Synthetic Near-Field Seismograms Due to Rupture-Propagation Fault Models

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Complete displacement potentials due to kinematic fault models with a bidirectional-bilateral and a fan-shaped rupture-propagations are derived in the Weyl integral representation. Dunkin's formulation of propagator-matrix method is used to compute the seismic responses of a flat-layered medium. Surface displacements are evaluated with the exact wavenumber discretization technique. Fast computation of synthetic near-field seismograms is made with a time-reducing device. A vertically traveling plane S-wave element (VTSE), which relates to contribution of a fault segment just below an observed station to wavenumber responses, in the Weyl integral representation is also participated in the computation. The synthetic near-field seismograms computed show that the contribution of VTSE to wavenumber responses is larger for a lower frequency range or a smaller epicentral distance. This is explained qualitatively by the saddle point approximation of the steepest descent method. Some numerical examples are presented to demonstrate the advantage of our method.

1. Introduction

The sources of seismic waves in the earth are not always localized in time and space. In many cases, a rupture usually propagates along a fault with an average velocity from 50 to 80% of an S-wave velocity. Haskell (1964) first derived dynamic near-field displacements for a simple rupture-propagation fault model in an infinite homogeneous elastic medium. Madariaga (1978) derived its analytic solution by using Cagniard de-Hoop technique (de-Hoop, 1958). Kawasaki *et al.* (1975) obtained the complete synthetic seismograms for rupture-propagation faults in a semi-infinite homogeneous medium using the Cagniard's technique (Cagniard *et al.*, 1962). Bouchon (1979) gave analytic solutions for displacement potentials of rupture-propagation faults with simple source geometries and simple rupture-propagation modes, and presented synthetic seismograms due to those dislocation sources in a layered half-space. Takeo (1983) extended them to those with an arbitrary source geometry but simple rupture-propagation modes. Kohketsu (1985) computed synthetic seismograms, based on the reflectivity method (Fuchs and Müller, 1971), for rupture-propagation faults by a numerical integration of seismic responses due to point dislocations distributed over a

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fault surface. Chouet (1987) presented an exact formulation for a rupture-propagation fault penetrating through a layered interface, although he did not show any numerical example.

In order to use the propagator matrices (Haskell, 1953), Bouchon (1979) derived conversion equations from vector S-wave potentials to scalar ones. Bouchon (1979) and Takeo (1983), however, ignored a vertically traveling plane S-wave element in the S-wave displacement potentials in their representations, which relates to contribution of a fault segment just below an observed station to wavenumber response. We will abbreviate the vertically traveling plane S-wave element to "VTSE" here. VTSE is the plane S-wave element that is specified by $k_x = k_y = 0$, where k_x and k_y are, respectively, the wavenumbers in x - and y -directions in a Cartesian coordinate system. Contribution of VTSE must be taken into account for the complete near-field seismogram synthesis, because VTSE does always exist except a few special cases as described later.

In the first step, we derive analytical expressions of displacement potentials for a rupture-propagation fault. The expressions in the present study can represent a kinematic dislocation fault with an arbitrary rupture direction and rupture velocity on a fault plane with an arbitrary strike, dip and slip angle. Although far-field expressions for rupture modes of the unilateral or bilateral rupture-propagation were derived by Ben-Menahem (1962), his expressions were not appropriate for seismogram synthesis in the near-field.

In the second step, we present a simple method to evaluate the exact contribution of VTSE to the surface displacements, which was ignored by Bouchon (1979) and Takeo (1983). We also discuss the contribution of VTSE on synthetic seismograms in terms of wavenumber spectral amplitudes.

Some numerical examples including a forward modeling to near-field seismograms of the 1961 Kushiro-Oki earthquake that occurred along a plate boundary are presented to demonstrate the advantage of the present method.

2. Displacement Potentials Due to the Rupture-Propagation Fault Model

We express seismic wavefields in terms of the P-wave and S-wave potentials ϕ and Ψ , from which the displacement vector is obtained through the relationship

$$\mathbf{u} = \nabla \phi + \nabla \times \Psi. \quad (1)$$

ϕ and Ψ are solutions of the wave equations

$$\nabla^2 \phi = \frac{1}{\alpha^2} \frac{\partial^2 \phi}{\partial t^2}, \quad \nabla^2 \Psi = \frac{1}{\beta^2} \frac{\partial^2 \Psi}{\partial t^2},$$

where α and β denote the P-wave and S-wave velocities, respectively. We start with expressions for a point dislocation and integrate them over a fault of a finite extent. Following Burridge and Knopoff (1964), displacement potentials due to a point dislocation source with a dip δ and a slip angle λ in an infinite elastic medium are expressed in a Cartesian coordinate (x, y, z) as

$$\begin{aligned}\phi &= \cos \lambda \sin \delta \left(\frac{\partial \phi^x}{\partial y_0} + \frac{\partial \phi^y}{\partial x_0} \right) - \cos \lambda \cos \delta \left(\frac{\partial \phi^x}{\partial z_0} + \frac{\partial \phi^z}{\rho x_0} \right) \\ &\quad + \sin \lambda \sin 2\delta \left(-\frac{\partial \phi^y}{\partial y_0} + \frac{\partial \phi^z}{\partial z_0} \right) + \sin \lambda \cos 2\delta \left(\frac{\partial \phi^y}{\partial z_0} + \frac{\partial \phi^z}{\partial y_0} \right), \\ \Psi &= \cos \lambda \sin \delta \left(\frac{\partial \Psi^x}{\partial y_0} + \frac{\partial \Psi^y}{\partial x_0} \right) - \cos \lambda \cos \delta \left(\frac{\partial \Psi^x}{\partial z_0} + \frac{\partial \Psi^z}{\partial x_0} \right) \\ &\quad + \sin \lambda \sin 2\delta \left(-\frac{\partial \Psi^y}{\partial y_0} + \frac{\partial \Psi^z}{\partial z_0} \right) + \sin \lambda \cos 2\delta \left(\frac{\partial \Psi^y}{\partial z_0} + \frac{\partial \Psi^z}{\partial y_0} \right),\end{aligned}$$

where ϕ^i and Ψ^i are displacement potentials due to a single force acting in the i -th direction. Expressions of ϕ^i and Ψ^i were derived by Bouchon (1979). A subscript, 0, indicates the source coordinate.

After some calculations (Bouchon, 1979), the displacement potentials for a point dislocation at (x_0, y_0, z_0) can be written in the form of the Weyl integral (Weyl, 1919) as

$$\begin{aligned}\phi &= \frac{iM_0}{8\pi^2 \mu k_\beta^2} \iint A \exp[-ik_x(x-x_0) - ik_y(y-y_0) - i\nu|z-z_0|] dk_x dk_y, \\ \Psi_j &= \frac{iM_0}{8\pi^2 \mu k_\beta^2} \iint B_j \exp[-ik_x(x-x_0) - ik_y(y-y_0) - i\gamma|z-z_0|] dk_x dk_y,\end{aligned}\quad (2)$$

where $k_\beta = \omega/\beta$, M_0 the scalar seismic moment, μ the rigidity, k_x and k_y are the wavenumbers in x - and y -directions, respectively. An exponent, $\exp[i\omega t]$, is omitted for simplicity. Vertical wavenumbers ν and γ are defined as

$$\nu = \sqrt{k_\alpha^2 - k_x^2 - k_y^2} \quad \text{Im } \nu \leq 0, \quad \gamma = \sqrt{k_\beta^2 - k_x^2 - k_y^2} \quad \text{Im } \gamma \leq 0,$$

with $k_\alpha = \omega/\alpha$. A and B_j ($j=x, y, z$) are written as

$$\begin{aligned}A &= \frac{2k_x k_y}{\nu} \cos \lambda \sin \delta - 2k_x \operatorname{sgn}(z-z_0) \cos \lambda \cos \delta \\ &\quad + \left(\nu - \frac{k_y^2}{\nu} \right) \sin \lambda \sin 2\delta + 2k_y \operatorname{sgn}(z-z_0) \sin \lambda \cos 2\delta, \\ B_x &= k_x \operatorname{sgn}(z-z_0) \cos \lambda \sin \delta + \frac{k_x k_y}{\gamma} \cos \lambda \cos \delta \\ &\quad - 2k_y \operatorname{sgn}(z-z_0) \sin \lambda \sin 2\delta + \left(\gamma - \frac{k_y^2}{\gamma} \right) \sin \lambda \cos 2\delta, \\ B_y &= -k_y \operatorname{sgn}(z-z_0) \cos \lambda \sin \delta + \left(\gamma - \frac{k_x^2}{\gamma} \right) \cos \lambda \cos \delta\end{aligned}\quad (3)$$

$$\begin{aligned}
 & +k_x \operatorname{sgn}(z-z_0) \sin \lambda \sin 2\delta + \frac{k_x k_y}{\gamma} \sin \lambda \cos 2\delta, \\
 B_z = & \left(\frac{k_y^2 - k_x^2}{\gamma} \right) \cos \lambda \sin \delta - k_y \operatorname{sgn}(z-z_0) \cos \lambda \cos \delta \\
 & + \frac{k_x k_y}{\gamma} \sin \lambda \sin 2\delta - k_x \operatorname{sgn}(z-z_0) \sin \lambda \cos 2\delta.
 \end{aligned}$$

Let us now consider a rectangular fault where a rupture starts at $(0, 0, z_1)$ and propagates over a fault length L_1 and a width W_1 with a uniform rupture velocity c_1 and a constant slip D_1 . The direction of rupture propagation is θ_1 , measured clockwise from a strike direction on a fault plane. We introduce new coordinates (ξ, η) fixed on the fault surface (Fig. 1). The source coordinates (x_0, y_0, z_0) are related to the coordinates (ξ, η) as

$$x_0 = \xi, \quad y_0 = \eta \cos \delta, \quad z_0 = \eta \sin \delta + z_1.$$

For simplicity, we restrict our attention to the radiation above the source. By operating

$$\int_0^{L_1} d\xi \int_0^{W_1} d\eta \exp \left[-i\xi \frac{\omega}{c_1} \cos \theta_1 - i\eta \frac{\omega}{c_1} \sin \theta_1 \right],$$

to (2), we obtain displacement potentials of the unilateral-unidirectional rupture-propagation fault, type I (Fig. 1), in the form of the Weyl integral as

Fig. 1. Geometry of the rectangular rupture-propagation fault. θ_i , c_i , and D_i indicate, respectively, a direction of rupture propagation, a rupture velocity and a slip on the i -th subfault. See text for explanation of other parameters.

$$\begin{aligned}
\phi &= \frac{-iD_1}{8\pi^2 k_\beta^2} \iint A \exp[-ik_x x - ik_y y + ivz] dk_x dk_y \\
&\quad \times I_L(L_1, \theta_1, c_1) I_w(W_1, \theta_1, c_1, v, z_2), \\
\Psi_j &= \frac{-iD_1}{8\pi^2 k_\beta^2} \iint B_j \exp[-ik_x x - ik_y y + i\gamma z] dk_x dk_y \\
&\quad \times I_L(L_1, \theta_1, c_1) I_w(W_1, \theta_1, c_1, \gamma, z_2),
\end{aligned} \tag{4}$$

where z_2 is the depth of the lower end of the width of the fault. I_L and I_w represent the effect of rupture-front propagation as

$$\begin{aligned}
I_L(L, \theta, c) &= \frac{\exp\left[ik_x L - i\frac{\omega}{c} L \cos \theta\right] - 1}{k_x - \frac{\omega}{c} \cos \theta}, \\
I_w(W, \theta, c, \kappa, z) &= \frac{\exp\left[ik_y W \cos \delta - i\frac{\omega}{c} W \sin \theta - i\kappa z\right] - \exp[-i\kappa z_1]}{k_y \cos \delta - \frac{\omega}{c} \sin \theta - \kappa \sin \delta}.
\end{aligned} \tag{5}$$

Setting $\theta=0^\circ$, (4) become identical to the expressions for the unilateral-unidirectional rupture-propagation fault by Takeo (1983).

Expressions for other types of the unilateral-unidirectional mode such as types II–IV (Fig. 1) in the form of the Weyl integral are obtained in the same way. We can therefore obtain the displacement potentials for a rupture-propagation mode, which is called the bilateral-bidirectional rupture-propagation fault, by combining four unilateral-unidirectional rupture-propagation faults as schematically illustrated in Fig. 1 as

$$\begin{aligned}
\phi &= \frac{-i}{8\pi^2 k_\beta^2} \iint A \exp[-ik_x x - ik_y y + ivz] dk_x dk_y \\
&\quad \times \{D_1 I_L(L_1, \theta_1, c_1) I_w(W_1, \theta_1, c_1, v, z_2) \\
&\quad - D_2 I_L(-L_2, \theta_2, c_2) I_w(W_1, \theta_2, c_2, v, z_2) \\
&\quad + D_3 I_L(-L_2, \theta_3, c_3) I_w(-W_2, \theta_3, c_3, v, z_3) \\
&\quad - D_4 I_L(L_1, \theta_4, c_4) I_w(-W_2, \theta_4, c_4, v, z_3)\}, \\
\Psi_j &= \frac{-i}{8\pi^2 k_\beta^2} \iint B_j \exp[-ik_x x - ik_y y + i\gamma z] dk_x dk_y \\
&\quad \times \{D_1 I_L(L_1, \theta_1, c_1) I_w(W_1, \theta_1, c_1, \gamma, z_2) \\
&\quad - D_2 I_L(-L_2, \theta_2, c_2) I_w(W_1, \theta_2, c_2, \gamma, z_2)
\end{aligned} \tag{6}$$

$$+ D_3 I_L(-L_2, \theta_3, c_3) I_w(-W_2, \theta_3, c_3, \gamma, z_3) \\ - D_4 I_L(L_1, \theta_4, c_4) I_w(-W_2, \theta_4, c_4, \gamma, z_3) \},$$

where z_3 is the depth of the upper end of the width of the fault. If propagating rupture fronts of four subfaults, types I-IV, link up each other, we impose the constraint as

$$\frac{\sin \theta_1}{c_1} = \frac{\sin \theta_2}{c_2}, \quad \frac{\sin \theta_3}{c_3} = \frac{\sin \theta_4}{c_4}, \quad \frac{\cos \theta_1}{c_1} = \frac{\cos \theta_4}{c_4}, \quad \frac{\cos \theta_2}{c_2} = \frac{\cos \theta_3}{c_3}.$$

We now define another rupture-propagation mode, called fan-shaped rupture-propagation fault. As shown in Fig. 2, let us consider a rupture which starts at $(0, 0, z_1)$ and radially propagates over a radius R with a rupture velocity c . We introduce new coordinates (χ, θ) to express the fan-shaped rupture-propagation (Fig. 2). The source coordinates (x_0, y_0, z_0) are related to the coordinates (χ, θ) as

$$x_0 = \chi \cos \theta, \quad y_0 = \chi \cos \delta \sin \theta, \quad z_0 = \chi \sin \delta \sin \theta + z_1.$$

By operating

$$\int_0^R d\chi \int_{\theta_1}^{\theta_2} d\theta \exp \left[-i \frac{\omega}{c} \chi \right],$$

to (2), we can obtain the expressions of the P-wave potential as

$$\phi = \frac{D}{8\pi^2 k_\beta^2} \iint A \exp[-ik_x x - ik_y y + iv(z - z_1)] dk_x dk_y \\ \times \int_{\theta_1}^{\theta_2} d\theta \frac{\exp \left[ik_x R \cos \theta + ik_y R \cos \delta \sin \theta - iv R \sin \delta \sin \theta - i \frac{\omega}{c} R \right] - 1}{k_x \cos \theta + k_y \cos \delta \sin \theta - v \sin \delta \sin \theta - \frac{\omega}{c}}. \quad (7)$$

S-wave potentials Ψ_j are obtained from (7) by replacing A with B_j , and replacing v with γ . If a circular rupture-propagation fault is considered, we obtain corresponding expressions, by setting $\theta_1 = 0^\circ$ and $\theta_2 = 360^\circ$ in (7). The integral with respect θ in (7) cannot be resolved analytically. Thus a lot of computation time is required for the fan-shaped rupture-propagation fault compared with that for a rectangular fault.

3. Seismogram Synthesis

In order to obtain synthetic seismograms for a rupture-propagation fault in a flat-layered medium, we introduce the propagator-matrix method (Haskell, 1953; Gilbert and Backus, 1966). For simplicity, we derive expressions of the surface displacement when the source is located in the half-space.

Following Bouchon (1979), we decompose (k_x, k_y) -components of the S-wave

Fig. 2. Geometry of a fan-shaped rupture-propagation fault. An area of rupture-propagation is defined by θ_1 and θ_2 . The fan-shaped rupture stops when its radius becomes R .

displacement potentials into SV-wave displacement potentials and SH-wave displacements as

$$\begin{aligned}\Psi^{SV} &= \frac{k_x}{k} \Psi_y - \frac{k_y}{k} \Psi_x, \\ V^{SH} &= i \left[k \Psi_z - \text{sgn}(z - z_0) \frac{\gamma}{k} (k_x \Psi_x + k_y \Psi_y) \right],\end{aligned}\quad (8)$$

where $k^2 = k_x^2 + k_y^2$.

We define a (k_x, k_y) -component of a displacement-stress vector as

$$\begin{aligned}f_{P-SV}(z) &= [u_{x'}(z), u_z(z), \tau_{x'z}(z), \tau_{zz}(z)]^t & \text{for P-SV,} \\ f_{SH}(z) &= [u_{y'}(z), \tau_{y'z}(z)]^t & \text{for SH,}\end{aligned}$$

where (x', y', z) are the different Cartesian source-coordinates from (x, y, z) , and a superscript, t , denotes the transpose. We should mention an angle between the x' -axis and the x -axis. The angle, ϑ , is determined by $\vartheta = \arctan(k_y/k_x)$. For example, the x' -axis is equivalent to the y -axis for $\vartheta = 90^\circ$ and the $-y$ -axis for $\vartheta = 270^\circ$. Each plane-wave element propagates along (x', z) -plane.

Applying the stress-free condition at the free surface and the radiation condition in the half-space, the (k_x, k_y) -components of the surface displacement are expressed as

$$T \begin{pmatrix} \phi \\ \Psi^{SV} \end{pmatrix} = A_{P-SV}^{-1} P_{P-SV}(z_n, 0) f_{P-SV}(0) \quad \text{for P-SV,} \quad (9a)$$

$$\begin{pmatrix} 0 \\ \mathbf{V}^{\text{SH}} \end{pmatrix} = \mathbf{A}_{\text{SH}}^{-1} \mathbf{P}_{\text{SH}}(z_n, 0) \mathbf{f}_{\text{SH}}(0) \quad \text{for SH}, \quad (9b)$$

where $\mathbf{P}_j$ is the product of the propagator matrices for the j -wavefield, $\mathbf{A}_j^{-1}$ the inverse of the layer matrix containing vertical phase factors for the j -wavefield, z_n the depth of the upper boundary of the half-space. A matrix $\mathbf{T}$ relates the displacement potentials to the wave vectors and is given by

$$\mathbf{T} = \begin{pmatrix} 0 & 0 & -ik_x & 0 \\ 0 & 0 & 0 & -ik_y \end{pmatrix}^t.$$

When we evaluate a (k_x, k_y) -component of the surface displacement, we use the Dunkin's formulation (Dunkin, 1965), which involves a decomposition of the propagator matrix into second order subdeterminants, to avoid the difficulty of numerical overflow for P-SV wavefields to a short wavelength limit.

The surface displacements with all the (k_x, k_y) -components are obtained by the discrete wavenumber summation (Bouchon and Aki, 1977). To remove singularities, such as surface wave poles, from the integration path, a complex frequency with a small negative imaginary part, ω_I , is introduced. This therefore produces the undesirable distorted solution. The distortion effect is corrected by multiplying the final time-domain solution by the factor $\exp[-\omega_I t]$ (Phinney, 1965; Bouchon and Aki, 1977).

4. The Weyl Integral and VTSE

In the previous section, we have introduced the conversion Eq. (8) to use the propagator-matrix method. These equations, however, cannot treat exactly VTSE, an element of $k_x = k_y = 0$, whereas VTSE always exists in the Weyl integral representation of spherical waves except a few special cases. In the following, we present a simple method, which does not use (8), to include the exact contribution of VTSE to the surface displacement. For simplicity, we consider here only the unilateral-unidirectional rupture-propagation fault as type I in Fig. 1.

Fig. 3. Relation between two horizontal components u_x , u_y , and S-wave displacements u_s . ϕ is the polarization angle of the S-wave due to VTSE.

Fig. 4. Two-dimensional display of wavenumber spectral amplitudes of the total wavefield at various frequencies. A model used in calculations of the wavenumber spectral amplitudes is shown in Tables 1 and 2. k_{a2} and k_{β_2} indicate branch points for the P-wave and S-wave in the half-space, respectively. k_s^p and k_s^s denote saddle points for direct P-wave and direct S-wave, respectively. Note that numerous branch points are not displayed at the points (k_x, k_y) that satisfy with $k_{a2}^2 = k_x^2 + k_y^2$ or $k_{\beta_2}^2 = k_x^2 + k_y^2$ for simplicity. Numerous saddle points are also not displayed. See text for explanations about the branch points and saddle points.

Fig. 5. Two-dimensional display of wavenumber spectral amplitudes of the P wavefields excited by only the P-wave potential. See explanations in Fig. 4.

Particle-motion direction due to VTSE is confined with a horizontal plane. As shown in Fig. 3, an S-wave displacement, u_s , in the half-space due to VTSE are decomposed into

$$u_s = u_x \cos \varphi + u_y \sin \varphi, \quad (10)$$

where u_x and u_y are the S-wave displacements in the half-space in x - and y -directions, respectively. Then an S-wave polarization angle, φ , is determined as

$$\tan \varphi = \frac{u_y}{u_x}. \quad (11)$$

From (1) and (4), horizontal displacements u_x and u_y due to VTSE for the unilateral-unidirectional rupture-propagation as type I in Fig. 1 are written as

$$\begin{aligned} u_x &= \frac{-D}{8\pi^2 k_\beta^2} \gamma B_y I_1(L_1, \theta_1, c_1) I_w(W_1, \theta_1, c_1, \gamma, z_2) \exp[ik_\beta z], \\ u_y &= \frac{D}{8\pi^2 k_\beta^2} \gamma B_x I_1(L_1, \theta_1, c_1) I_w(W_1, \theta_1, c_1, \gamma, z_2) \exp[ik_\beta z]. \end{aligned} \quad (12)$$

From (11) and (12), the S-wave polarization angle for VTSE is represented as

$$\tan \varphi = \frac{B_x}{-B_y} = \frac{\sin \lambda \cos 2\delta}{-\cos \lambda \cos \delta}. \quad (13)$$

Equation (13) shows that the polarization angle of the S-wave due to VTSE is determined by only focal parameters δ and λ . Equation (13) is valid for other rupture-propagation faults because rupture-front propagation terms (5) in the kinematic fault model are

Table 1. Model used in calculations of wavenumber spectral amplitudes as shown in Figs. 4 and 5.

Point source	
Fault strike	N240°E
Dip	60°
Slip	-170°
Depth	5 km
Receiver position	
Epicentral distance	14 km
Source-to-receiver azimuth	N60°W

Table 2. Structure model used in calculations of wavenumber spectral amplitudes as shown in Figs. 4 and 5.

Layer No.	α (km/s)	β (km/s)	ρ (g/cm ³)	Thickness (km)
1	1.1	0.4	1.7	1
2	5.0	2.9	2.4	—

independent of the focal parameters. The S-wave polarization angle defined by (13) cannot be determined for the strike-slip vertical fault ($\delta=90^\circ$ and $\lambda=0^\circ, \pm 180^\circ$) or the 45° dip-slip fault ($\delta=45^\circ$ and $\lambda=\pm 90^\circ$). In these cases, however, it is evident from (3) that displacement amplitude of VTSE is zero. We replace V^{SH} in (9b) with u_s determined by (10) after the y' -axis is taken parallel to the direction of the S-wave polarization. Thus we can evaluate exactly the surface displacements with contribution of VTSE. Contribution of all of the plane S-wave elements except VTSE is evaluated via (8).

Effects of VTSE on synthetic seismograms are now investigated by 2-D display of wavenumber spectral amplitudes. Figure 4 shows wavenumber spectral amplitudes at various frequencies of the total wavefield due to a point dislocation source with a step time history. A model used in computations of the wavenumber spectral amplitudes is shown in Tables 1 and 2. Although a location of branch point $k_{\beta_2}=\omega/\beta_2$ (β_2 is an S-wave velocity in the half-space) determines an overall feature of the total wavenumber spectrum, we shall remember the role of a branch point $k_{\alpha_2}=\omega/\alpha_2$ (α_2 is a P-wave velocity in the half-space). As shown in Fig. 5, this determines an overall feature of the wavenumber spectrum due to only P-wave potentials. Contribution of regions of a larger wavenumber k than the branch point k_{β_2} to wavenumber responses is very small (Fig. 4). Positions of these branch points are close to origins of the (k_x, k_y) -planes, when an interested frequency becomes low (Fig. 4). Therefore contribution of VTSE to wavenumber responses cannot be neglected in low frequencies.

Figure 6 shows a comparison between the complete surface displacements due to a point dislocation source with VTSE (top traces) and those without VTSE (bottom traces). In previous studies, a very small numeral were used as a substitute of zero for the horizontal wavenumber in cases where plane SH-waves are incident vertically on a surface layer (e.g., Horike, 1988). In Fig. 6, we also show surface displacements for which we replace exact VTSE with "approximate" VTSE, which are plane wave responses in the case when $k_x=k_y=10^{-15}$ (middle traces in Fig. 6). All the plane P-wave components and all the plane S-wave components except VTSE are included in the three traces in Fig. 6. Outlines of the numerical experiments shown in Fig. 6 are schematically illustrated in Fig. 7. A model used in the seismogram syntheses is shown in Tables 3 and 4. A dislocation time function is a cosine-ramp function with a rise time, τ_r , of 2 s or 4 s. Epicentral distances, r , are 1 and 5 km in the direction of the fault strike. In the case of $\tau_r=4$ s and $r=1$ km, final offsets of surface displacements without exact VTSE (middle and bottom traces) reduce almost half of that with exact VTSE (top trace).

Figure 8 shows amplitude spectra of the three surface displacements displayed in Fig. 6. VTSE mainly affects low frequency components of surface displacements. Neglecting VTSE results in underestimating the power of spectral densities in the low frequency range. Difference between the three amplitude spectral densities in the low frequency range tends to increase when the rise time is longer and when the epicentral distance is smaller.

We have examined here the effect of VTSE on synthetic seismograms only due to a point dislocation source. Therefore seismograms with VTSE may become almost the same as those without VTSE when a fault with a finite extent is considered, because a large part of the contribution to wavenumber responses probably come from other

Fig. 6

Fig. 7

Fig. 6. Comparison between complete surface displacements due to a point dislocation source with VTSE and those without VTSE. Upper, middle, and lower traces in each set represent surface displacements for exact VTSE, "approximate" VTSE and no VTSE, respectively. A model used in the seismogram syntheses is shown in Tables 3 and 4.

Fig. 7. Schematic illustrations of numerical experiments in Fig. 6. Using the Weyl integral representation, a spherical S-wave (thick and shaded curve) excited by a point dislocation source is decomposed into plane S-wave elements (solid lines). (Top) Contribution of VTSE is included exactly to seismogram syntheses by the simple method presented in this study. (Middle) Contribution of VTSE is not included. (Bottom) VTSE (dashed line) is replaced with "approximate" VTSE (solid line), and contribution of "approximate" VTSE is included. In the three cases, contributions of all the plane S-wave elements except VTSE are included in seismogram syntheses.

Table 3. Model used in seismogram syntheses as shown in Fig. 6.

Point source	
Fault strike	N0°E
Dip	30°
Slip	40°
Depth	5 km
Receiver position	
Epicentral distance	1 km/5 km
Source-to-receiver azimuth	N0°E

Table 4. Structural model used in seismogram syntheses as shown in Fig. 6.

Layer No.	α (km/s)	β (km/s)	ρ (g/cm ³)	Thickness (km)
1	3.0	1.9	1.9	1
2	5.0	2.9	2.4	—

plane-wave elements rather than VTSE.

We now attempt to explain the reason why VTSE gives only insignificant contribution to wavenumber responses for distant receivers in terms of the conventional steepest descent method. In the concept of the steepest descent method, the most important contribution to wavenumber responses stems from near the saddle point. A position of the saddle point corresponding to a direct P-wave or a direct S-wave on the Riemann sheets is given by

$$k_s = \omega \sin \vartheta_h / c, \quad (14)$$

where ϑ_h is the take-off angle from a source, c the elastic wave velocity (Aki and Richards, 1980). In $r=0$ km, therefore $\vartheta_h=0^\circ$, the saddle points for the direct P- and S-waves are located just at the origins in all the (k_x, k_y) -planes. This means that VTSE contributes strongly to wavenumber responses when a fault with a finite extent is located just below the receiver, because the saddle points are located at a point where VTSE is generated. On the other hand, in a distant receiver from the source region, the saddle points for the direct P- and S-waves are taken away from the origins. Therefore contribution of VTSE to wavenumber responses becomes small in a distant receiver (Fig. 8).

As shown in Fig. 8, the contribution of VTSE to wavenumber responses is large in the low frequency range. This fact is also explained using (14). In the low frequency range ($\omega \ll 2\pi$), the saddle points for the direct P- and S-waves are located near the origin; contribution of VTSE is large. In the high frequency range ($\omega \sim 2\pi$), the saddle points are located away from the origin; contribution of VTSE becomes small. Consequently, surface displacements due to sources with a short rise time are perhaps less sensitive to VTSE as shown in Fig. 6.

As is well known, the steepest descent method is valid for the far-field approximation or the high frequency approximation. The idea mentioned above is only one possible

Fig. 8. Amplitude spectra of the surface displacements displayed in Fig. 6. Spectral densities at 0 Hz are calculated from averages of surface displacements in time interval of 10 s.

explanation. Although Takenaka (1990) and Ohori *et al.* (1992) also represented another method to include VTSE in the seismogram synthesis, they did not discuss the effect of VTSE on surface displacements.

5. Numerical Examples of the Synthetic Near-Field Seismograms Due to Rupture-Propagation Faults

We present some numerical examples to demonstrate the advantage of our method. We study here the effect of rupture-propagations on the near-field seismograms.

Figure 9 shows a good agreement of synthetic seismograms by the method presented in this study with those by Cagniard's technique (Kawasaki *et al.*, 1975). Focal parameters and source-to-receiver geometry are also shown in Fig. 9.

Top traces in Fig. 10 are near-field seismograms from the Kushiro-Oki earthquake ($M_s 7.1/m_B 7.3$ (Abe, 1981)) of Aug. 11, 1961, recorded with a strong-motion seismograph at a station Kushiro (KUS) which is located in the southeast part of Hokkaido, Japan. The natural periods are 6 s for both the horizontal components and 5 s for the vertical component. The damping factor is 0.5. Lower traces in Fig. 10 show synthetic seismograms due to various rupture-propagation modes in Fig. 11. Amplitude scales common for both the observed and synthetic seismograms are shown at the lower right corner of Fig. 10. A closed circle is attached in the synthetic seismogram when the

Fig. 9. Comparison of displacement seismograms by the method proposed in this study with those by the Cagniard method (Kawasaki *et al.*, 1975). Fault parameters and source-to-receiver geometry are also shown in right-side of the figure. P-wave and S-wave velocities in the half-space are 6 and 3.5 km/s, respectively. The dislocation time function is a ramp function with a rise time of 2 s. Seismograms are convolved with a long-period seismograph response with a natural period of 30 s and a damping factor of 0.5. Numerals on the left shoulder of seismograms entitled as the Cagniard method indicate a maximum amplitude in 10^{-2} m.

one-sided amplitude exceeds 3 cm. Table 5 shows the source model used. The distance and azimuth from the fault plane to KUS are not changed. Only the initial break point is allowed to be changed. Table 6 shows a structural model used in the seismogram syntheses. This model is based on a seismic refraction survey in the southern part of Hokkaido, Japan (Okada *et al.*, 1973). The S-wave velocity structure is determined by $\alpha = \sqrt{3} \beta$. The effect of anelastic attenuation is included by making the P- and S-wave velocity complex (Takeo, 1987). Since the epicentral distance is roughly comparable to the source dimension obtained by Miyajima and Abe (1982), it is expected that the near-field waveforms are sensitive to the rupture-propagation mode.

Although the quality of observed seismograms is poor, we can see that an observed S-wave amplitude in the N-S component exceeds an upper limit 3 cm of the trace amplitude (Fig. 10). Miyajima and Abe (1982) estimated the average dislocation of 1.2 m for the 1961 Kushiro-Oki earthquake from a teleseismic P-wave and a tide level difference in a few months before and after the mainshock. If we assume an average dislocation of 1.2 m, therefore, an appropriate fault model must reproduce the observed large S-wave amplitude in the N-S component. Among the fault models in Fig. 11, models A5, A6, A8, A9, A11, and A12 reproduce such large S-wave amplitude. Models that do not have a component of a rupture propagating toward KUS (toward west) do not excite a large S-wave amplitude compatible with the observation. Model A11, which is almost equivalent to that proposed by Miyajima and Abe (1982), is the best

Table 5. Source model of the 1961 Kushiro-Oki earthquake.

Strike direction*	N235°E
Dip angle*	24°
Slip angle*	113°
Fault length*	40 km
Fault width*	25 km
Rupture velocity*	4.0 km/s
Average dislocation*	1.2 m
Rise time*	2 s
z_2^{**}	45 km
z_3^{**}	35 km
Azimuth from NWC [†] to KUS	N272°E
Distance from NWC [†] to KUS	48.8 km

* Estimated by Miyajima and Abe (1982). ** z_2 and z_3 are the depths of the lower and upper ends of the width of the fault plane, respectively. [†] NWC means the northern west corner of the fault plane. Strike, dip, and slip angle are measured in the same way as Aki and Richards (1980).

Table 6. Structural model used in the forward modeling to the 1961 Kushiro-Oki earthquake.

Layer No.	α (km/s)	β (km/s)	ρ (g/cm ³)	Q_p	Q_s	Thickness (km)
1	5.9	3.4	2.3	800	400	14
2	6.6	3.8	2.9	1,000	500	16
3	7.5	4.3	3.2	1,500	1,000	—

model among all the models considered in the present analysis comparing the observed peak-to-peak amplitudes of the P-wave in the W-E and D-U components and of the S-wave in the D-U component with the theoretical ones. However, model A11 does not reproduce the shape down-swing of the observed S-wave (Fig. 10 (b)).

We recognize again that as shown by many previous authors, near-field seismograms are very sensitive to the effect of a rupture-propagation on a fault with a finite extent. This is demonstrated by synthetic seismograms in Fig. 10.

6. Discussion

We have used the Weyl integral representation in the present study, an alternative to which is the Sommerfeld integral representation (Sommerfeld, 1954) of spherical waves for obtaining surface displacements due to a point source as Sato (1969) and Harkrider (1976) used. For a point source, computation time for the Sommerfeld integral is shorter than that for the Weyl integral, because only a single integration with respect to a horizontal wavenumber is required for the Sommerfeld's one but a double integration with respect to two horizontal wavenumbers, k_x and k_y , must be carried out for the Weyl's one (Bouchon, 1981). On the contrary, for a rupture-propagation fault, we likely consume much more computation time for the Sommerfeld integral than for the Weyl integral, because analytic expressions for the displacement potentials due to the

Fig. 10. (a) Comparison between observed (top traces) and synthetic seismograms (lower traces) due to fault models from A1 to A6. An average dislocation of 1.2 m is assumed in all the models. An amplitude scale common for both the observed and synthetic seismograms is shown at the right lower corner of the figure. The source model used is shown in Table 5. The structural model used is shown in Table 6. (b) Comparison between observed (top traces) and synthetic seismograms (lower traces) due to fault models from A7 to A11. (c) Comparison between observed (top traces) and synthetic seismograms (lower traces) due to fault models from A12 to A15.

rupture-propagation fault are not obtained for the Sommerfeld integral representation. For example, each set of synthetic seismograms as shown in Fig. 10 can be computed within a few seconds, because our computer program based on the Weyl integral representation is highly tuned up for a vector array processor (HITAC S-820/80, 3 GFLOPS).

We have introduced the Dunkin's formulation (Dunkin, 1965) to avoid a difficulty of a numerical overflow for P-SV wavefields in a short wavelength. If one hopes to avoid completely such kind of numerical difficulty, the reflection-transmission matrix method (Kennett and Kerry, 1979) should be used.

As is well known, one cannot obtain the synthetic seismogram for a receiver which locates just above a point source in a layered medium so far as the Sommerfeld integral representation is used. This may mean that one cannot obtain the rigorous near-field seismograms due to a rupture-propagation fault, when receivers located just above a fault of a finite extent. The present method can evaluate the complete seismic responses.

7. Conclusion

We have derived new complete representation of the displacement potentials due to kinematic fault models with a bidirectional-bilateral and a fan-shaped rupture-propagations. We have used the exact discretization (Bouchon and Aki, 1977) to evaluate the Weyl integral over the horizontal wavenumbers and the propagator-matrix method (Haskell, 1953) to include the seismic responses of a flat-layered medium. We have employed the Dunkin's formulation (Dunkin, 1965) to avoid the numerical overflow for P-SV wavefields in a band of short wavelengths.

We have proposed a simple but exact method in order to evaluate wavenumber responses including VTSE. As shown in Figs. 6 and 8, we have found that contribution

Fig. 11. Various rupture-propagation fault models for the 1961 Kushiro-Oki earthquake. All the models have the same rupture velocity of 4.0 km/s. See also Table 5.

of VTSE to wavenumber responses is larger for a lower frequency range or a smaller epicentral distance. This is explained qualitatively by the saddle point approximation of the steepest descent method. We shall not neglect the contribution of VTSE when the epicentral distance is very small or when the averaged rise time is very long.

We have recognized again that near-field seismograms are very sensitive to the effect of a rupture-propagation on a fault with a finite extent as shown by previous investigators. This has been demonstrated by synthetic seismograms for the 1961 Kushiro-Oki earthquake off the southeast coast of Hokkaido, Japan, as shown in Fig. 10. We have found that a bilateral rupture-propagation fault model A11 (Fig. 11 and Table 5) among all the models considered reproduces well the peak-to-peak amplitudes of the observed seismograms except the large, sharp down-swing of the S-wave (Fig. 10(b)).

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