The Behavior of Head Waves Propagating along Irregular Interfaces

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A wide range of experiments and observations have revealed that the Earth's crust can be considered as an elastic medium consisting of many layers. Thus head waves should play an important role in seismic refraction experiments, because they are expected to be observed as first arrivals in multi-layered media.

When we derive the shape of an irregular interface separating layers from travel times or waveforms of head waves, it is essential to know their behavior on the interface. Werth (1967) investigated it approximately by using ray-theoretical seismograms. Stephen (1984) calculated finite difference (FD) seismograms for seafloor survey models and observed synthetic 'diving head waves' (Cerveny and Ravindra, 1971). In this correspondence, we will also carry out FD calculations for land survey models, with the aim of illustrating the behavior of pure head waves propagating along irregular interfaces.

At first we consider a two-dimensionally synclined interface, which separates an upper layer with an S wave velocity \( \beta_1 \) of 1.0 km/s and a density \( \rho_1 \) of 2.0 g/cm³ from a lower halfspace with \( \beta_2 \) of 3.0 km/s and \( \rho_2 \) of 2.5 g/cm³. Upon this two-dimensional model we impose a Cartesian coordinate system \( (x, z) \), where \( x \) is horizontal distance and \( z \) is depth. The depth of the interface varies as

\[
h(x) = D + \frac{C}{2} \left[ 1 + \cos(2\pi(x - x_0)/w) \right] \quad \text{if} \quad x_0 - \frac{w}{2} < x < x_0 + \frac{w}{2} \\
= D \quad \text{otherwise},
\]

where \( D \) (average depth) = 2 km, \( C \) (maximum deviation from \( D \)) = 4 km, \( x_0 \) (distance of the center) = 15 km, and \( w \) (width of syncline) = 10 km (see Fig. 1). A line source extending perpendicularly to the \( x-z \) plane is located at the origin. This radiates an SH displacement pulse having a time function:

\[
s(t) = \sin(2\pi t/T) - \frac{1}{2} \sin(4\pi t/T) \quad \text{if} \quad 0 < t < T \\
= 0 \quad \text{otherwise},
\]

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where \( T \) (duration time) = 1 s.

The equation of motion for SH waves is solved as described by Yamanaka et al. (1989). The explicit second-order FD scheme by Korn and Stoeckl (1982) is applied, and the absorbing boundary condition of Reynolds (1978) is imposed on the sides and the bottom of the FD grid. This condition is also imposed on the top surface in order to prevent reflected waves from contaminating seismograms.

A pure head wave can be regarded as a diffracted wave emerging through an interface from a transient wave in the lower layer (this will be called a ‘parent wave’ hereinafter). When we interpret this in terms of the asymptotic ray theory (Červený and Ravindra, 1971), a head wave from a flat interface, for example, arises from the first-order term of ray expansion series. Since this term holds \( \omega^{-1} \), its waveform is represented by the single integral of an input signal.

Figure 1 shows snapshots of displacements from 4 to 9 s after the source origin time. The first-order diffraction mentioned above takes place at a horizontal part of the interface, and a clear head wave is seen in the upper layer on the first two snapshots. As soon as it reaches the downward slope of the syncline, its wavefront becomes curved following the shape of the slope. On the downward slope the apparent head wave is transmitted from the parent wave without diffraction, because there is a geometrical ray path to surface receivers. Finally, the head wave becomes very weak after it passes through the deepest point of the syncline. The head wave emerges in the opposite direction to its parent wave on the upward slope. Higher-order expansions are necessary in asymptotic theories to explain this backward propagation.

These phenomena clearly appear in the record section of Fig. 2, where calculated seismograms are plotted with amplitudes multiplied by \( x^2 \) and the head wave is indicated by solid dots. On the way to the syncline the head wave has an integrated waveform due to the first-order diffraction, while it shows a large amplitude and a distorted waveform on the downward slope, especially near the deepest point. On the upward slope and at farther distances, its waveform looks like a double integral of Eq. (2), and so the head wave there should arise from the second-order term of ray series. This second-order diffraction strongly reduces the amplitude of the head wave.

We picked the travel times of the first arrivals from all the traces in Fig. 2. These are reduced with a velocity of 3 km/s, and plotted with small crosses in Fig. 3. The solid line in the figure shows the travel time curve calculated by

\[
t = \frac{x}{\beta_2} + \frac{h(x) + D - d}{\beta_1},
\]

while the shaded line shows the one by

\[
t = \frac{1}{\beta_2} \int_0^x \left( \left( \frac{dh}{dx} \right)^2 + 1 \right)^{1/2} dx + \frac{h(x) + D - d}{\beta_1};
\]

\( d \) is the depth of the source (0 km). Equation (3) corresponds to a synthetic travel time.
Fig. 2. Record section of synthetic seismograms on the surface of the syncline model at distances between 1 and 30 km. Head waves are indicated by dots.

used in the time term method (Scheidegger and Willmore, 1957; Yoshii and Asano, 1972). Equation (4) is the travel time calculated with the assumption that the parent wave propagates just along the interface; this assumption is reasonable on the basis of Fermat's principle. If the interface is flat, i.e., $h(x) = D$, both Eqs. (3) and (4) will give us a same travel time.

The non-diffracted wave emerging from the downward slope arrived at distances between 11 and 17 km much earlier than estimated by Eqs. (3) or (4), because the geometrical ray path exists as a shortcut. Equation (4) presents a good estimate for the travel time of the second-order diffracted wave from the upward slope. When the parent wave is passing by the syncline, it is expected from Fermat's principle that it propagates along the syncline. This is also expected from the fact that it has a curved wavefront in the lowermost snapshot of Fig. 1. Thus the travel time of the head wave
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Fig. 3. Small crosses show the travel times of the first arrivals picked from the traces in Fig. 2. A time reduction is applied with a velocity of 3 km/s. The solid and shaded lines show travel times estimated by Eqs. (3) and (4), respectively.

should follow Eq. (4) beyond the syncline, but the parent wave recovers the delay due to the syncline and the travel time comes close to the one estimated by Eq. (3) at farther distances. This cannot be understood with simple asymptotic theories.

The source for Fig. 4 is located at a depth d of 4 km, and an anticlinal interface is introduced into the same two-layer model as the previous example with D = 5 km, C = -4 km, x₀ = 10 km, and w = 10 km. In this case the parent wave first encounters an upward slope, and a weak head wave emerges with a doubly integrated waveform. After the parent wave passes the shallowest point, the head wave appears from a downward slope with a larger amplitude. Figure 5 shows that the waveform of the head wave is changing from a double integral of Eq. (2) to a single one, while the parent wave is climbing down the slope. Beyond the anticline the head wave becomes very weak, and shows an extremely expanded waveform.

Figure 6 illustrates that Eq. (4) again gives us a good estimate for the travel time of the head wave from the upward slope. The head wave from the downward slope again arrives at the surface much earlier than estimated by Eqs. (3) or (4). The travel times at points 5 km or more away from the anticline are estimated well by Eq. (3), because Fermat's principle expects a straight path under the interface rather than a path along the anticline.

In summary, the behavior of head waves propagating along simple irregular interfaces is as follows:

(a) A head wave from an upward slope has a doubly integrated waveform, and its travel time can be estimated by Eq. (4).

(b) Geometrical ray tracing is necessary for a head wave from a downward slope. Its travel time is much earlier than estimated by Eqs. (3) or (4).

(c) The effect of a syncline gives an offset in the travel time of a head wave passing it, which reduces gradually at farther distances. The effect of an anticline is
Fig. 4. Snapshots of displacements from 3 to 8 s after the source origin time. A line source is located at (0, 4 km) in a two-layer model with the anticlinal interface indicated by a solid line. Positive displacements are filled in with black.
Fig. 5. Record section of synthetic seismograms on the surface of the anticline model at distances between 1 and 25 km. Head waves are indicated by dots.

Fig. 6. Small crosses show the travel times of the first arrivals picked from the traces in Fig. 5. A time reduction is applied with a velocity of 3 km/s. The solid and shaded lines show travel times estimated by Eqs. (3) and (4), respectively.
so small that we can neglect it except for close points.

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