Seismic Responses of Three-Dimensionally Sediment-Filled Valleys due to Incident Plane Waves

Michihiro Ohori,1,* Kazuki Koketsu,2 and Tadao Minami2

1 Technical Research Institute, Obayashi Corporation, Kiyose, Tokyo 204, Japan
2 Earthquake Research Institute, The University of Tokyo, Bunkyo-ku, Tokyo 113, Japan

It is a very important problem to study the effects of three-dimensional (3-D) topographical and geological irregularities on seismic motions. We first formulate this problem by introducing the Aki-Larner method (ALM) and check the validity and accuracy of our formulation by comparing with the results by the boundary integral equation method (BIEM). Secondly, we calculate the seismic responses of three-dimensionally sediment-filled valleys due to vertically incident plane SH-waves and compare them with those by 1-D and 2-D analyses. The 3-D responses both in the frequency and time domains show larger amplitudes and higher predominant frequencies than the 1-D and 2-D ones. We can find these amplification characteristics caused by 3-D irregularities even in thin valleys.

1. Introduction

It is important to evaluate long-period ground motions for design of large-scale structures with long resonance periods, e.g., high-rise buildings, long-spanned bridges, and large tanks. So earthquake engineers have become more and more interested in surface motions with a predominant period longer than 1 s. Many observations showed that these long-period ground motions were affected by thick sedimentary layers which sometimes have irregular interfaces. Therefore, it is important to study effects of interface irregularities on seismic responses.

In the last two decades, many different methods for studying these problems have been developed. Most of these methods treat two-dimensional (2-D) problems, but we can find only a few cases in which the extension to three-dimensional (3-D) problems has been carried out, e.g., the ray theory by Lee and Langston (1983); the boundary integral equation method (BIEM) by Sánchez-Sesma et al. (1984); the eigenfunction expansion method by Lee (1984); the finite element method (FEM) by Suzuki and Hakuno (1984); the boundary element method by Tong and Kuribayashi (1988). Most of the above works treat axisymmetric problems except the work with FEM.

As shown by Bard and Bouchon (1980, 1985), the Aki-Larner method (ALM)
proposed by Aki and Lamer (1970) is a very powerful and reliable means for solving 2-D problems. Moreover, even for 3-D problems, as Shinozaki (1990) stated, ALM holds greater computational efficiency than the other methods. Recently, Horike et al. (1990) extended ALM to 3-D problems, but their formulation still leaves the question how to treat the vertical incidence of S-waves. They proposed an approach to solve this problem numerically, but their method may still lead to numerical instability. Therefore, we first present the 3-D formulation with the use of new vector potentials and check its validity and accuracy by comparing its results with those obtained by another method. We then calculate the seismic responses of sediment-filled valleys due to vertically incident plane SH-waves. The results of axisymmetric and non-axisymmetric cases are shown and compared with those obtained by 1-D and 2-D analyses.

2. Method

We consider an $m$-layer structure consisting of homogeneous, isotropic layers separated by irregular interfaces. As for material properties $\rho_i$, $\alpha_i$, and $\beta_i$ indicate the mass density, P-wave velocity, and S-wave velocity of the $i$-th layer, respectively. The 3-D wave equation is given by

$$
\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = (\lambda + 2\mu) \text{grad} \cdot \text{div} \mathbf{u} - \mu \text{curl} \cdot \text{curl} \mathbf{u},
$$

where $\mathbf{u}$ represents a displacement vector and $\lambda$ and $\mu$ are Lame's constants. We first introduce potentials $\phi$ and $\psi$ such as

$$
\mathbf{u} = \text{grad} \phi + \text{curl} \psi,
$$

and assume a steady-state excitation, defined by the factor exp($-j\omega t$). We then obtain the 3-D Helmholtz equations

$$
\{ \mathcal{P}^2 \phi + h^2 \phi = 0 \quad (h = \omega / \alpha) \\
\mathcal{P}^2 \psi + k^2 \psi = 0 \quad (k = \omega / \beta)
$$

where $\mathcal{P}^2$ is the Laplacian operator, and $h$ and $k$ are wavenumbers for P- and S-waves.

If we take the usual representation for the vector potential

$$
\psi = \text{curl}(0, 0, 1\psi) + (0, 0, 2\psi),
$$

we have the harmonic solutions

$$
1\psi = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left[ 1 C(k_x, k_y) \exp(+j_3\nu Z) \\
+ 2 D(k_x, k_y) \exp(-j_3\nu Z) \right] \exp\{j(k_x X + k_y Y)\} dk_x dk_y,
$$

$$
2\psi = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left[ 1 C(k_x, k_y) \exp(+j_3\nu Z) \\
+ 2 D(k_x, k_y) \exp(-j_3\nu Z) \right] \exp\{j(k_x X + k_y Y)\} dk_x dk_y,
$$

J. Phys. Earth
where \( s_v \) is a vertical wavenumber for S-wave given by

\[
s_v = \left\{ \frac{(\omega/\beta)^2 - k_x^2 - k_y^2}{j\sqrt{k_x^2 + k_y^2 - (\omega/\beta)^2}} \right\}^{1/2}
\]

for \((\omega/\beta)^2 \geq k_x^2 + k_y^2\)

\[
\left\{ \frac{(\omega/\beta)^2 - k_x^2 - k_y^2}{\sqrt{k_x^2 + k_y^2 - (\omega/\beta)^2}} \right\}^{1/2}
\]

for \((\omega/\beta)^2 < k_x^2 + k_y^2\),

and \( k_x \) and \( k_y \) are horizontal wavenumbers in \( X \)- and \( Y \)-direction, respectively. By using the solutions of Eq. (5) the displacement can be expressed as

\[
u = (k_x s_v, k_y s_v, k_x^2 + k_y^2) \psi + (j k_y, -j k_x, 0)^2 \psi.
\]

However, Eq. (7) vanishes in the case of a vertically incident S-wave, i.e., \( k_x = k_y = 0 \). In other words, the solutions of Eq. (5) cannot exactly treat vertically incident S-waves. Horike et al. (1990) avoided the above problem by giving very small non-zero values to \( k_x \) and \( k_y \), but this may lead to another numerical problem.

We here introduce a new representation for the vector potential

\[
\psi = (0, \psi_z, z \psi),
\]

which has the same harmonic potentials as Eq. (5). In our case the displacement is expressed as

\[
u = (0, -j k_x, -j k_y) \psi + (j k_y, 0, +j k_x) z \psi,
\]

and so we are free from the problem because Eq. (9) does not vanish when \( k_x = k_y = 0 \).

The P-wave potential \( \phi \) also has a harmonic solution similar to Eq. (5). In the \( i \)-th layer (except the \( m \)-th layer)

\[
\phi_i = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} [A_i(k_x, k_y) \exp(+j \nu_i Z) + B_i(k_x, k_y) \exp(-j \nu_i Z)] \exp\{j(k_x X + k_y Y)\} d k_x d k_y,
\]

where \( \nu_i \) is a vertical wavenumber for P-waves given by

\[
\nu_i = \left\{ \frac{(\omega/\beta)^2 - k_x^2 - k_y^2}{1/2} \right\}^{1/2}.
\]

On the other hand, we neglect the upward scattered waves in the lowest layer (the \( m \)-th layer) so that

\[
\phi_m = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} A_m(k_x, k_y) \exp(+j \nu_m Z) \exp\{j(k_x X + k_y Y)\} d k_x d k_y
\]

\[+ \exp(-j \nu_0 Z) \exp\{j(k_0 X + k_0 Y)\},
\]

where

\[
\nu_m = \left\{ \frac{(\omega/\beta_m)^2 - k_x^2 - k_y^2}{1/2} \right\}^{1/2}
\]

\[
\nu_0 = \left\{ \frac{(\omega/\beta_0)^2 - k_x^2 - k_y^2}{1/2} \right\}^{1/2}.
\]

In Eqs. (10) and (12), \( A_i(k_x, k_y) \), \( B_i(k_x, k_y) \), and \( A_m(k_x, k_y) \) are unknown amplitudes of the P-wave field. The second term of Eq. (12) indicated by an underline represents
Fig. 1. Assumption of horizontally periodic irregularity of surface and/or interfaces.

the wave field which is produced by an incident plane P-wave having the horizontal wavenumbers \((k_{0x}, k_{0y})\).

Assuming the periodicity of irregularities with lengths of \(L_x\) in X-direction and \(L_y\) in Y-direction, as shown in Fig. 1(c), we discretize continuous horizontal wavenumbers and truncate the discrete wavenumbers by \((2N_x + 1)\) in X-direction and by \((2N_y + 1)\) in Y-direction:

\[
\begin{align*}
    k_q &= k_{0x} + 2\pi/L_x \times q \quad (q = 0, \pm 1, \pm 2, \ldots, \pm N_x), \\
    k_r &= k_{0y} + 2\pi/L_y \times r \quad (r = 0, \pm 1, \pm 2, \ldots, \pm N_y), \\
    \Delta k_x &= 2\pi/L_x, \\
    \Delta k_y &= 2\pi/L_y.
\end{align*}
\]  

We also discretize \(A_i(k_x, k_y), p_{vi} \) etc. as \(A_{qr}^{(i)}\), \(p_{vi}^{(i)} \) etc., and we can then approximate the infinite integrals in Eqs. (10) and (12) by the finite sums

\[
\phi_i = \sum_{q = -N_x}^{+N_x} \sum_{r = -N_y}^{+N_y} [A_{qr}^{(i)} \exp(+j p_{vi}^{(i)} Z) + B_{qr}^{(i)} \exp(-j p_{vi}^{(i)} Z)] \exp[j(k_q X + k_r Y)],
\]

in the \(i\)-th layer (except the \(m\)-th layer), and
\[ \phi_m = \sum_{q=1}^{N_x} \sum_{r=1}^{N_y} \left[ A_{qr}^{(m)} \exp(\pm j \nu_{qr}^{(m)} Z) \exp\{j(k_q X + k_r Y)\} \right] + \exp(-j \nu_0 Z) \exp\{j(k_{0x} X + k_{0y} Y)\}, \] (16)

in the lowest layer (the \(m\)-th layer). In the above expressions
\[ A_{qr}^{(i)} = A_i(k_q, k_r) \Delta k_x \Delta k_y, \]
\[ B_{qr}^{(i)} = B_i(k_q, k_r) \Delta k_x \Delta k_y, \]
\[ \nu_{qr}^{(i)} = \left\{ (\omega / \xi_i)^2 - k_q^2 - k_r^2 \right\}^{1/2}. \] (17)

For \( \psi_1 \) and \( \psi_2 \), we can obtain similar discrete expressions as Eqs. (15) and (16).

1. Consider multi-layer structures with surface or interface having curvatures due to an incident plane wave in the steady state.
2. Express the scattered field in the form of double infinite integrals with respect to continuous horizontal wave-numbers.
3. Assuming the periodicity of irregularities for \(X\) and \(Y\) direction, discretize continuous horizontal wave-numbers.
4. Approximate above double infinite integrals in (2) by the double finite sums with respect to discrete horizontal wave-numbers.
5. Applying the boundary conditions, obtain the boundary continuous equations with respect to \(X\) and \(Y\).
6. Taking double Fourier transform of above equations in (5) in \(X\) and \(Y\) direction, obtain \((2N_x + 1)(2N_y + 1) \times 9\) simultaneous linear equations.
7. Solving these equations for unknown potential amplitudes, calculate the displacement at arbitrary points.
8. To obtain time domain responses, repeat above procedure from (1) to (7) with frequency changing.

Fig. 2. Flow chart of the Aki-Larner method.
We apply the continuity conditions at every interface to Eqs. (15) and (16) etc. and take the double Fourier transform of boundary continuous equations so that we obtain \((2N_x + 1)(2N_y + 1) \times 6\) simultaneous linear equations for unknown potential amplitudes in the first layer and the half-space. We similarly obtain \((2N_x + 1)(2N_y + 1) \times 3\) equations from traction-free condition at the free surface. Solving them for unknown potential amplitudes, we can calculate displacements at arbitrary points from Eqs. (2), (15), and (16) etc. The above procedure is illustrated by the flow chart in Fig. 2. To increase the stability in our computation, we introduce the complex frequency as shown by Bard and Bouchon (1980) and Kohketsu (1987) in 2-D problems.

3. Validity and Accuracy

To show the validity of this procedure and check its accuracy, we compare the results by our method with those by BIEM of Sánchez-Sesma et al. (1984). The seismic motions of a 3-D axisymmetric sediment-filled valley due to a vertically incident plane SV-wave and P-wave were calculated. The interface shape is given by the equation

\[
Z(X, Y) = \begin{cases} 
\frac{H}{2}(1 + \cos \pi r) & \text{for} \quad 0 \leq r \leq 1 \\
0 & \text{otherwise,}
\end{cases}
\]  

(18)

where

\[
r = \sqrt{X^2 + Y^2} / R,
\]

and \(H\) denotes the maximum depth of a valley, and \(2R\) denotes the horizontal width of a valley. The model used in Fig. 3 has the width \(2R = 10\) km and the maximum depth \(H = \ldots\) km. The model is an axisymmetric sediment-filled valley used in the analyses.

Fig. 3. An axisymmetric sediment-filled valley used in the analyses.
Table 1. Soil properties.

<table>
<thead>
<tr>
<th>Density (t/m³)</th>
<th>P-velocity (km/s)</th>
<th>S-velocity (km/s)</th>
<th>Damping (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alluvium</td>
<td>ρ₁=1.8</td>
<td>α₁=1.87</td>
<td>β₁=1.0</td>
</tr>
<tr>
<td>Half-space</td>
<td>ρ₂=2.4</td>
<td>α₂=3.0</td>
<td>β₂=1.73</td>
</tr>
</tbody>
</table>

Fig. 4. Responses on the surface due to a vertically incident plane SV-wave at a frequency of 0.173 Hz. The horizontal axis is normalized by R (=half width of valley).

Fig. 5. Responses on the surface due to a vertically incident plane P-wave at a frequency of 0.3 Hz. The other conditions are the same as in Fig. 4.

Fig. 4. Responses on the surface due to a vertically incident plane SV-wave at a frequency of 0.173 Hz. The horizontal axis is normalized by R (=half width of valley).

Fig. 5. Responses on the surface due to a vertically incident plane P-wave at a frequency of 0.3 Hz. The other conditions are the same as in Fig. 4.

depth of valley $H=2.5$ km. The material properties are shown in Table 1. In our calculation, we choose the numerical parameters described in the last section as follows: $N_x=N_y=9$, $L_x=L_y=24$ km. The FFT was carried out with 64 terms for deriving the simultaneous linear equations. The normalized frequency (the ratio of the valley width to incident wavelength) was taken as $\eta=2R/\lambda=1.0$, where $\lambda$=incident wavelength. So, our calculations were carried out at a frequency of 0.173 Hz for SV-wave incidence and of 0.3 Hz for P-wave incidence. The imaginary part of the circular frequency used here was corresponding to 1 and 3% complex damping factor for the cases of SV- and P-wave incidence, respectively.

Figure 4 shows the surface responses due to a vertically incident plane SV-wave. The horizontal axis is normalized by the half width of valley, R. The results by our method are in good agreement with those by BIEM. The BIEM solution for SV-wave incidence has already been checked by Tong and Kuribayashi (1988).

Figure 5 shows the surface responses due to a vertically incident plane P-wave. We cannot find the agreement of our result with BIEM solution for P-wave incidence as much as those for SV-wave incidence. This phenomenon may be caused by the use of the complex frequency for numerical stability. Its effect appears in frequency-domain responses but can easily be removed in time-domain responses as stated by Tong and Kuribayashi (1988).
Fig. 6. Responses on the surface due to a vertically incident plane SH-wave at frequencies normalized by \( f/1(=\beta_1/4H) \), (a) 1.0, (b) 1.375, (c) 1.75, and (d) 2.0. The horizontal axis is normalized by \( R (=\text{half width of valley}) \).


Through the above comparison, we have confirmed the validity and sufficient accuracy of our method.

4. Results

4.1 Axisymmetry case

We first calculate the responses of the sediment-filled valley in Fig. 3 due to vertically incident plane SH-waves in the frequency and time domains and compare the results with those by the 1-D analyses (Haskell method) and 2-D analyses (2-D ALM). In the 2-D analyses, the Y-component on the X-axis is calculated for SH-wave incident problems and the Y- and Z-components on the Y-axis are calculated for SV-wave incident problems. Numerical parameters are chosen as follows: \( N_x = N_y = 9 \), \( L_x = L_y = 24 \text{ km} \). Our calculations presented here were made for 24 frequencies varying from \( f_1/8 \) to \( 3f_1 \) by a step of \( f_1/8 \) (\( f_1 = \beta_1/4H \)). The imaginary part of complex \( J. \text{Phys. Earth} \)
Seismic Responses of Three-Dimensionally Sediment-Filled Valleys 217

Figure 6 shows the responses on the surface for particular frequencies obtained by the 1-D, 2-D, and 3-D calculations. The normalized frequencies \( f/f_1 \) are set to (a) 1.0, (b) 1.375, (c) 1.75, and (d) 2.0. In the case with \( f/f_1 = 1.0 \) (Fig. 6(a)), the responses by the 1-D analyses show rather larger amplification than those by the 2-D and 3-D ones. At the center of the valley, the Y-component amplitude by the 1-D analyses is 1.27, 1.74, and 2.18 times larger than those by the 2-D SH, 2-D SV, or 3-D ones, respectively. In the case of \( f/f_1 = 1.75 \) (Fig. 6(b)), the responses by the 2-D analyses show larger amplification than those by the 1-D and 3-D ones. In the case of \( f/f_1 = 2.0 \) (Fig. 6(c), (d)), the response by the 3-D analyses generally shows larger amplification than those by the 1-D and 2-D ones. However, the Z-component amplitude on the Y-axis of the 2-D SV analyses is larger than that by the 3-D ones.

Figure 7 shows the Y-component transfer function at the center of the valley obtained by the 1-D, 2-D, and 3-D analyses with respect to the normalized frequency. It is found that the predominant frequencies and maximum peak levels become higher and larger as the number of model dimensions increases. Those by the 3-D analyses are 2 times higher and 1.78 times larger than those by the 1-D ones.

Figures 8 and 9 show the time-domain responses of the model on the surface to the Ricker wavelet with the predominant periods of \( T_1 (= 4H/\beta_1) \) s and \( 0.7 \times T_1 \) s. In
Fig. 7. Y-component transfer function at the center of the valley in Fig. 3 due to a vertically incident SH-wave. The frequency axis is normalized by $f_1 (= \beta_1 / 4H)$.

Fig. 8. Comparison of time-domain responses for model in Fig. 3 obtained by 1-D, 2-D, and 3-D analyses due to a vertically incident plane SH-wave. The predominant period of the incident Ricker wavelet is $T_1 (= 4H / \beta_1)$ s. The time axis is normalized by $T_1$.

In the case of $T_1$ s (Fig. 8), the largest amplitude appears in the responses by the 2-D analyses but in the case of $0.7 \times T_1$ s (Fig. 9), the largest amplitude appears in that by the 3-D ones. These features can be expected from the frequency-domain responses in Figs. 6 and 7.
4.2 Non-axisymmetry case

We next calculate the responses of non-axisymmetric sediment-filled valleys due to vertically incident plane SH-wave. Following Horike et al. (1990), the valley is defined by

\[ Z(X, Y) = \begin{cases} 
\frac{H}{4} (1 + \cos \pi A)(1 + \cos \pi B) & \text{for } |A| \leq 1 \text{ and } |B| \leq 1 \\
0 & \text{otherwise,}
\end{cases} \]  

(19)

where

\[ A = X/R_x, \quad B = Y/R_y, \]

and \( H \) denotes the maximum depth of the valley, \( R_x \) is the half width of the valley along the \( X \)-axis, and \( R_y \) is that along the \( Y \)-axis.

We take \( H = 2.5 \text{ km}, 2R_x = 10 \text{ km}, \) and \( 2R_y = 20 \text{ km} \) (Model-B in Fig. 11) or 2\( R_y = 40 \text{ km} \) (Model-C). For comparison we also consider an axisymmetric case (Model-A) and a 2-D model (Model-D). Analytical models are shown in Fig. 11. Material properties are the same as in Table 1.

Figure 11 shows the responses of the four models at the normalized frequencies (a) 1.0, (b) 1.375, and (c) 1.75. In our computation, the imaginary part of complex
circular frequency is set to $\pi f_1/8$. In the case of $f/f_1 = 1.0$ (Fig. 11(a)), the response of Model-C is almost the same as that of Model-D. However, from Fig. 11(b) and (c), we can clearly find the differences between responses of Model-C and Model-D. The responses of Model-A and Model-B also show greater amplification with increment of frequency. At the center of the valley, the surface amplitude becomes larger in the order of Model-C, Model-D, Model-B, and Model-A in the case of $f/f_1 = 1.375$ and larger in the order of Model-B, Model-A, Model-C, and Model-D in the case of $f/f_1 = 1.75$. The large amplification caused by 3-D irregularities appears clearly when frequency becomes higher than $f_1$.

5. Conclusion

If the Aki-Larner method (ALM) is extended straightforward to 3-D problems, we will encounter a theoretical difficulty in the case of vertical incidence of plane S-waves. Since this is considered the most basic and important case in earthquake engineering, we first presented a new formulation of the 3-D ALM. We then calculated
the seismic motions of a typical 3-D sediment-filled valley due to a vertically incident plane SV-wave and P-wave and our solutions agree well with the results of BIEM.

We next calculated seismic responses of a variety of sediment-filled valleys due to vertically incident plane SH-waves, and compared them with each other. The conclusions of this computation are summarized as follows.

1. The frequency-domain responses show that the predominant frequency and the maximum amplitude become larger as the number of dimensions increases. Especially, at the center of the valley, the 3-D axisymmetric valley has a 2 times higher predominant frequency and a 1.74 times larger maximum amplitude than the 1-D model.

2. The time-domain responses show that the largest amplification occurs in 3-D analyses, the second in 2-D ones, and the third in 1-D ones. This feature becomes remarkable when the frequency of the incident wave is higher than the first resonance frequency at the center of the valley.

3. In the rather thin valleys, such as those considered in this paper, 3-D irregularities induce great amplification at higher frequencies. This means that 1-D or 2-D modeling may lead to smaller responses than 3-D modeling.

We would like to express sincere thanks to Prof. Kojiro Irikura of Kyoto University.
and Dr. Hiroshi Takenaka of Obayashi Corporation for useful discussions and suggestions throughout this study. Thanks are also given to Mr. Wong Zifa, a graduate student of the University of Tokyo, for the critical reading of the manuscript.

The computations were carried out by HITACHI M-280H at the Earthquake Prediction Data Center, the Earthquake Research Institute, and by HITACHI M-682H at the Computer Center, the University of Tokyo.

REFERENCES


