Maximum Responses of Viscoelastic Multilayers
Subjected to Inhomogeneous
Plane Wave Incidence

Makoto Sato
Faculty of Engineering, Hiroshima University,
Higashi-Hiroshima 724, Japan

The frequency response functions of the viscoelastic horizontal multilayers subjected to time harmonic plane P or SV wave incidence are generally governed by the acoustic impedance ratios of the base rock to that of each layer and by the incident angle of the waves to the layer interfaces and loss factor $Q^{-1}$ of each layer. And the critical angles in the inhomogeneous wave fields play more important roles in the response characteristics than those of elastic wave fields and generalized Snell’s law should be applied.

In order to clarify the surface response characteristics of the viscoelastic layers, the factor termed “maximum response ratio” is introduced, which is evaluated by the ratios of the maximum surface response of viscoelastic layer in a properly chosen frequency range to that of elastic layer at the first resonant frequency in this frequency range. It was found that in the inhomogeneous wave field subjected to plane harmonic SV wave incidence, the maximum response ratio of horizontal component has eminent value at the normal incidence of SV wave, but that of vertical components has maximum value in the neighborhood of critical angles of SV wave.

1. Introduction

The ground motions which vibrate horizontally and vertically due to an earthquake are mostly caused by Rayleigh wave and shear wave propagation. Dynamic earthquake responses to multilayer systems are mainly analyzed by multiple reflection method for SH wave (type-II S-wave) propagation in two-dimensional approach. In this case only horizontal component of ground motion is obtained. However, the responses to SV wave (type-I S-wave) propagation contain not only both components of horizontal and vertical motions by body waves but also motions due to inhomogeneous wave when incident angle of SV wave to a layer interface exceeds a critical angle. Therefore the dynamic responses to SV wave are of importance from the point of view of earthquake-resistant design.

The frequency response functions of the viscoelastic horizontal multilayers subjected to time harmonic plane P or SV wave incidence are generally governed by

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the acoustic impedance ratios of the base rock to that of each layer and by the incident angle of the waves to the layer interfaces, and by the loss factor $Q^{-1}$ (Borcherdt, 1973) of each layer. Especially, it must be considered that the transmitted and the reflected waves generated by incident P or SV waves at arbitrary angles are always converted to inhomogeneous waves in the viscoelastic layers (Borcherdt, 1973; Henry, 1967). Therefore, the critical angles in the inhomogeneous wave fields play more important roles in the response characteristics than those of elastic wave fields and generalized Snell's law (Wennerberg, 1985) should be applied.

In the present paper, for the two-dimensional approach, in order to clarify the surface response characteristics of the viscoelastic multilayers to every incident angle of plane harmonic SV waves, the parameter termed "maximum response ratio" is introduced, which is defined as the ratios of the maximum response spectrum within a necessary frequency range of viscoelastic layer to the response spectrum at the first resonant frequency of the corresponding elastic layer. For a viscoelastic multilayer, once the parameter is obtained the incident angle generating the maximum response can be estimated easily.

The parameter is calculated to the simple model layers, to the realistic subsoil-multilayers and to the two-layered models corresponding to realistic layers, and the response characteristics of viscoelastic layers are discussed.

2. Viscoelastic Wave Fields

The equation of motion, with body forces neglected, for time harmonic plane waves in a homogeneous and isotropic linearly viscoelastic materials are

$$\mu \nabla^2 \mathbf{u} + (\lambda + 2\mu) \nabla(\nabla \cdot \mathbf{u}) + \rho \omega^2 \mathbf{u} = 0,$$

where $\mu$ and $\lambda$ are complex material properties that reduce in the Lamé's constants respectively in the limiting elastic case and these are independent of the circular frequency $\omega$, and $\mathbf{u} = u_0 \exp(-i\omega t)$ denotes the time harmonic particle displacement vector. The constant density is denoted by $\rho$, and the gradient operator is denoted by $\nabla$.

Generally, the displacement vector $\mathbf{u}$ is expressed in terms of scalar potential function $\phi$ and vector potential function $\psi$, by

$$\mathbf{u} = \nabla \phi + \nabla \times \psi.$$

The displacement vector obtained from Eq. (2) is a solution of Eq. (1), provided that

$$\nabla^2 \phi + k_p^2 \phi = 0,$$

and

$$\nabla^2 \psi + k_T^2 \psi = 0,$$

where $k_p$ and $k_T$ are the complex-valued wavenumbers of the dilatational and the distortional wave, respectively.

When both Eqs. (3a) and (3b) are expressed in the form of

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where \( k^2 = \omega^2 \rho / \bar{M}, \bar{M} = \lambda + 2\mu \) or \( \bar{\mu} \).

The general solution of Eq. (4) is obtained by the form of

\[
G = G_0 \exp(-A \cdot r) \exp(ip \cdot r) = G_0 \exp(ik \cdot r),
\]

and

\[
k = P + iA,
\]

where \( G_0 \) is a complex constant, \( r \) is a position vector, and \( k \) is the complex wave-number vector consisting of the propagation vector \( P \) and attenuation vector \( A \) (Krebes, 1983) which express the directions of the normals to both lines of constant phase and lines of constant amplitude respectively.

From Eq. (5b), the following relations are obtained:

\[
P \cdot P - A \cdot A = \text{Re}\{k^2\},
\]

and

\[
P \cdot A = |P| |A| \cos \zeta = \text{Im}\{k^2\}/2,
\]

where \( \text{Re}\{\ldots\} \) and \( \text{Im}\{\ldots\} \) denote the real part and imaginary part, respectively, and the angle \( \zeta \) denotes the angle between the vector \( P \) and \( A \) and the angle is termed attenuation angle or inhomogeneity.

In the Eq. (5), the vector \( P \) and \( A \) corresponding to the attenuation angle \( \zeta \) and wavenumber \( k \) respectively are expressed by

\[
|P|^2 = [\text{Re}\{k^2\}] + ([\text{Re}\{k^2\}]^2 + [\text{Im}\{k^2\}]^2)/(\cos^2 \zeta)^{1/2})/2,
\]

and

\[
|A| = \text{Im}\{k^2\}/2 |P| \cos \zeta.
\]

The angle \( \zeta \) must be less than \( \pi/2 \), and when \( \zeta = 0 \) we call the wave homogeneous, otherwise Eq. (5a) represents an inhomogeneous wave propagation (Borcherdt, 1982).

3. Loss Factor \( Q^{-1} \)

The attenuation of time harmonic body waves is often expressed in terms of a dimensionless parameter \( Q^{-1} \), defined to be the fractional energy loss per cycle. For the homogeneous wave, loss factor \( Q^{-1} \) is expressed by

\[
Q^{-1} = \Delta E/2\pi E = - \text{Im}\{\bar{M}\}/\text{Re}\{\bar{M}\},
\]

and

\[
\bar{M} = \text{Re}\{\bar{M}\}(1 - iQ^{-1}),
\]

where \( \Delta E \) is the energy dissipated per unit cycle per unit volume, and \( E \) is the elastic strain energy per unit cycle per unit volume (Borcherdt, 1973).

For P and SV wave, the relations between loss factor \( Q^{-1} \) and complex wave-
number $k$ in the homogeneous wave field are given by

$$Q_p^{-1} = -\text{Im}\{\tilde{\mathbf{Z}} + 2\tilde{\mu}\}/\text{Re}\{\tilde{\mathbf{Z}} + 2\tilde{\mu}\} = \text{Im}\{k_p^2\}/\text{Re}\{k_p^2\},$$  \hspace{1cm} (9a)

and

$$Q_T^{-1} = -\text{Im}\{\tilde{\mu}\}/\text{Re}\{\tilde{\mu}\} = \text{Im}\{k_T^2\}/\text{Re}\{k_T^2\},$$  \hspace{1cm} (9b)

where $Q_p^{-1}$ and $Q_T^{-1}$ are the loss factors of P and SV wave, respectively, and $k_p$ and $k_T$ are the complex wavenumbers of those waves, respectively. We assume that the change of the ratio of $Q_p^{-1}$ to $Q_T^{-1}$ will be less effective to wave behaviors than another material property change and assume that the ratio is in the range of 0.2–0.8 (Silva, 1976; Sato, 1987).

For inhomogeneous wave field, layer properties such as loss factor $(Q_N^{-1})$, wave-velocity $(C_N)$, and wavenumber $(k_N)$ are computed as a function of the attenuation angle $\zeta$ (Sato, 1987), and the inhomogeneous wave factor is expressed by

$$Q_N^{-1} = Q_Z^{-1}(1 + 2H)/(1 + H),$$  \hspace{1cm} (10a)

where

$$H = (Q_Z^{-1}/Z_{HZ})^2 \tan^2 \zeta, \quad Z_{HZ} = \sqrt{(1 + Q_Z^{-2})},$$  \hspace{1cm} (10b)

and the subscript $Z$ stands for the subscript P or T corresponding to P or SV wave, respectively.

In both homogeneous and inhomogeneous wave fields, the complex wave velocity is expressed by

$$C_Z = [C_{HZ}S_Z/2(1 + Q_Z^{-2})^{1/2}](1 - iQ_Z^{-1}/S_Z),$$  \hspace{1cm} (11a)

where for the homogeneous wave field

$$S_Z = 1 + (1 + Q_Z^{-2})^{1/2},$$  \hspace{1cm} (11b)

and for the inhomogeneous wave field

$$S_Z = 1 + (1 + Q_Z^{-2}/\cos^2 \zeta_Z)^{1/2}.$$  \hspace{1cm} (11c)

For the homogeneous wave fields, the wave velocity $C_Z$ coincides with homogeneous wave velocity $C_{HZ}$ which reduces to the elastic wave velocity when the loss factors vanish (Krebes, 1983).


For the plane interface of the viscoelastic layers shown in Fig. 1, the generalized Snell’s law (Wennerberg, 1985; Sato, 1987) in the inhomogeneous wave fields is expressed for the transmitted waves in the form of

$$|P_{Ti}| \sin \gamma_i = |P_{P_i}| \sin \theta_i = |P_{T_i}| \sin \gamma_i,$$  \hspace{1cm} (12a)

and

$$|A_{Ti}| \sin(\gamma_i - \zeta_{Ti}) = |A_{P_i}| \sin(\theta_i - \zeta_{P_i}) = |A_{T_i}| \sin(\gamma_i - \zeta_{T_i}).$$  \hspace{1cm} (12b)
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Fig. 1. Inhomogeneous wave field. When an inhomogeneous plane SV wave propagates to plane layer interface, transmitted and reflected P and SV inhomogeneous wave generates. $P$, propagation vector; $A$, attenuation vector; $\theta$, angle of P wave propagation vector; $\gamma$, angle of SV wave propagation vector; $\zeta$, attenuation angle of P or SV wave. Subscript P and T denote the P and SV wave, respectively, and subscript t and r denote the transmitted and reflected wave, respectively.

where the subscript P and T correspond to P and SV wave, respectively, the subscript i and t correspond to incident wave and transmitted wave, respectively, and the angle $\gamma$ and $\theta$ are angles between the normal to the interface and the propagation vector of SV and P wave, respectively, and when $\gamma = 0$ or $\theta = 0$, the incident wave is termed normal incidence. While the reflected waves are considered, we may replace the subscript t with r representing the reflected wave.

If the attenuation angles $\zeta$ of transmitted and reflected wave are obtained, the propagation vector $P_Z$ and the attenuation vector $A_Z$ ($Z = P$ or T) are evaluated by using Eq. (6) and the following relation which is given from Eqs. (9) and (10) is obtained:

$$k_Z^2 = \frac{\omega^2 \rho}{\tilde{M}_Z} = \left(\frac{\omega}{C_{HZ}}\right)^2 \frac{2}{\tilde{S}_Z}(1 + iQ_Z^{-1}) .$$

(13)

In the inhomogeneous wave field, the critical angles of an incident wave are such angle that one of the propagation vectors of the reflected and transmitted waves is parallel to the layer interface considered. But since the wavenumber vector $P$ and $A$ are a function of the attenuation angle $\zeta$, the attenuation angle and the critical angles as well as the angles of propagation vector of the reflected and transmitted wave are not obtained explicitly as in the elastic layer. They are evaluated by solving a simultaneous equation of Eq. (12) (Sato, 1987). The propagation vector of the reflected and transmitted wave lying along the interface occurs only for discrete critical incident angles, and not for a continuous range of incident angle as in the elastic
supercritical incidence (Borcherdt, 1982).

5. Maximum Response Ratio

The frequency response functions of given layers are usually expressed by the spectra or amplitude ratios of displacement or acceleration for the prescribed incident angle (Sato, 1987; Silva, 1976), therefore the response characteristics for an arbitrary incident angle of P or SV waves cannot be discussed easily. Furthermore, the maximum response of a layer to every possible incident angle in the range of usual earthquake frequency plays an important role in earthquake-resistant engineering. In order to express plainly the maximum response spectrum at a given layer surface for every possible incident angle (0°–90°), it may be convenient to introduce the parameter termed maximum response ratio, and the ratios are plotted to every incident angle within a necessary frequency range.

In a properly chosen frequency range, the maximum response ratio ($\eta'$) is defined by the ratio of the maximum spectrum ($\eta_{max}$) at the viscoelastic multilayer surface to the spectrum at the corresponding elastic layer surface ($Q^{-1} = 0$).

The maximum spectrum ($\eta_{max}$) at the prescribed incident angle is searched for the frequency range of 0–40 Hz corresponding to the range of usual earthquake frequency, and the spectrum of the corresponding elastic layer surface is specified by the horizontal response spectrum ($\eta_1$) to the first resonant frequency of the elastic layer at the normal incidence of SV wave.

Thus, for SV wave incidence with the incident angle $\gamma$, the maximum response ratio of a viscoelastic layer is given by

$$\eta'(\gamma) = \frac{\eta_{max}(\gamma)}{\eta_1(0)},$$

where $\eta_1(0)$ is the horizontal response spectrum to the first resonant frequency of the corresponding elastic layer surface at $\gamma_0 = 0$ (normal incidence), and $\gamma_0$ shows the incident angle of SV wave to the base rock of the layers.

It is necessary to pay attention that $\eta_{max}$ is not always obtained at the same resonant frequency of the layer considered when the incident angle changes.

6. Applications

In order to consider the effects of an acoustic impedance ratio and a loss factor change of viscoelastic layers, an extended Haskell matrix is employed (Brekhovskikh, 1960; Haskell, 1953; Sato, 1982, 1987), and three types of model which represent simple model layers, realistic layers, and two-layered model corresponding to the realistic layers are considered. The material properties and thickness of these layers are listed in Tables 1–3. In these applications, incident waves in the base rock are restricted to homogeneous SV wave, i.e., $A_{\xi\xi} = 0$ in the base rock, but in the viscoelastic layers all reflected and transmitted waves are generally inhomogeneous waves.

Figure 2 shows the viscoelastic model layer ML-1–ML-5 composed of surface layer and base rock subjected to plane SV wave incidence with incident angle $\gamma_0$. The
### Table 1. Material properties and thickness of model layers.

<table>
<thead>
<tr>
<th>Model layer</th>
<th>S-wave velocity $C_T$ (m/s)</th>
<th>Thickness $d$ (m)</th>
<th>Density $\rho$ (t/m$^3$)</th>
<th>Poisson ratio $\nu$</th>
<th>Loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>ML-1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface layer</td>
<td>200</td>
<td>25</td>
<td>2.1</td>
<td>0.45</td>
<td>0.10</td>
</tr>
<tr>
<td>Base rock</td>
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<td>0.40</td>
<td>0.10</td>
<td>0.04</td>
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<td></td>
<td></td>
</tr>
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<td>0.50</td>
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<tr>
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<td>0.40</td>
<td>0.30</td>
<td>0.24</td>
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<tr>
<td>ML-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface layer</td>
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<td>25</td>
<td>2.1</td>
<td>0.45</td>
<td>0.10</td>
</tr>
<tr>
<td>Base rock</td>
<td>300</td>
<td>2.1</td>
<td>0.40</td>
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<td>0.04</td>
</tr>
<tr>
<td>ML-4</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Surface layer</td>
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<td>2.1</td>
<td>0.45</td>
<td>0.30</td>
</tr>
<tr>
<td>Base rock</td>
<td>500</td>
<td>2.1</td>
<td>0.40</td>
<td>0.10</td>
<td>0.04</td>
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<tr>
<td>ML-5</td>
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<td></td>
<td></td>
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<tr>
<td>Surface layer</td>
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<td>25</td>
<td>2.1</td>
<td>0.45</td>
<td>0.50</td>
</tr>
<tr>
<td>Base rock</td>
<td>500</td>
<td>2.1</td>
<td>0.40</td>
<td>0.10</td>
<td>0.04</td>
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### Table 2. Material properties and thickness of realistic layers.

<table>
<thead>
<tr>
<th>Realistic layer</th>
<th>S-wave velocity $C_T$ (m/s)</th>
<th>Thickness $d$ (m)</th>
<th>Density $\rho$ (t/m$^3$)</th>
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<th>Loss factor</th>
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<td>RL-1</td>
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<tr>
<td>Surface layer</td>
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<td>1.4</td>
<td></td>
<td>0.09</td>
</tr>
<tr>
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<td>3.2</td>
<td>1.9</td>
<td></td>
<td>0.11</td>
</tr>
<tr>
<td>3rd layer</td>
<td>160</td>
<td>1.4</td>
<td>1.4</td>
<td></td>
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</tr>
<tr>
<td>4th layer</td>
<td>319</td>
<td>4.5</td>
<td>2.1</td>
<td>0.45</td>
<td>0.16</td>
</tr>
<tr>
<td>5th layer</td>
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<td>1.2</td>
<td>2.1</td>
<td></td>
<td>0.15</td>
</tr>
<tr>
<td>6th layer</td>
<td>329</td>
<td>1.1</td>
<td>2.1</td>
<td></td>
<td>0.16</td>
</tr>
<tr>
<td>7th layer</td>
<td>282</td>
<td>3.3</td>
<td>1.8</td>
<td></td>
<td>0.14</td>
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<td>2.1</td>
<td>0.40</td>
<td>0.23</td>
<td>0.09</td>
</tr>
<tr>
<td>RL-2</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface layer</td>
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<td>3.3</td>
<td>1.9</td>
<td></td>
<td>0.11</td>
</tr>
<tr>
<td>3rd layer</td>
<td>317</td>
<td>5.9</td>
<td>2.1</td>
<td></td>
<td>0.16</td>
</tr>
<tr>
<td>4th layer</td>
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<td>2.3</td>
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<td>0.45</td>
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<td>1.8</td>
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<tr>
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<td>0.40</td>
<td>0.23</td>
<td>0.09</td>
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</table>
Table 3. Material properties and thickness of two layered model.

<table>
<thead>
<tr>
<th>Model layer</th>
<th>S-wave velocity $C_f$ (m/s)</th>
<th>Thickness $d$ (m)</th>
<th>Density $\rho$ (t/m$^3$)</th>
<th>Poisson ratio $\nu$</th>
<th>Loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>TL-1</td>
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<tr>
<td>Surface layer</td>
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<td>1.84</td>
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<td>0.11</td>
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<tr>
<td>Base rock</td>
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<td>2.10</td>
<td>0.40</td>
<td>0.23</td>
</tr>
<tr>
<td>TL-2</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Surface layer</td>
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<td>26.1</td>
<td>1.95</td>
<td>0.45</td>
<td>0.12</td>
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<tr>
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<td></td>
<td>2.10</td>
<td>0.40</td>
<td>0.23</td>
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</table>

Fig. 2. Simple model layer ML-1–ML-5 and incident SV wave. Material properties of these models are listed in Table 1. The incident SV wave in the base rock is homogeneous wave. $\gamma_0$, incident angle of SV wave; $d$, layer thickness.

material properties of the model layer ML-1–ML-5 are shown in Table 1. Solving the Eq. (12) with given parameters of the model in the restriction of $\theta_{Ti} = \theta_{Ti} = 0$ in the base rock, it is found that ML-1 has no critical angles ($\gamma_{CV}$) for inhomogeneous wave, but elastic critical angle ($\gamma_{CE}$) for the reflected P wave of the elastic layer corresponding to ML-1 is $24.1^\circ$.

For a special situation of the maximum response ratios, horizontal response component ($\eta_x'$) and vertical response component ($\eta_z'$) are shown in Fig. 3(a) and (b), respectively, where the ratios are plotted to every incident angle ($\gamma_0 = 0^\circ$–$90^\circ$) when the ratios are composed for only each first three resonant frequencies of ML-1. True maximum response ratios of this model are shown in Fig. 4, thus it is clear that the ratios are composed of those response ratios containing first three and higher resonant frequencies. Also it is clear from a deformation of smooth curve at $\gamma_0 = 24.1^\circ$ of Figs. 3 and 4 that the elastic critical angle has effects on the maximum response ratios of the viscoelastic layer.

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Fig. 3. Maximum response ratios of first three resonant frequencies of model layer ML-1. (a) is horizontal component ($\eta_{X'}$) and (b) is vertical component ($\eta_{Z'}$). The ratios are plotted for only each first three resonant frequencies. It is clear from a deformation of smooth curve at $\gamma_0 = 24.1^\circ$ (elastic critical angle $\gamma_{CE}$) that the elastic critical angle has effects on the ratios. —— 1st resonant freq; —— 2nd resonant freq; —— 3rd resonant freq.

Fig. 4. Maximum response ratios for horizontal ($\eta_{X'}$) and vertical ($\eta_{Z'}$) components of model layer ML-1. The ratios are composed of first three and higher resonant frequencies and are true maximum response ratios of this model.
Fig. 5. Maximum response ratios of model layer ML-2. (a) is horizontal component ($\eta_{X'}$) and (b) is vertical component ($\eta_{Z'}$). This model has two viscoelastic critical angles of $\gamma_{CV}=14.7^\circ$ for the transmitted P wave and $\gamma_{CV}=64.7^\circ$ for the transmitted SV wave and three elastic critical angles of $\gamma_{CE}=17.8^\circ$ for the transmitted P wave, $\gamma_{CE}=48.5^\circ$ for the transmitted SV wave, and $\gamma_{CE}=24.1^\circ$ for the reflected P wave. The ratios show that the effect of the elastic critical angle $\gamma_{CE}=24.1^\circ$ appears very sensitively.

Fig. 6. Maximum response ratios of model layer ML-3. (a) is horizontal component ($\eta_{X'}$) and (b) is vertical component ($\eta_{Z'}$). This model has one viscoelastic critical angle of $\gamma_{CV}=48.6^\circ$ and has the same elastic critical angles as model layer ML-2. The ratios have two sharp and large peak ratios between $\gamma_0=40^\circ$ and $\gamma_0=50^\circ$. The first peak ratio does not correspond to any critical angles, thus the peak ratio will cast some doubts on the numerical accuracy.

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Fig. 7. Maximum response ratios of model layer ML-1, ML-4, and ML-5. (a) is horizontal component ($\eta_X'$) and (b) is vertical component ($\eta_Z'$). The differences of the ratios are compared considering the loss factor change. The horizontal components decrease smoothly as the loss factor increases, but the vertical components have a tendency to converge to a constant ratio as the loss factor increases. — ML-1; — ML-4; — ML-5.

To investigate the effects of inhomogeneous critical angles to the maximum response ratio, the viscoelastic model layer ML-2 and ML-3 are introduced, and the maximum response ratios are shown in Figs. 5(a), (b) and 6(a), (b), respectively. The material properties of these model layers are shown in Table 1. The model layer ML-2 has two viscoelastic critical angles of $\gamma_{CV}=14.7^\circ$ for the transmitted P wave and $\gamma_{CV}=64.7^\circ$ for the transmitted SV wave and ML-3 has a critical angle of $\gamma_{CV}=48.6^\circ$ for the transmitted SV wave, when the incident wave is homogeneous SV wave only. The elastic layers corresponding to both model layers have three elastic critical angles of $\gamma_{CE}=17.8^\circ$ for the transmitted P wave, $\gamma_{CE}=48.5^\circ$ for the transmitted SV wave, and $\gamma_{CE}=24.1^\circ$ for the reflected P wave.

We can see that the viscoelastic response ratios show a smooth transition, unlike the elastic response which exhibits a sharp transition when going from subcritical to supercritical incident angles. It is considered that the effect of the elastic critical angle ($\gamma_{CE}=24.1^\circ$) for the reflected P wave appears more sensitively than the effect of other critical angles. But there are some special cases like ML-3, where two sharp and large ratios occur between $\gamma_0=40^\circ$ and $\gamma_0=50^\circ$. The first peak ratio does not correspond to any critical angles, but the incident angle corresponding to the second peak ratio is comparatively close to the critical angles ($\gamma_{CV}=48.6^\circ$ or $\gamma_{CE}=48.5^\circ$) for the transmitted SV wave. Unnatural first peak ratio will cast some doubts on the numerical accuracy, but we could not indicate the reason.

Figure 7(a), (b) shows the maximum response ratios of the model layer ML-1,
ML-4, and ML-5 (see Table 1) and the differences of the response ratios are compared considering the loss factor change of surface layer. The horizontal response decreases smoothly as the loss factor of surface layer increases, but the vertical responses have a tendency to converge to a constant value in an incident angle when the attenuation increases.

To investigate the responses of realistic viscoelastic subsoil-multilayers, two examples of layer RL-1 and RL-2 (see Table 2) are chosen, and to examine the usefulness of simplifying the treatments of complex realistic layers, two-layered model TL-1 and TL-2 (see Table 3) composed of only a surface layer and base rock are introduced by using the averaged value of the material properties (\(C_z\), \(Q_z^{-1}\), and \(\rho\)) weighted by a layer thickness corresponding to the realistic layers. These averaged values are evaluated by the following forms,

\[
C_{HT}^{*} = D / \left\{ \sum_j (d_j / C_{HTj}) \right\}, \quad (15a)
\]

\[
Q_{Z}^{-1}^{*} = 2 \sqrt{R (1 + R)} , \quad (15b)
\]

\[
1 / \sqrt{R} = \sum_j [ (d_j / D) / \{ C_{HZj} Q_{Tj}^{-1} / 2 \sqrt{S_{Zj}} \} ] , \quad (15c)
\]

and

Fig. 8. Maximum response ratios of realistic elastic layer RL-1 and the two-layered elastic model TL-1. (a) is horizontal component (\(\eta_X\)) and (b) is vertical component (\(\eta_Z\)). Material properties of these models are listed in Tables 2 and 3. The reason why the horizontal maximum response ratio of RL-1 exceeds unit at normal incidence is that the response of higher resonant frequency is greater than that of the first resonant frequency. The maximum response ratios change so drastically in the range of supercritical incident angle (\(\gamma_{CE} \geq 24.1^\circ\)) and in the higher resonant frequencies, that the two-layered model TL-1 is not suitable to investigate the realistic layers as an elastic layer. —— RL-1: —— TL-1.

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Fig. 9. Maximum response ratios of realistic viscoelastic layer RL-1 and the two-layered viscoelastic model TL-1. (a) is horizontal component ($\eta_X^*$) and (b) is vertical component ($\eta_Z^*$). Material properties of these models are listed in Tables 2 and 3. The horizontal components of these models decrease monotonously with an increase of incident angles, and the vertical components form a dome-like shape attaining the peak value at the incident angle of about $45^\circ$. In the viscoelastic responses, the ratios of the realistic layers and the two-layered models are much the same in comparison with the elastic ratios, therefore in this situation the two-layered model is suitable to assume the response characteristics of the realistic layers. — RL-1; — TL-1.

$$\rho^* = \frac{\sum \rho_j d_j}{D},$$

where superscript * denotes averaged value, $D$ is total layer thickness except base rock, $j$ is the number of layer, and $\sum_j$ implies the summation of the number of layer.

The maximum response ratios as an elastic layer ($Q^{-1}$ = 0) of RL-1 and TL-1 are shown in Fig. 8(a), (b). The reason why the horizontal maximum response ratio of RL-1 exceeds unit at normal incidence is that the response of higher resonant frequency is greater than that of the first resonant frequency. The maximum response ratios change so drastically in the range of supercritical incident angle ($\gamma_{CE} \geq 24.1^\circ$) and in the higher resonant frequencies, that the two-layered model TL-1 is not suitable to investigate the realistic layers as an elastic layer.

The maximum response ratios for the given viscoelastic layers are shown in Figs. 9(a), (b) and 10(a), (b). The horizontal response curves of these layers decrease monotonously with an increase of incident angles, and the vertical response curves form a dome-like shape attaining the peak value at the incident angle of $45^\circ$. In the viscoelastic responses, the response curves of the realistic layers and the two-layered models are much the same in comparison with the elastic response curves, therefore in
Fig. 10. Maximum response ratios of realistic viscoelastic layer RL-2 and the two-layered viscoelastic model TL-2. (a) is horizontal component ($\eta_X'$) and (b) is vertical component ($\eta_Z'$). Material properties of these models are listed in Tables 2 and 3. The ratios of RL-2 slightly differ from that of TL-2, because RL-2 has the eminent value at the resonant frequencies beyond the second resonant frequencies. —— RL-2; —— TL-2.

In this situation the two-layered model is suitable to assume the response characteristics of the realistic layers. But the response curves of RL-2 slightly differ from that of TL-2, because RL-2 has the eminent value at the resonant frequencies beyond the second resonant frequency.

7. Conclusion

In this paper, a parameter termed "the maximum response ratio" was introduced and the surface response characteristics of viscoelastic layers to plane harmonic homogeneous SV wave with arbitrary incident angle were studied by using the extended Haskell matrix. The results may be summarized as follows.

1) The maximum response ratio of a layer is not always obtained at the same resonant frequency when the incident angles change, and the maximum response ratios of a given layer are obtained by maximum ratios of each resonant frequency.

2) Although the elastic response ratio curves exhibit a sharp transition when going from subcritical to supercritical incident angles, the curves of viscoelastic response ratio show a smooth transition.

3) For the realistic elastic layers, there are some cases where horizontal maximum response ratio exceeds unit at normal incidence, because the response of higher resonant frequency is greater than that of the first resonant frequency.

4) To the maximum response ratios of the viscoelastic layer, the effect of the
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elastic critical angle for the transmitted P wave appears very sensitively.

5) There are some special cases where sharp and large responses occur in the neighborhood of the critical angle for the transmitted SV wave.

6) In the viscoelastic layers, the horizontal maximum response ratios decrease monotonously with increase of incident angle, but the vertical ratios form a dome-like shape for increase of the incident angle.

7) The two-layered model gives good approximation for the response of realistic layer in viscoelastic case compared with elastic one.

REFERENCES


