Modeling Seismic Wave Propagation in Complex Media

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We review the studies on modeling seismic wave propagation in complex media, which were carried out during the past 10 years by researchers at Japanese institutions. Special emphasis is placed on those works which are perceived as important but had little exposure outside Japan. We can say that the fundamental development of method for seismogram synthesis in the (3, 1) dimension was completed, where the first and second numbers in parentheses are the space dimensions of the wavefield and the heterogeneity of the medium. However, seismologists have been eager to model seismic propagation in the (2, 2), (3, 2), and (3, 3) dimensions during the last 10 years. Modeling seismic wave propagation in a full (3, 3) dimension is now limited to simple small-scale problems because of the extensive computation and large memory, even when using a supercomputer. Nevertheless, in laterally and vertically heterogeneous media, we have to calculate 3-dimensional wavefields in order to analyze real seismic records quantitatively. There exist two potential breakthroughs. One is to assume the medium is axisymmetric and the other to model (3, 2) dimensional wave propagation (the so-called 2.5 dimensional problem). Numerical methods have the potential to simulate seismic wave propagation in realistic environments of substantial spatio-temporal extent. Since we are not used to processing such an abundance of information, we have to investigate methods for analyzing and interpreting large volumes of computational data. These studies may well challenge our conception of seismic wave propagation.

1. Introduction

In the last decade, the study of wave propagation in complex media has become increasingly important in seismology. Modeling wavefields by analytical and numerical techniques has now become one of the most valuable tools for the study of seismic source processes, detailed analysis of subsurface structure and prediction of strong motion. Recently general reviews of theoretical and numerical studies on wave propagation were presented by Koketsu and Takenaka (1989) and Koketsu (1991), including current research outside Japan. In this paper we concentrate exclusively on research carried out within Japanese institutions during the past 10 years. In particular,
special emphasis is placed on those works which are perceived as important by the
author but which have had little exposure outside Japan.

In mentioning theories and computational methods for studying seismic wave
propagation, dimension is one of the most important factors to characterize them. There
are two kinds of dimensions pertaining to wave propagation problems. One is the
dimension of the medium parameters, e.g., P-wave velocity, S-wave velocity and density,
and the other is that of the seismic wavefield itself, i.e., the number of independent
spatial coordinate variables. In order to model seismic wavefields efficiently, these
dimensions must be distinguished from each other. The dimension of a wavefield is
always equal to or greater than that of the medium parameters. We will hereafter use
the terminology \((m, n)\) dimension, where the dimensions of the wavefield and the me-
dium parameters are \(m\) and \(n\), respectively.

Synthetic seismograms in the \((3,1)\) dimension, which simulate seismic wave
excitation and propagation in a horizontally layered medium, have become standard
tools for routine work in the analysis of earthquake sources. Kohketsu (1985), Sasatani
(1985) and Takeo (1985) have developed efficient 1-D reflectivity codes for solid-solid
interfaces. Kohketsu (1985) is based on the propagator matrix formulation of Kind
(1979), while the others are based on the reflection/transmission matrix formulation of
Kennett and Kerry (1979). Ogawa et al. (1986) extended the method proposed by Sato
and Hirata (1980), which uses numerical integration in the complex wavenumber domain,
to calculate synthetic seismograms at the ocean bottom due to underwater explosions.

We can say that the fundamental development of method for seismogram synthesis
in the \((3,1)\) dimension has been completed as a result of the above studies, except
modeling of source fields (e.g., Hirata, 1992) and gradual variation of medium
parameters. However, seismologists have been eager to model seismic propagation in
the \((2,2)\), \((3,2)\), and \((3,3)\) dimensions during the last 10 years. Thus we will con-
centrate our attention on this modeling in the subsequent sections.

2. Analytical and Ray Theoretical Methods

Analytical methods are of limited utility in solving realistic problems in seismic
wave propagation since they demand an oversimplified specification of the material
properties and medium geometry. However, they are indispensable tools for examining
the partitioning of energy between the various wave types, offering physical insight into
scattering processes and providing a check on the consistency of the more flexible
numerical modeling schemes.

Momoi (1982, 1985, 1987) studied the scattering of \((2,2)\) dimensional Rayleigh
waves incident on some rough surfaces and derived some qualitative relations for the
energy partition among several kinds of Rayleigh waves and body waves by use of an
almost exact method. He obtained an integral equation by using the Fourier transform
technique and numerically solved it after deforming the integration path so that the
integrands vary more gently. Fujii et al. (1984) and Fujii (1986) also obtained almost
exact reflection and transmission coefficients for the \((2,2)\) dimensional Rayleigh waves
at wedge corners of various wedge angles and various inclined discontinuities, by deriving
integral equations, distorting the integration path and solving them numerically. Hisada

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et al. (1991) proposed an analytical method for computing the Love waves excited at a vertical interface between a homogeneous quarter-space and multi-layered quarter-space by a horizontally incident SH wave from the former. Although they neglected the diffracted waves in the homogeneous quarter space and the body waves in the multi-layered quarter space, their results simulated the Love waves generated at the interface very well for a shallow source. Their method takes much less computation time than popular numerical methods such as the boundary element method.

Ray theoretical methods are semi-analytic, but they are not applicable in the low frequency regime due to the high frequency approximation inherent in the ansatz. However, they can be applied to any inhomogeneous media. Yomogida (1989) has already reviewed recent developments in ray theoretical methods with emphasis on the Gaussian beam method. We here review several more recent studies pertaining to ray theoretical methods in Japan.

Watanabe (1991) studied a way to determine the optimal width of (3, 1) dimensional Gaussian beams for various frequencies, propagation lengths, velocities, beam end points, etc. and Matsui (1992) also investigated some fundamental problems related to the optimal choice of beam width for (2, 2) dimensional wavefields. Yoshida (1992) calculated the (3, 2) dimensional Green's functions by a ray theoretical method to take into account the effects of realistic ocean-floor topography on long-period teleseismic waveforms. He represented the ocean-floor by the plane elements of appropriate dip and performed three-dimensional ray tracing to compute the Green's functions. Sekiguchi (1992) calculated the amplitudes of body wave traveling through realistic models of a subducting plate using the Gaussian beam method.

Several studies have applied ray theoretical methods to strong motion synthesis in sedimentary basins. Seki and Nishikawa (1990) extended the method of Sánchez-Sesma et al. (1988) in an approximate way to include an elastic boundary and take obliquely incident SH-waves into account. Kagawa et al. (1991) used the Gaussian beam method to simulate the (2, 2) dimensional SH waves in the sedimentary basins. Matsui et al. (1990) investigated how surface waves are excited in sedimentary basins through stacking of body waves by use of the (2, 2) and (3, 3) dimensional Gaussian beam method. A similar surface-wave synthesis was also performed by Kinoshita (1985) using a simpler ray-theoretical method. He modeled pseudo-Love waves observed in the Fuchu area, Tokyo, by stacking train of plane waves totally reflected in a dipping layer. Zheng et al. (1992) simulated the effect of a thin dipping layer on SH-waves using another ray-theoretical method, called the "Wave Front Tracing Method," which can approximately incorporate diffracted waves.

3. Numerical Method

For modeling seismic wave propagation the following three conditions should be satisfied: the pertinent dynamic equations (wave equations), the boundary conditions, and the radiation conditions. These three conditions can be satisfied exactly only through the apparatus of analytical procedures. This is tractable only for simple geometries. For a realistically complex geometry, numerical methods are mandatory. However, since numerical methods cannot exactly satisfy some or all of the three conditions, these
intractable conditions are approximated in a computational manner. Fletcher (1984) has classified numerical methods in three groups based on the methods of weighted residuals: (1) Interior method, (2) Boundary method, (3) Mixed method. Methods in category (1) can construct an approximate solution so that the boundary conditions are satisfied exactly, while a solution constructed by (2) satisfies the differential equation exactly. An approximate solution calculated by (3) does not exactly satisfy either the differential equation or the boundary conditions. Koketsu and Takenaka (1989) and Koketsu (1991) have applied Fletcher's classification to numerical methods for synthetic seismograms which are currently available for irregularly layered media.

Although the classification based on the methods of weighted residuals is mathematically strict and often useful, we here choose a rougher classification from a practical point of view: (1) Interior method, (2) Boundary method, (3) Hybrid method. Interior methods discretize a medium, solution and differential operators in time (or frequency) and throughout the spatial domain, and then obtain a solution by solving the resulting linear equations. Therefore interior methods are applicable for modeling wavefields in arbitrary heterogeneous structures because the medium parameters are distributed on the numerical grid points (Fig. 1(a)). In boundary methods, the differential equations and boundary conditions are transformed into boundary integral equations involving unknown functions, e.g., displacement and traction, force density, or wavevector. The

![Fig. 1.](image)

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boundary integral equations are then discretized and solved by various numerical techniques. Figure 1(b) shows an example of the discretization of a boundary. Hybrid methods are combinations of several different methods. In the following subsections, we review current studies in each of these three groups.

3.1 Interior method

In interior methods, the finite difference method (FDM) and the finite element method (FEM) have been very popular in various areas of seismology as in other fields of science and technology. FEM has been preferred by earthquake engineers and FDM by seismologists and exploration geophysicists. Zama (1982) used FEM to study the propagation of the $(2, 2)$ dimensional Rayleigh waves through some irregular structures (a three-quarter homogeneous space, a step on a homogeneous half-space and a model of the subduction zone beneath northeastern Japan). Suetsugu (1989) applied an FEM scheme to calculate the $(2, 2)$ dimensional displacement field in the heterogeneous near-source region which includes a slab penetrating the lower mantle beneath the Okhotsk Sea. Shima et al. (1985) assumed a solution calculated by a $(2, 2)$ dimensional FEM as an impulse response and synthesized the seismograms at the fill-in surface by convolution integral using the reference seismograms observed at the basement and non-disturbed ground surface. Toshinawa and Ohmachi (1992) developed a simplified 3-dimensional finite element method employing the eigenfunctions for fundamental-mode surface waves as interpolation functions to simulate Love-wave propagation in a 3-dimensional sedimentary basin and applied this method to the 1990 Near Izu-Ohashima, Japan earthquake ($M=6.5$). Yoshida (1993) investigated the propagation of Love waves across a symmetrical mountain root structure with a high dip angle by use of a 2-dimensional FDM. Yuan et al. (1986) simulated the 3-dimensional strong motions observed during the Nov. 15, 1976 Ninghe, China earthquake ($M=6.4$) by use of an axisymmetric finite difference method incorporating a faulting model. Yamanaka et al. (1992) also used an axisymmetric finite difference method to model Love-wave propagation in the Kanto plain, Japan.

Most recently, another interior method, the pseudospectral method (PSM) has also been investigated both in Japan and abroad to enhance its applicability to wave excitation and propagation problems of complex subsurface structures (e.g., Murayama et al., 1991; Furumura, 1992). The PSM is a spectral method formulated by collocation. Spectral methods have recently emerged as a viable alternative to FDM and FEM for the numerical solution of partial differential equations, and have proved particularly useful in numerical fluid dynamics where large spectral hydrodynamic codes are now regularly used to study turbulence, numerical weather prediction and ocean dynamics. In seismology the PSM was introduced by Kosloff et al. (1984) to elastic wave calculations. The PSM can use a much coarser grid than the finite difference counterpart, as few as two grid points per shortest seismic wavelength, while the second-order and fourth-order explicit finite difference methods require more than ten and five grid points per wavelength, respectively. Therefore the PSM may in the near future enable us to calculate $(3, 3)$ dimensional synthetic seismograms for regional-scale structure, which is beyond the realm of other modeling methods.

In all interior methods, there are two intrinsic problems. One is the difficulty of
incorporating a point source because of the mathematical singularity near the source. In the case of teleseismic wave calculations, since the plane-wave approximation can be applied to spherical waves, this difficulty can be avoided by use of the reciprocal theory (Okamoto and Miyatake, 1989), but it cannot be bypassed in the case of near-field calculation. One way to overcome this problem is to approximate a point source by a volume source with finite extent (FDM: Aboudi, 1971; FEM: Geller et al., 1979; PSM: Furumura, 1992). Furumura (1992) have examined the effect of source volume on seismic waveforms by PSM. To circumvent the source singularity problem, another possibility is to place a "source box" around the source at a certain distance away from the source (Alterman and Karal, 1968). The exact values of the wavefield on the outer wall of the source box can be computed from a closed-form solution. These wavefields can then be used as the source field outward. Any field reflected from the surrounding inhomogeneity impinging on the box can be made to transmit directly inside the box. From a mathematical point of view this technique enables us to solve homogeneous equations instead of inhomogeneous ones. This technique has been scarcely used by Japanese seismologists.

Another problem is how to impose a radiation condition, i.e. the problem of an open boundary. Interior methods cannot satisfy the radiation conditions in an exact sense, and radiation conditions must be imposed in an approximate manner. Many studies have been carried out to devise "artificial boundary conditions" which approximate open boundaries having a radiation condition. The results of this effort can be found in the literature related to various fields, such as acoustics, gas dynamics, hydrodynamics, electrical engineering, civil engineering, seismology, exploration geophysics, meteorology and plasma physics. Although the geometry and governing equations in these fields are sometimes different, the techniques are quite similar. Most recently, Givoli (1991) has reviewed this subject in a uniform manner, while referring to the literature of all the pertinent fields.

Some Japanese seismologists have also challenged the problem of an open boundary to develop numerical techniques for satisfying a radiation condition for some interior methods. Suzuki and Hakuno (1984) extended Cundall's method (Cundall et al., 1978) to the 3-dimensional problems by FEM. This method, which is similar to Smith's method (Smith, 1974, 1975), solves the problem by superposing two reflected waves from Dirichlet and Neumann boundaries. This method is theoretically complete, while Smith's method is not. Recently, Ohminato (1990) has proposed a new method achieving the radiation condition. He has developed the method of weighted residuals in the frequency domain, where he used the trigonometric and basic spline functions as horizontal and vertical trial functions and succeeded in incorporating the radiation condition for the bottom boundary of the computational domain into a mixed Galerkin formulation as a natural boundary condition.

In PSM artificial reflections do not occur, but wrapround appears from the outer boundaries of the numerical mesh. To suppress this, the absorbing boundary condition of Cerjan et al. (1985) is often applied. It is based on gradual reduction of the amplitude in a strip of nodes along the boundaries of the mesh. The reduction of the amplitude in the strip reduces wrapround, but the reduction of the amplitude in the strip generates an artificial reflection, so there is a trade-off between the wrapround and the artificial
reflection. Most recently Furumura (1992) proposed a new procedure to eliminate the
wraparound. Although his procedure requires one more calculation using PSM, it
generates no artificial reflection.

3.2 Boundary method

Koketsu and Takenaka (1989) and Koketsu (1991) have classified boundary
methods used in seismology by use of two key concepts of the methods of weighted
residuals, i.e. the "trial function" and "weight function" (see their papers or the in-
troduction of Koketsu et al. (1991)). Although it may be one of the best approaches
for an unified treatment of all boundary methods to base on the formulation of the
method of weighted residuals, in the present paper we do not follow their approach
but focus on the practical aspects of boundary methods. In seismology the following
methods are widely used: (1) Rayleigh-Fourier method, (2) Boundary integral equation
method (BIEM), (3) Wave function expansion method (WFEM), (4) Boundary element
method (BEM), (5) T-matrix method.

The Rayleigh-Fourier method is based on the Rayleigh hypothesis that the scattered
field near and on an interface can be expanded in terms of plane waves propagating
or exponentially damped in a single direction, namely, away from the interface. The
wavefield is expanded in terms of up- and down-going plane waves. Their expansion
coefficients are determined by satisfying the boundary conditions at the interface, i.e.,
the continuity of displacement and traction. The boundary condition is represented in
the horizontal wavenumber domain instead of the space domain, because the matrix
equation for expansion coefficients is better conditioned in the horizontal wavenumber
domain.

The Aki-Larner method (Aki and Larner, 1970) is the most widely used version
of the Rayleigh-Fourier method in Japan. Since it uses the fast Fourier transform (FFT),
the Aki-Larner method sometimes is referred to as Rayleigh-FFT method (Jiracek,
1973). Kohketsu (1987) has extended the Aki-Larner method to (2,2) dimensional
multi-layered problems. He used the propagator matrix technique for layers with
irregular interfaces. Horike (1987) also proposed a similar technique. The propagator
matrix technique is subject to an "exponential dichotomy" (Mattheij, 1985). This class
of numerical problem may cause, if left unchecked, loss of numerical precision for
evanescent waves. To avoid this exponential dichotomy, Takenaka(1990) has shown
how the reflection/transmission matrix approach of Kennett (1983) can be adapted to
irregularly layered media. This procedure eliminates the problem of evanescent waves,
but requires a number of large-scale matrix inversions, which introduce a different range
of numerical complications. Koketsu et al. (1991) presented an alternative formulation
of the reflection/transmission matrix approach, which exploited the propagation
invariants of Kennett et al. (1990) to simplify the calculations. The formulation of
propagation invariants for irregularly layered media has been generalized in terms of
integral operators by Takenaka et al. (1993). Recently, the Aki-Larner method has been
extended to solve (3,3) dimensional problems with 2-dimensionally irregular interfaces
by Horike et al. (1990), Uebayashi et al. (1992), Ohori et al. (1990) and Takenaka
(1990). Almost all of these works are based on the propagator matrix technique, but
Takenaka (1990) used the reflection/transmission matrix approach. Ohori et al. (1990)
and Takenaka (1990) exactly treated vertical incidence of S waves, while Horike et al. (1990) and Uebayashi et al. (1992) treated it only approximately. Although the Aki-Larner method is a computationally cheap method, in modeling realistic structures it requires quite huge memory for computation as well as other boundary methods. Thus Ohori et al. (1993) estimated possible reduction of the memory consumption in the Aki-Larner method by truncating the non-diagonal elements of the submatrices of the matrix equation to be solved, which is a technique also applied by Ohminato (1990) to an interior method.

When an interface is too steep, the Aki-Larner method may not give a correct solution (Shinozaki, 1988; Axilrod and Ferguson, 1990). This may be a consequence of the extreme ill-conditioning of the matrix equation for the expansion coefficients. To ameliorate this problem, Baba et al. (1988) modified the (2, 2) dimensional formulation of the Aki-Larner method. The lateral extremities of the irregularly layered structure are constrained to match the respective plane layered structure so that the matrix equation is stabilized. In order to overcome this stability problem completely, we have to look for basis functions for the wavefield other than the plane wave solution used in the Aki-Larner method. The boundary integral equation method (BIEM) adopts Green’s function for body forces distributed along the interface, sometimes referred to as the “source method” or “indirect boundary element method.” Many authors, e.g., Sánchez-Sesma and Esquivel (1979), have succeeded in calculating seismic wavefields scattered by steep interfaces using the BIEM. However, the Green’s function has a singular point at the source, so that they located the secondary sources in the vicinity of the interface, not on the interface itself (Kupradze, 1963). This treatment may lead to a subtle numerical problem. Thus, Bouchon (1985) and Campillo and Bouchon (1985) replaced the Green’s function with its finite Fourier expansion (“discrete wavenumber representation”). In Japan, Sasatani (1988) applied the formulation of Campillo and Bouchon (1985) to obtain the seismic response of sediment-filled valleys to incident plane SH-waves. Takenaka and Koketsu (1990) again adapt the reflection/transmission matrix approach to BIEM using the propagation invariant. This method can be combined with the reflection/transmission matrix approach of the Aki-Larner method.

The wave function expansion method (WFEM) is similar to BIEM, in fact, which is sometimes classified into BIEM. WFEM expresses unknown scattered fields in terms of wave functions (e.g., the Bessel functions, Hankel functions and spherical surface harmonics) which satisfy a wave equation and appropriate radiation conditions, instead of Green’s functions. WFEM has the advantage that it avoids the introduction of singular integral equations and Green’s functions which may be in general more difficult to construct than complete systems of solutions. This method has been used by some seismologists and earthquake engineers outside Japan. For example, Eshraghi and Dravinski (1989 a, b) utilize WFEM to solve (2, 2) dimensional multi-layered problems and Sánchez-Sesma (1983) and Sánchez-Sesma et al. (1989) applied this method to study the seismic response of axisymmetric 3-dimensional valleys on the surface of an elastic half-space. However, Japanese seismologists have scarcely used WFEM.

The boundary element method (BEM) is one of the most widely used boundary methods in Japan, particularly in the fields of engineering, e.g., electromagnetic engineering, mechanical engineering and civil engineering. Some earthquake engineers
and seismologists have also developed procedures for BEM and applied them to simulate
the seismic response of irregularly layered media (see Niwa and Hirose, 1985; Kawase
et al., 1985; Kawase, 1988; Fukushima and Kawase, 1986; Kaneko et al., 1988; Tong
and Kuribayashi, 1988; Hisada et al., 1988, 1992; Fujiwara and Takenaka, 1993;
Yamamoto et al., 1990; and Saito and Ino, 1990). The formulation of BEM is based
on the representation theorem (e.g., Aki and Richards, 1980). In BEM the Green's
functions both for displacement and traction are used as weight functions. The boundary
integral equations in the space domain are discretized by a similar technique to FEM
which uses interpolation functions as trial functions to obtain a system of linear equations
for the unknown displacement and traction on the boundaries. Kawase (1988) proposed
a procedure, called the "discrete wavenumber boundary element method" (DWBEM),
where the finite Fourier expansions of the Green's functions are utilized in the place
of Green's functions so that the analytical term-by-term evaluation can be done for the
element integrations which may contain a singularity in the integrand. One of the biggest
disadvantage of BEM is the time-consuming effort to evaluate the Green's function for
a half-space, so Kawase and Aki (1989) proposed an economical evaluation of a
discrete-wavenumber Green's function by just taking its imaginary part and reproducing
the real part using the causality condition. Hisada et al. (1992) and Fujiwara and
Takenaka (1993) presented efficient methods to simulate the secondary surface waves
excited near the edge of a basin structure using the normal-mode Green's functions.

The T-matrix method is very popular in the United States, but has scarcely used
in Japan. That may be why the boundary element method has become so popular in
Japanese engineering. The T-matrix method is based on the "extinction theorem": the
wavefield on an interface acts as sources that extinguish the incoming field in the medium.
The T-matrix method involves two steps: solving a system of surface integral equations
resulting from the extinction theorem, and using the result to compute the unknown
fields with Green's second identity. It is an exact approach when the interface is
so smooth that Green's theorem can be applied. Nevertheless, for various medium
configurations, it can be demonstrated that the T-matrix methods and the Rayleigh-
Fourier methods are theoretically equivalent (e.g., Kazandjian, 1991). Since the sys-
tem of surface integral equations (actually, this becomes a matrix equation in ac-
tual computations) is represented in the horizontal wavenumber domain, the T-matrix
method cannot derive a correct solution for a rather steep interface as well as the
Rayleigh-Fourier method. Moreover, the T-matrix method requires more computation
time than the Rayleigh-Fourier method (Axilrod and Ferguson, 1990). This proves the
Rayleigh-Fourier method may be preferable in practical applications.

3.3 Hybrid method

Most hybrid methods are combinations of interior methods and boundary methods,
because both local heterogeneities and radiation conditions can be taken into account.
We list up some examples of such combinations which have been studied in Japan.

(1) FEM+BEM  Fukuwa et al. (1985), Nakamura et al. (1988), Takemiya et
al. (1990), Takemiya and Tomono (1992)

(2) FEM+Aki-Lamer method  Motosaka and Urao (1989)
In the above list the thin layer element method is an interior method, which is efficient for calculating wavefields in the horizontally layered media approximately \((2,1)-D:\) Lysmer and Drake, 1972; \((3,1)-D:\) Tajimi and Shimomura, 1976) and sometimes classified into FEM, and the particle method is also an interior method, which models a medium as point-masses connected by the springs each other (Harumi et al., 1978).

4. Discussion

Recent advances in high performance computers are allowing us to model wave propagation in more realistic media. Calculation for \((2,2)\) dimension is now routinely used for the analysis of realistic wave propagation problems. Many theoretical and numerical methods have been developed, particularly for layered media with 1-dimensionally irregular interfaces (2-dimensionally layered media), a kind of \((2,2)\) problem, as mentioned above. These methods are now used in strong motion seismology by many earthquake engineers. However, modeling seismic wave propagation in a full \((3,3)\) dimension is now limited to simple small-scale problems because of a large memory requirements, even when using a supercomputer. Nevertheless, since real seismic wavefields are 3-dimensional wavefields propagating in laterally and vertically heterogeneous media, we have to calculate wavefields for such problems in order to analyze real seismic records quantitatively. There exist two potential breakthroughs. One is assuming the medium to be axisymmetric. Axisymmetric calculations take only the same memory and computational time as \((2,2)\) calculations. Yuan et al. (1986) and Yamanaka et al. (1992) applied axisymmetric finite difference methods, and Sato and Hasegawa (1990) proposed an axisymmetric finite element method combined with a thin layer element method. Kohketsu (1987) and Baba (1989) developed axisymmetric versions of the Rayleigh-Fourier method, where Baba (1989) used the Laplace transform instead of the Fourier transform. Tong and Kuribayashi (1988) applied an axisymmetric BEM to calculate the seismic response of axisymmetric sediment-filled valleys. Most recently Furumura and Takenaka (1992a) and Okamoto (1992) modeled \((3,2)\) dimensional wave propagation, the so-called 2.5 dimensional problem, without solving a full \((3,3)\) dimensional elastodynamic equation. The work of Furumura and Takenaka (1992a) is based on the PSM and that of Okamoto (1992) the FDM (Fig. 2).

The numerical methods mentioned above have the potential to simulate seismic wave propagation in realistic environments of extensive spatio-temporal extent. Moreover, some explicit schemes of interior methods can compute multi-attribute wavefields in a single execution of one program (Chen and McMechan, 1992). For example, in a calculation of 2-D PSM the wavefields and seismograms produced include any or all of the following: two components of each of acceleration, velocity and displacement; two components of normal strain; shear strain; two components of normal stress; shear stress; and the dilatation and rotation of the displacement. Since we are not used to processing such an abundance of information, we also have to investigate the mode of analysis and interpretation of the large volume of computational data.

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Computer graphics made from the output data may become one of the most powerful tools by which such abundant information can be viewed efficiently. Furumura and Takenaka (1992 b) made such animation from the results calculated by the PSM (Fig. 3). The animation gives us the “Wavefront World” instead of the “Ray World.” The “Wavefront World” may well challenge our conception of seismic wave propagation.

In this paper, our review was limited to studies on modeling wave propagation in isotropic and elastic (or weakly anelastic) media. Because studies for more general media are still few in Japan though they exists (e.g., anisotropic media: Kawasaki and Koketsu, 1990; porous media: Motosaka and Ohtsuka, 1992). However, with the increasing interest in more general media in recent years, techniques which are useful for the
modeling of wave propagation in isotropic and elastic media have been extended to more general media, particularly anisotropic media, and some useful codes have been developed outside Japan (e.g., Mallick and Frazer, 1991). We believe that studies on modeling wave propagation in more general media will become an active research area in Japan in the near future.

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