A SIMPLE METHOD TO CALCULATE SYNTHETIC LONG-PERIOD SEISMOGRAMS INCORPORATING FIRST-ORDER ASYMPTOTICS

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A simple method is suggested to calculate synthetic long-period seismograms by normal mode summation. Effects of the lateral heterogeneity are incorporated by the first-order asymptotics which are valid up to the order of $l^{-1}$ ($l$: angular order). Synthetic seismograms of a laterally heterogeneous earth model can be calculated by those of a spherically symmetric earth model with varying frequency, epicentral distance, amplitude, and azimuth.

1. Introduction

Normal mode summation is a natural way to synthesize long-period seismograms. For a spherically symmetric earth model, theory for normal modes (free oscillations) has been established. Dziewonksi et al. (1981) have used nearly 5,000 modes (period longer than 45 s) to synthesize seismograms and applied them to the routine determination of seismic mechanisms (moment tensors). If the assumption of spherical symmetry is removed (e.g., diurnal rotation and lateral heterogeneity), calculations of normal modes become difficult (Dahlen, 1968; Woodhouse and Dahlen, 1978). The difficulty is pronounced when the coupling between multiplets is included. Recently, Park and Gilbert (1986) and Morris et al. (1987) have shown that calculations of normal modes with the coupling are possible for a rotating, laterally heterogeneous earth model. However, such calculations are tedious and not suitable for inversion of earth structure. To avoid this difficulty, Woodhouse and Dziewonksi (1984) have used an approximate method which enables them to obtain laterally heterogeneous earth models. Effects of the lateral heterogeneity are incorporated by the frequency and epicentral distance perturbations. These are mode dependent and are linearly related to the lateral heterogeneity. In this method the coupling between multiplets on the same dispersion branch is implicitly incorporated (Mochizuki, 1986 a).

Davis and Henson (1986), Romanowicz and Roult (1986), Romanowicz (1987), and Park (1987) have derived the first-order asymptotics for free oscillations of a laterally heterogeneous earth model. These are valid up to the order of $l^{-1}$ ($l$:
angular order), while Woodhouse and Dziewonski (1984) have relied on the 0-th order asymptotics. However, the first-order asymptotics are extremely complicated, and the physical interpretation is not clear, especially for perturbed frequencies that behave as

$$\tan(\nu \Delta - \pi/4 + \chi)$$

where $\nu = l + 1/2$, $\Delta$ is the epicentral distance, and $\chi$ is dependent on source parameters and instruments. In this paper, simplification of their results is suggested, and the correspondence between free oscillations and surface waves is established.

2. Previous Works

2.1 Born approximation

We start from the Born approximation (Woodhouse and Girnius, 1982; Woodhouse, 1983):

$$s(t) = \sum_k \left[ \sum_m R_k^m S_k^m + it \sum_{mn} R_k^m H_{mn}^{(k)} S_m^m + \delta A_k \right] \exp(i \omega_k t).$$

In Eq. (2) seismic displacement (or acceleration) is expressed as the summation of normal modes; $k$ denotes multiplet, and $m$ is the azimuthal order. The first term of Eq. (2) corresponds to a spherically symmetric earth model, and the rest is the effect of aspherical perturbations. The receiver and source vectors are defined by

$$R_k^m = \sum_{N=1}^{1} R_{kN} Y_l^{Nm}(\theta_s, \phi_s)$$

$$S_k^m = \sum_{N=2}^{2} S_{kN} Y_l^{Nm}(\theta_r, \phi_r)$$

where $(\theta_s, \phi_s)$ and $(\theta_r, \phi_r)$ are the coordinates of the receiver and the source, respectively. $Y_l^{Nm}$ are generalized spherical harmonics of Phinney and Burr ridge (1973), and the asterisk denotes complex conjugation. Explicit expressions for $R_{kN}$ and $S_{kN}$, which are dependent on the polarization of the instrument and the moment tensor, respectively, are given in Woodhouse and Girnius (1982). The perturbation matrix elements $H_{mn}^{(k)}$, in the second term of Eq. (2), which express the coupling in a multiplet $k$, are related linearly to aspherical perturbations. The matrix elements for isotropic perturbations (lateral heterogeneity) are given in Woodhouse and Dahlen (1978).

The second term of Eq. (2) increases (or decreases) secularly with time $t$, and Woodhouse (1983) has rewritten the first and second terms of Eq. (2) as

$$\sum_{km} R_k^m S_k^m \exp[i(\omega_k + \dot{\omega}_k)t]$$

where
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\[ \lambda_{k} = \frac{\sum R_{m}^{m} H_{mm}^{(k)}, S_{k}^{m'}}{\sum_{m} R_{k}^{m} S_{k}^{m}} \]  

(5)
is the multiplet location parameter defined by JORDAN (1978). The multiplet location parameter is dependent not only on the perturbation matrix elements but on source parameters and instruments. In Eq. (4), the multiplet location parameter is interpreted as the perturbation to the degenerate frequency \( \omega_{k} \) of the multiplet. However, this interpretation is not convenient, when we consider the first-order asymptotics as shown below.

The third term of Eq. (2) is the contribution from different multiplets. The coupling between multiplets on the same dispersion branch has been considered (MOCHIZUKI, 1986 a; ROMANOWICZ, 1987; PARK, 1987). However, for example, the spheroidal-toroidal coupling has been neglected. The validity of this assumption is discussed in the final section.

2.2 Asymptotic approximation (l^0)

JORDAN (1978) has shown that the multiplet location parameter is approximated by, in the asymptotic limit of large \( l \),

\[ \lambda \sim \delta \omega \]  

(6)

with

\[ \delta \omega = (1/2\pi) \int_{0}^{2\pi} \delta \omega_{\text{local}} \, ds \]  

(7)

where index \( k \), which denotes multiplet, is omitted for simplicity. We use \( \sim \) to denote the average over the great circle containing the source and receiver. The local frequency perturbation \( \delta \omega_{\text{local}}(\theta, \phi) \) may be related to the phase velocity of surface waves. The multiplet location parameter is independent of source parameters and instruments at this level of approximation, and the interpretation given in Eq. (4) is natural and convenient.

MOCHIZUKI (1986 a) has shown that coupling between multiplets on the same dispersion branch leads to the apparent shift of the epicentral distance, which was first derived by WOODHOUSE and DZIEWONSKI (1984):

\[ \delta \theta = (a A/v U)(\delta \omega - \delta \tilde{\omega}) \]  

(8)

with

\[ \delta \omega = (1/A) \int_{0}^{A} \delta \omega_{\text{local}} \, ds \]  

(9)

where \( U \) is the group velocity, and \( a \) is the earth's radius. We use \( \sim \) to denote the average over the minor arc connecting the source and receiver.
2.3 Asymptotic approximation (l\(^{-1}\))

Davis and Henson (1986) and Romanowicz and Roult (1986) have derived the first-order asymptotics for the multiplet location parameter, which are valid up to the order of l\(^{-1}\). Romanowicz (1987) and Park (1987) have extended the results to include the coupling between multiplets on the same dispersion branch.

We first discuss a simple case of an azimuthally symmetric source and a vertical instrument; \(S_{k_0}\) and \(R_{k_0}\) are not zero in Eq. (3). The multiplet location parameter is approximated by (Romanowicz and Roult, 1986; Romanowicz, 1987)
\[
\lambda \sim \delta \omega + (\hat{D}/2\nu) \tan \Psi
\] (10)

with
\[
\Psi = \nu \Delta - \pi/4.
\] (11)

Here \(\hat{D}\) depends on the second transverse derivative of the local frequency (Romanowicz, 1987, Eq. 45). The second term of Eq. (10) fluctuates as a function of the angular order. For general sources and receivers, expressions for the multiplet location parameter become more complicated.

3. An Alternative Interpretation of First-Order Asymptotics

To clarify the physical meaning of first-order asymptotics, we consider a simple case of an azimuthally symmetric source and a vertical instrument, and return to the expression from which Eq. (10) is derived:
\[
\cos \Psi + it[\delta \omega \cos \Psi + (\hat{D}/2\nu) \sin \Psi].
\] (12)

Romanowicz and Roult (1986) have rewritten Eq. (12) as
\[
\cos \Psi \exp(it),
\] (13)

and have interpreted \(\lambda\) given in Eq. (10) as the frequency perturbation. Alternatively, following Romanowicz (1987, Eq. 79), Eq. (12) may be rewritten as
\[
(1/2)[1-(\hat{D}/2\nu)t+i\delta \omega t] \exp(-it\Psi)
+ (1/2)[1+(\hat{D}/2\nu)t+i\delta \omega t] \exp(it\Psi)
\sim (1/2)[1-(\hat{D}/2\nu)t] \exp[i(\delta \omega t - \Psi)]
+ (1/2)[1+(\hat{D}/2\nu)t] \exp[i(\delta \omega t + \Psi)].
\] (14)

The first term of Eq. (14) corresponds to odd orbits of surface waves, and the second term to even orbits. We may interpret Eq. (14) as that the frequency perturbation is \(\delta \omega\), and that the amplitude perturbation is
\[
\mp \frac{\hat{D}}{2\nu} t \quad \text{(for odd and even orbits)}.
\] (15)

The frequency perturbation is the same as that of the 0-th order approximation. The
amplitude perturbation given in Eq. (15) increases (or decreases) secularly with time. If the effects of coupling between multiplets on the same dispersion branch are included (ROMANOWICZ, 1987), the amplitude perturbation may be written as

\[ \mp \frac{D}{2v} t + \frac{a \Delta}{U} \frac{\dot{D} - \ddot{D}}{2v} \]  

(for odd and even orbits). \hfill (16)

The above results are derived from a normal mode approach, and the correspondence to surface waves is discussed. The travel time of each orbit is expressed as

\[ \begin{align*}
(a/U)[\Delta + (n-1)\pi] & \quad \text{(odd } n) \\
(a/U)[-\Delta + n\pi] & \quad \text{(even } n).
\end{align*} \hfill (17)\]

Substituting Eq. (17) into Eq. (16), we have the amplitude perturbation for each orbit:

\[ \begin{align*}
- \frac{a}{U} \frac{\Delta \dot{D} + (n-1)\pi \ddot{D}}{2v} & \quad \text{(odd } n) \\
- \frac{a}{U} \frac{\Delta \ddot{D} - n\pi \dot{D}}{2v} & \quad \text{(even } n).
\end{align*} \hfill (18)\]

This expression coincides with one obtained by WOODHOUSE and WONG (1986) using a surface wave approach. For the first orbit \((n = 1)\), the amplitude perturbation reduces to

\[ -\frac{a \Delta}{U} \frac{\dot{D}}{2v} \]  

which is, as expected, dependent only on the minor arc average (ROMANOWICZ, 1987).

Equations (18) and (19), which have been derived by WOODHOUSE and WONG (1986) and ROMANOWICZ (1987), are valid only for the travel time of surface waves given in Eq. (17). On the other hand, Eq. (16), which is the main result of this paper, is valid for general \(t\) and useful for calculating synthetic seismograms as discussed in the next section.

For general sources and instruments, many other terms appear in the first-order asymptotics. For example, the amplitude perturbation of the first orbit for a vertical instrument and general moment tensors may be written as (ROMANOWICZ, 1987)

\[ -\frac{a \Delta}{U} \frac{\ddot{D}}{2v} + \frac{\overline{E}}{v} \frac{2B_2(A_0 - A_2) + A_1B_1}{(A_0 - A_2)^2 + A_1^2}. \]  

Here \(\overline{E}\) depends on the first transverse derivative of the local frequency (ROMANOWICZ, 1987, Eq. 68), and \(A_i (i = 0, 1, 2)\) depend on the moment tensor and the azimuth of receiver from source. In our notation, \(A_i\) are related to \(S_{k(\pm)}\) of Eq.
(3). The derivatives of $A_i$ with respect to the azimuth are $iB_i$. We separate Eq. (20) into three parts for each $i$:

$$-\frac{aA}{U} \left( \frac{D}{2v} + \frac{\bar{E}}{v} \frac{iB_i}{A_i} \right).$$  \hspace{1cm} (21)

This is justified because we rely on the linearized approximation given in Eq. (2). The first term in the bracket is the same as Eq. (19). The second term may be interpreted in terms of the azimuthal perturbation given by

$$-\frac{aA}{U} \frac{\bar{E}}{v}.$$  \hspace{1cm} (22)

The azimuthal perturbation, which is valid for general $t$, is similar in form to the amplitude perturbation given in Eq. (16):

$$-\frac{\bar{E}}{v} t + \frac{aA}{U} \frac{\bar{E} - \bar{E}}{v}$$  \hspace{1cm} (for odd and even orbits).

For general instruments and general moment tensors, the azimuth of source from receiver is also perturbed:

$$-\frac{\bar{F}}{v} t + \frac{aA}{U} \frac{\bar{F} - \bar{F}}{v}$$  \hspace{1cm} (for odd and even orbits)

where $\bar{F}$ and $\bar{F}$ may be defined similarly to $\bar{E}$ and $\bar{E}$ (WOODHOUSE and WONG, 1986, Eq. 50). For each orbit, the azimuthal perturbation is similar to Eq. (18).

4. **Synthetic Seismograms**

WOODHOUSE and DZIEWONSKI (1984) have developed a method to calculate synthetic seismograms by normal mode summation. Synthetic seismograms are calculated exactly for a spherically symmetric earth model. The effects of aspherical perturbations are incorporated by the 0-th order asymptotics; the frequency perturbation $\delta$ and the epicentral distance shift $\delta$. We extend their method to incorporate the first-order asymptotics. Again we consider a simple case of an azimuthally symmetric source and a vertical instrument, because extension to more general cases is straightforward. The displacement is proportional to the Legendre function $P_l (\cos \Delta)$, which may be written as (e.g., AKI and RICHARDS, 1980)

$$P_l = Q_l^{(1)} + Q_l^{(2)}$$  \hspace{1cm} (25)

with

$$Q_l^{(1,2)} = 1/2 [P_l \pm (2i/\pi)Q_1].$$  \hspace{1cm} (26)

Here $Q_l$ is the Legendre function of the second kind. For large $l$, $Q_l^{(1)}$ and $Q_l^{(2)}$ behave like traveling waves of odd and even orbits, respectively;
To calculate synthetic seismograms for a laterally heterogeneous earth model, it is necessary to multiply the right-hand side of Eq. (25) by the amplitude perturbation given in Eq. (16):

\[
Q_{l}^{(12)} \sim (2\pi v \sin \Delta)^{-1/2} \exp(\mp i\Psi) .
\]  

(27)

This method has the advantages that it is exact for a spherically symmetric earth model and that overtones are easily treated. For general sources and receivers, the azimuthal perturbations given in Eqs. (23) and (24) are necessary in addition to the frequency perturbation, the distance shift, and the amplitude perturbation.

5. Discussion

In previous studies, the spheroidal-toroidal coupling has been neglected. The validity of this assumption is discussed below. For large \( l \), the spheroidal-toroidal coupling is of the order of \( l^{-1} \) compared to the coupling between multiplets on the same dispersion branch considered above (see perturbation matrix elements given in WOODHOUSE (1980) for isotropic perturbations). Therefore, the effects of the spheroidal-toroidal coupling may be comparable to those of the amplitude and azimuthal perturbations that are consequences of the first-order asymptotics. If anisotropy is introduced, the situation becomes more complicated; the spheroidal-toroidal coupling is of the same order as the coupling between multiplets on the same dispersion branch (MOCHIZUKI, 1986 b). At present, no systematic method is available to incorporate the spheroidal-toroidal coupling, and this is an interesting subject for future study. It is emphasized that anisotropy should be included to consider the spheroidal-toroidal coupling.

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REFERENCES


