2r\Delta \cos \Theta \]
\[ = (r \sin i_h \cos \phi_h - \Delta \cos \phi)^2 + (r \sin i_h \sin \phi_h)^2 + (-r \cos i_h + \Delta \sin \phi)^2, \]
\[ r^2 = h^2 + y^2 + x^2; \quad \cos \Theta = \sin i_h \cos \phi_h \cos \phi + \cos i_h \sin \phi = s \cos(\phi - \phi_0) \]
\[ s = [\cos^2 i_h + \sin^2 i_h \cos^2 \phi_h]^{1/2}; \quad \phi_0 = \sin^{-1} \left[ \frac{\cos i_h}{s} \right]. \]
\[(B.15)\]

As before we expand

\[ R = r - \Delta \cos \Theta + \frac{\Delta^2}{2r} \sin^2 \Theta + \frac{\Delta^3}{2r^2} \cos \Theta \sin^2 \Theta + O \left( \frac{\Delta^4}{r^3} \right). \]
\[(B.16)\]

The far-field approximation now bears the result

\[ I = \int_0^{2\pi} d\phi \int_0^a \frac{\exp(-ik_c R)}{R} \Delta d\Delta = \frac{\exp(-ik_c r)}{r} \int_0^{2\pi} d\phi \int_0^a \{ \exp(ik_c \Delta \cos \Theta) \} \Delta d\Delta. \]
\[(B.17)\]

Finally

\[ I = \frac{\exp(-ik_c r)}{r} \int_0^a \Delta d\Delta \int_{\phi_0}^{\phi_0 + 2\pi} d\phi \exp[isk_c \Delta \cos(\phi - \phi_0)] \]
\[ = 2\frac{\exp(-ik_c r)}{r} \int_0^a J_0(k_c s) s d\Delta \]
\[(B.18)\]

or

\[ I = 2\pi a^2 \left[ \frac{J_1(k_c a s)}{k_c a s} \right] \exp\left( -ik_c r \right). \]

However, in the Fresnel approximation we find from (B.16) that the integral to be evaluated is

\[ I = a^2 \frac{\exp(-ik_c r)}{r} \int_0^{2\pi} d\phi \int_0^1 \left\{ \exp\left( -i \frac{\Delta^2}{2r^2} \right) \right\} \{ \exp(i\Phi) \} \Delta d\tau, \]
\[(B.19)\]

with
The integral (B.20) may be developed in an infinite series, by expanding both \( \exp(\nu t \cos \phi) \) and \( \exp((1/2)w \tau^2 \cos \phi) \) with the aid of the Jacobi identity

\[
\exp(iz \cos \phi) = J_0(z) + 2 \sum_{n=1}^{\infty} i^n J_n(z) \cos n\phi.
\]

Multiplying the two expansions together we find that

\[
\exp(iz \cos \phi) = J_0(z) + 2 \sum_{n=1}^{\infty} i^n J_n(z) \cos n\phi.
\]

Multiplying the two expansions together we find that

\[
\exp[i(\nu \tau \cos \phi + w \tau^2 \cos 2\phi)]
\]

\[
= 4 \sum_{n=0}^{\infty} \sum_{n'=0}^{\infty} (-i)^n (-i)^{n'} J_n(\nu \tau) J_{n'}(w \tau^2) \cos 2n\phi \cos n'(\phi - \pi),
\]

where the prime on the summation sign implies that the terms in \( n=0 \) and \( n'=0 \) are each to be taken with the factor \( 1/2 \). We substitute this double series into Eq. (B.20) and integrate with respect to \( \phi \) term by term. This gives

\[
I = 4\pi a^2 \exp(-ikr) \sum_{n=0}^{\infty} \left\{ \exp\left(-\frac{i}{2}u \tau^2\right) \right\} J_n(\nu \tau) J_{2n}(w \tau) d\tau,
\]

where the term \( n=0 \) begins with the factor \( 1/2 \).

**Special cases**

1. Far-field:

   \[
   w = u = 0, \quad n = 0, \quad I = 2\pi a^2 \exp(-ikr) \int_0^1 J_0(\nu \tau) d\tau,
   \]

   same as (A.18).

2. \( i_n = \phi_n = \frac{\pi}{2}, \quad s = 0, \quad w = v = 0, \quad n = 0, \)

   \[
   I = 2\pi a^2 \frac{\exp(-ikr)}{y} \int_0^1 \left\{ \exp\left(-\frac{i}{2}u \tau^2\right) \right\} d\tau,
   \]

   same as Eq. (B.13) with \( A_0 = 0 \).

Finally we return to (4.5) and evaluate the integrals in (2.10) by means of the Lommel functions. Instead of (4.5) we write

\[
R = y + \frac{(x-x_1)^2 + z_1^2}{2y} = \left( y + \frac{x^2}{2y} \right) + \frac{x_1^2 + z_1^2}{2y} - \frac{xx_1}{y},
\]

\[
Q = \exp\left(-ik\frac{y^2}{2y}\right) \left\{ \exp\left(-ik\frac{x^2}{2y}\right) \right\} \int_{-L/2}^{L/2} \int_{-W/2}^{W/2} \exp\left[ -\frac{ik}{2} (\xi^2 + \eta^2) + il\xi \right] d\xi d\eta
\]

\[
(B.23)
\]
where

\[ k = \frac{k_c}{y}, \quad l = \frac{k_c x}{y}. \]

Using

\[ \exp(i\sigma) = \cos \sigma + i \sin \sigma = \sqrt{\frac{\pi\sigma}{2}} \left[ J_{-1/2}(\sigma) + i J_{1/2}(\sigma) \right], \quad (B.24) \]

the integral in Eq. (4.2) becomes

\[
Q = \frac{\pi}{2} \frac{\exp(-ik_c y)}{y} \left\{ \exp\left( -\frac{ik_c x^2}{2y} \right) \int_{-L/2}^{L/2} (\xi)^{1/2} \left[ J_{-1/2}(\xi) + i J_{1/2}(\xi) \right] \right. \\
\times \exp\left( -\frac{i}{2} k \xi^2 \right) d\xi \\
\left. \int_{-W/2}^{W/2} \exp\left( -\frac{i}{2} k \eta^2 \right) d\eta \right\}. \quad (B.25) \]

Noticing that

\[
\int_{-\nu}^{\nu} \omega^{1/2} J_{1/2}(\omega) \exp\left( -i\omega^2 \frac{u}{2v^2} \right) d\omega = \sqrt{\frac{2}{\pi}} \int_{-\nu}^{\nu} \sin \omega \exp\left( -i\omega^2 \frac{u}{2v^2} \right) d\omega \equiv 0, \quad (B.26)\]

we may represent our interference integral in terms of Lommel functions of orders 1/2 and 3/2

\[
Q = \frac{2\pi}{k_c} \{\exp(-ik_c x)\} \left[ U_{1/2}\left(\frac{k_c l^2}{4y}; \frac{k_c L x}{2y}\right) + i U_{3/2}\left(\frac{k_c l^2}{4y}; \frac{k_c L x}{2y}\right) \right] \\
\times \left[ U_{1/2}\left(\frac{k_c W^2}{4y}; 0\right) + i U_{3/2}\left(\frac{k_c W^2}{4y}; 0\right) \right], \quad (B.27) \]

where

\[ \chi(x, y) = y + \frac{x^2}{2y} + \frac{L^2 + W^2}{8y}. \]

Explicitly,

\[
U_{1/2}\left(\frac{k_c l^2}{4y}; \frac{k_c L x}{2y}\right) = \sum_{m=0}^{\infty} (-)^m \left( \frac{L}{2x} \right)^{2m+1/2} J_{2m+1/2}\left(\frac{k_c L x}{2y}\right) \]
\[
U_{3/2}\left(\frac{k_c l^2}{4y}; \frac{k_c L x}{2y}\right) = \sum_{m=0}^{\infty} (-)^m \left( \frac{L}{2x} \right)^{2m+3/2} J_{2m+3/2}\left(\frac{k_c L x}{2y}\right) \]
\[
U_{1/2}(2A; 0) + i U_{3/2}(2A; 0) = \sqrt{2} \{\exp(iA)\} \int_{0}^{\sqrt{2A}/\pi} \exp\left( -\frac{1}{2} \pi^2 \tau^2 \right) d\tau. \quad (B.28) \]
Ground accelerations were recorded in Israel during the Dead-Sea earthquake of 23 April, 1979 ($M_L = M_S = 4.5$, $M_B = 5.1$). Table 6 shows the similarity of the observations with that of local and global scaling laws. The local scaling law derived earlier by BEN-MENAHEM et al. (1982) corresponds to the data quite well. The magnitude dependence via the factor $\exp(1.2M_L)$ is that of case II in Table 2 for $M_{\theta} \leq 1$; 

$$\tau < \frac{L}{V_t} (1 - M_{\theta} \cos \theta).$$

(C.1)

### Table 6. The fit of observed local peak accelerations (April 23, 1979$^*$, $M_L=4.5$) with local and global empirical laws.

<table>
<thead>
<tr>
<th>Epicentral distance $d$ (km)$^{*2}$</th>
<th>Recorded peak acceleration (cm/s$^2$)$^{*2}$</th>
<th>LOCAL</th>
<th>GLOBAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$a_{\text{hit}} = 17.8(\exp(1.2M_L))(R+25)^{-1.32}\exp(-R/400)$</td>
<td>$a_{\text{hit}} = 1.080(\exp(0.5M_L))(R+25)^{-1.32}$</td>
</tr>
<tr>
<td>28</td>
<td>11.1 (DL)$^{*3}$</td>
<td>19.1</td>
<td>(DONOVAN, 1974)</td>
</tr>
<tr>
<td>50</td>
<td>10.9 (D)</td>
<td>12.2</td>
<td></td>
</tr>
<tr>
<td>67</td>
<td>24.0 (A)</td>
<td>9.5</td>
<td></td>
</tr>
<tr>
<td>95</td>
<td>Less than 5 (K)</td>
<td>5.8</td>
<td></td>
</tr>
</tbody>
</table>

$^*$1 USCGS, BISC, $M_B = 5.0 - 5.1$; $M_S = 4.5$. Source parameters of this event: $L = 5-7$ km; $W = 1-2$ km; $\bar{U} = 4-7$ cm.

$^{*2}$ BEN-MENAHEM, 1986.

$^{*3}$ Rock type: DL, Dolomite and limestone; D, Dolomite; A, Alluvium; K, Kurkar.
With the calculated values of $L$, $\theta_i$ ($i = 1, 2, 3, 4$) (Table 2; BEN-MENAHEM, 1986), $M_\beta$ (BEN-MENAHEM and VERED, 1976) and $\tau$ (BEN-MENAHEM, 1977b), the conditions in (C.1) hold for the above event.

REFERENCES


