Cooling of the Earth's Mantle by Plate Subduction
—Importance of Pressure- and Stress-Dependent Rheology—

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Numerical simulation of 2-D mantle convection with variable viscosity is performed to examine the effect of plate subduction on cooling of the Earth's mantle. We employ temperature-, pressure-, and stress-dependent viscosity in order to realize plate-like motion and subduction of a cold thermal boundary layer at the surface. Our model predicts that the velocity of surface plates strongly depends on the viscosity of the mantle interior, at least when the viscosity of the mantle strongly depends on the pressure. As a result, the heat transfer of the Earth's mantle strongly couples with the temperature, and the Earth cools slowly. This is consistent with past mantle temperatures derived from petrological data. It also suggests the possibility that the Earth has a higher Urey ratio than that inferred from measurements of terrestrial heat flow combined with geochemical data.

1. Introduction

The viscosity of the Earth, or the planetary mantle, is an important mechanism controlling heat release of the Earth (Tozer, 1972). Several workers focused their attention on the effect of temperature-dependent viscosity on the heat transport of the mantle. Christensen (1984a, 1985a) examined the relation between the Nusselt number (Nu) and Rayleigh number determined by the viscosity of the interior of the mantle (internal Rayleigh number, Ra\_T) using two-dimensional (2-D) convection models with strongly temperature-dependent rheology. He found a weak dependence of Nu on Ra\_T. He therefore concluded that the viscosity of the surface rigid layer controls heat removal from the mantle, at least when the mantle does not have plate tectonics. He also pointed out the possibility that his conclusion may change for a mantle with plate tectonics. Numerical studies for higher Rayleigh number convection (Hansen and Yuen, 1993) and for convection in a spherical shell (Ratcliff et al., 1996), and experimental work on convection in the large aspect-ratio box (Gianandrea and Christensen, 1993) found a stronger dependence of Nu on Ra\_T.

Using simple 2-D models, Gurnis (1989) pointed out that the dynamics of real lithosphere (horizontal uniform velocity and subduction) significantly affect heat removal from the mantle. He suggested that terrestrial heat loss strongly depends on the viscosity
of the interior of the mantle rather than on that of the surface layer. In his model, the plate-like motion of the surface layer was realized by assuming a weak (low viscosity) zone at the subduction region. However, the efficiency of heat transport may also depend on the assumed viscosity of the weak zone in his model. To understand the behavior of the plate in the cooling mantle, a model with fewer artificial assumptions is needed.

In this study, we have carried out numerical simulations to reveal heat removal from the mantle with plate subduction using a fully dynamical convection model for cooling. The plate-like motion of the surface layer in our convection model is realized by introducing realistic temperature-, pressure-, and stress-dependent rheology instead of the artificial weak zone. Especially, we will emphasize the important role of pressure-dependent rheology combined with temperature- and stress-dependent rheology as a controlling mechanism of the mantle cooling rate through plate velocity.

2. Models and Numerical Methods

We employed a 2-D time-dependent whole mantle convection model to study the roles of realistic rheology in the Earth's thermal evolution. We use a Boussinesq viscous fluid with an infinite Prandtl number in a rectangular box. The mechanical boundary conditions are set to be stress-free. The fluid layer is heated internally by heat production elements and, at its base, by the cooling of the core. Cooling of the core included heat release from inner core growth. The fluid layer is cooled from the top with constant temperature. We assume that the internal heat source decays exponentially with time. The amount of the heat source is determined by present-day concentrations of radioactive isotopes (Hart and Zindler, 1989). We also assume that the core is nearly in thermodynamic equilibrium (the core has very high, effective thermal conductivity and a homogeneous temperature) (Steinbach et al., 1993; Honda and Iwase, 1996).

Equations to be presented here have non-dimensional forms except for viscosity. The equation of motion is

\[
\left( \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial x^2} \right) \left[ \eta \left( \frac{\partial^2 \Psi}{\partial z^2} - \frac{\partial^2 \Psi}{\partial x^2} \right) \right] + 4 \frac{\partial^2}{\partial x \partial z} \left( \eta \frac{\partial^2 \Psi}{\partial x \partial z} \right) = -\alpha(z) Ra \frac{\partial T}{\partial x}, \tag{1}
\]

where \( \eta \) is effective viscosity and \( \Psi \) is a stream function defined by

\[
u = \left( \frac{\partial \Psi}{\partial z}, -\frac{\partial \Psi}{\partial x} \right), \tag{2}
\]

where \( \nu \) is flow velocity. \( Ra \) is the thermal Rayleigh number at the surface based on internal heating as

\[
Ra = \frac{\rho_0 \alpha(0) g Q_0 h^2}{k \kappa(0) \eta}. \tag{3}
\]

The explanations and values of the parameters which are not mentioned here are found in Table 1. The equation of energy is
Table 1. Model parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(g)</td>
<td>Gravitational acceleration</td>
<td>(10 \text{ m s}^{-2})</td>
</tr>
<tr>
<td>(h)</td>
<td>Thickness of the mantle</td>
<td>(1,800 \text{ km})</td>
</tr>
<tr>
<td>(w)</td>
<td>Width of the mantle</td>
<td>(4,800 \text{ km})</td>
</tr>
<tr>
<td>(\rho_0)</td>
<td>Density of the mantle</td>
<td>(4,500 \text{ kg m}^{-3})</td>
</tr>
<tr>
<td>(k)</td>
<td>Thermal conductivity at the surface</td>
<td>(3.78 \text{ W m}^{-1} \text{ K}^{-1})</td>
</tr>
<tr>
<td>(\kappa_0)</td>
<td>Thermal diffusivity at the surface</td>
<td>(0.7 \times 10^{-6} \text{ m}^{2} \text{ s}^{-1})</td>
</tr>
<tr>
<td>(\alpha_0)</td>
<td>Thermal expansivity at the surface</td>
<td>(2.5 \times 10^{-5} \text{ K}^{-1})</td>
</tr>
<tr>
<td>(Q_0)</td>
<td>Present internal heating of the mantle</td>
<td>(2.12 \times 10^{-8} \text{ W m}^{-3})</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>Decay constant of internal heating</td>
<td>(8.78 \times 10^{-18} \text{ s}^{-1})</td>
</tr>
<tr>
<td>(t_0)</td>
<td>Duration before present condition of internal heat</td>
<td>(3,000 \text{ Myr})</td>
</tr>
<tr>
<td>(\eta_0)</td>
<td>Gravitational and latent heat release by inner core nucleation</td>
<td>(2.52 \times 10^{-3} \text{ W m}^{-2})</td>
</tr>
<tr>
<td>(\eta_0)</td>
<td>Ratio of the heat capacity (volume) of the core against mantle</td>
<td>0.25</td>
</tr>
<tr>
<td>(\eta_0)</td>
<td>Reference viscosity</td>
<td>(1 \times 10^{25} \text{ Pa s})</td>
</tr>
<tr>
<td>(n)</td>
<td>Stress exponent</td>
<td>3</td>
</tr>
<tr>
<td>(\sigma_r)</td>
<td>Rheology transition stress</td>
<td>4 MPa</td>
</tr>
<tr>
<td>(\sigma_r)</td>
<td>Yield stress</td>
<td>70 MPa</td>
</tr>
<tr>
<td>(b)</td>
<td>Factor of temperature dependence of the mantle viscosity</td>
<td>20.383</td>
</tr>
<tr>
<td>(c)</td>
<td>Factor of depth dependence of the mantle viscosity</td>
<td>((3 \times 10^5 \text{ for } 1,317 \text{ K}))</td>
</tr>
<tr>
<td>(\eta_p)</td>
<td>Maximum zero-stress viscosity of the plate</td>
<td>((10^5 \text{ for } 1,800 \text{ km}))</td>
</tr>
<tr>
<td>(T_r)</td>
<td>Transition temperature from the mantle to the lithosphere viscosity</td>
<td>(900 \text{ K})</td>
</tr>
<tr>
<td>(\tau)</td>
<td>Factor of temperature dependence of the lithosphere viscosity</td>
<td>(90 \text{ K})</td>
</tr>
<tr>
<td>(\eta_\tau/\eta_0)</td>
<td>Factor of the viscosity jump at 660 km</td>
<td>30</td>
</tr>
<tr>
<td>(\eta_\tau/\eta_0)</td>
<td>Depth of the viscosity jump</td>
<td>660 km</td>
</tr>
</tbody>
</table>

\[ \frac{\partial T}{\partial t} + \nabla \cdot \mathbf{u} T = \frac{\partial}{\partial x} \left( \kappa_x(z) \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial z} \left( \kappa_z(z) \frac{\partial T}{\partial z} \right) + Q(t), \]  

where \(T\) is temperature, and \(\kappa_x\) and \(\kappa_z\) are thermal diffusivities for the \(x\) and \(z\) directions. \(Q(t)\) is internal heating as

\[ Q(t) = Q_0 \exp[-\lambda(t-t_0)], \]

where \(t_0\) is the duration of thermal evolution before the present condition in the model.

We employ depth-dependent thermal expansivity (Choperas and Boehler, 1989) and conductivity (Anderson, 1987). The parameterization is taken to be the same as that of Steinbach et al. (1993). The thermal expansivity depends on the depth of the model \(z\) as

\[ \alpha(z) = (1.0 + z)^{-3}. \]

This decreases by a factor of 8 from the top to the bottom of the mantle. The thermal diffusivity is expressed by

\[ \kappa_x(z) = (1.0 + z)^{1.5} \quad \text{and} \quad \kappa_z(z) = (1.0 + z)^{1.5}. \]
This increases by a factor of 2.3 from the top to the bottom of the mantle. In order to take into account the effect of the decrease of the surface area of the mantle with the depth on the horizontally averaged temperature, especially the temperature jump at the core-mantle boundary, vertical thermal diffusivity is modified to

$$\kappa_z(z) = S(Z)(1.0 + z)^{1.5},$$

where $S(Z)$ is a normalized surface area of the mantle at each depth $(Z)$ as

$$S(Z) = (R - Z)^2/R^2,$$

with

$$Z = H \cdot z/h,$$

where $R$ is the radius of the Earth (6,400 km), $H$ is the thickness of the real mantle (3,000 km), $z$ is non-dimensional depth in the model, and $h$ is a normalization depth of the model. $S(Z)$ is normalized by the surface area of the Earth ($S(0) = 1$). We have checked the effect of this modification, and can obtain a horizontally averaged temperature similar to that of convection in a 3-D spherical shell by 2-D calculation with the modified $\kappa_z$.

We assume that composite Newtonian and non-Newtonian rheology dominates the flow of the upper mantle (Christensen, 1984b; Christensen and Yuen, 1984; van den Berg et al., 1993). In order to realize plate-like motion of the surface viscous layer, we assume that the yield stress criterion determines maximum flow stress (Cserepes, 1982). The effective viscosity is governed by

$$\eta = \sigma_y/2\dot{\varepsilon}_II, \quad \sigma = \sigma_y$$

$$\eta = \left[ \frac{1}{\eta_{New}} + \left( \frac{\dot{\varepsilon}_II^{-1}}{\eta_{New}\sigma_t^{-1}} \right)^{1/n} \right]^{-1} \quad \sigma < \sigma_y,$$

where $\dot{\varepsilon}_II$ is the second invariant of the strain rate tensor. The explanations and values of parameters are listed in Table 1. Here, $\eta_{New}$ is the viscosity at zero stress depending on temperature and depth as

$$\eta_{New} = \eta_m + \frac{(\eta_m + \eta_p)}{2} \tanh \left( \frac{T - T_{tr}}{\tau} \right),$$

where

$$\eta_m = \eta_0 \exp[-bT + cz].$$

Equation (12) gives strong temperature dependence on the viscosity in the low-temperature region. The upper bound of the viscosity is given by the value of $\eta_p$. A linearized Arrhenius law (Eq. (13)) is employed to easily control the magnitude of the viscosity variation.

For the lower mantle, we employed Newtonian rheology (Karato and Li, 1993; Nakakuki and Fujimoto, 1994; van den Berg et al., 1993) as

$$\eta = \eta_L \exp(-bT + cz).$$
The viscosity increases by a factor of 30 at a depth of 660 km (Hager, 1984). The lateral viscosity variation from 900 to 1,600 km depth is removed using an averaged layer temperature to determine the viscosity. We do this for the following reason: If lateral viscosity variation due to temperature dependence was introduced, cold and high-viscosity fluids would accumulate beneath the subduction location fixed at the side boundary of the model. This would cause plate subduction to stagnate. Such a situation probably does not occur in the real mantle, because the location of the subduction migrates. In order to avoid an artificial effect from fixed subduction, we remove the lateral viscosity variation in this layer.

The set of the equations is solved by a finite difference method described in Nakakuki et al. (1994). The computation is performed on uniform grids with 256 by 96 grid spacing for the x and z directions, respectively.

In order to obtain a solution with plate subduction, we introduce a stage to make an initial condition. We first set the temperature of the mantle to be uniform (1,790 K) with a temperature change of 500 K at the core-mantle boundary. Second, the cooling calculation is performed while requiring a constant velocity (2 cm year$^{-1}$) of the surface boundary for 500 Myr. In this time, the internal heat source has already started to decay. After that, the top boundary is set to be stress-free and calculation of the thermal evolution by free convection is started. The origin of the evolution time (0 Myr) is taken to be at this point.

3. Results

Figure 1 shows snap shots of stream lines, temperature, temperature anomaly from horizontally averaged temperature and viscosity at 800 Myr. Stress weakening occurs at the top-right corner in the surface layer. The moving surface plate and subducting slab are generated as shown by a straight stream line (i.e., the uniform velocity along the top and right-hand side boundaries) (Fig. 1(a)). The surface plate and the sinking slab have the same moving speed. The surface plate is thus driven by slab pull.

In the upper mantle, temperature anomaly localizes upon the surface and descending boundary layer, and is very small in the other region (Fig. 1(b), (c)). Small-scale convection is generated beneath the lithosphere (Fig. 1(a)). This causes thinning of the lithosphere in the model. The heat flow through the old lithosphere becomes higher than that inferred from the inverse-root-age law of oceanic heat flow (Fig. 2). In the lower mantle, there exist broad and strong thermal anomalies. These are caused by depth-dependent viscosity and thermodynamic properties (Steinbach et al., 1993; Hansen and Yuen, 1993; Cadek et al., 1994). The widely spreading hot temperature anomaly could be related to large-scale seismic low-velocity regions found in the lower mantle (Fukao, 1993) as several authors have pointed out (Steinbach et al., 1993; Hansen and Yuen, 1993; Cadek et al., 1994; Tackley, 1996).

Figure 3 shows the evolution histories of averaged temperature of the mantle and core (Fig. 3(a)), heat flux at the top and bottom boundaries of the mantle (Fig. 3(b)), and averaged velocity of the surface and interior (root mean square velocity) of the mantle (Fig. 3(c)). The temperature of the model means potential temperature because we use a Boussinesq approximation. The temperature of the mantle decreases rapidly
Fig. 1. Snap shots of the calculated results for 800 Myr. (a) Stream lines in each 50 non-dimensional unit (3.5 \times 10^{-5} m^2 s^{-1}) interval, (b) temperature in each 0.005 non-dimensional unit (111 K) interval, (c) temperature anomaly from horizontally averaged temperature in each 0.005 non-dimensional unit (111 K) interval, (d) ten's logarithm of viscosity in each 0.5 interval. The dimension of the box is 4,800 km (horizontal) by 1,800 km (vertical). Solid lines show the positive values and dashed lines show the negative values in (a) and (c). The positive values of the stream lines mean clockwise motion of the flow. The positive values of the temperature anomalies are of hot ones.

(average of 80 K/Gyr) before 10 Gyr, and after that, with a nearly constant rate of 50 K/Gyr. The core cools more rapidly (60 to 80 K/Gyr) than the mantle. Heat flow at the surface rapidly decreases with time as the velocity of the surface layer becomes slower. The drastic changes in the first few hundred million years are affected by the initial spin-up of the convection.

The heat flow and surface velocity smoothly change in the period before 1,300 Myr (Period I). In Period I, slab-pull exerted by the subducting lithosphere is the main driving force as mentioned above. The velocity of the subducting slab is dominated by
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Fig. 2. Surface heat flux in 800 Myr. Vertical axis shows the heat flux and the horizontal axis shows distance from the left boundary. The solid line shows the value obtained from the model. The dashed line is an inverse-root-age (horizontal distance) curve which is fitted to the obtained value between 0 km (0 Ma) to 1,600 km (100 Ma). The difference between two curves from 0 to 200 km is caused by the resolution of the calculation.

the viscous resistance of the lower mantle. The plate is decoupled from the underlying asthenosphere. The fluctuation of convection under the plate does not affect the motion of the plate. The smooth change in lower mantle viscosity with the cooling results in decreases in the plate velocity. The heat flow and surface velocity fluctuate after 1,300 Myr (Period II). In Period II, the slab-pull force becomes small because the speed of the subducting slab slows significantly due to the high viscosity of the lower mantle (Fig. 4(a)). The driving force of the plate changes into ridge-push force and viscous drag from the underlying mantle. The plate couples with the underlying mantle, and non-Newtonian rheology magnifies the time-dependent behavior of the plate and mantle. A large change in the surface velocity at 2,700 Myr results from the initiation of surface plate subduction (Fig. 4(b)).

The relationship between the Nusselt number (Nu) and internal Rayleigh number (Ra_T) is shown in Fig. 5. We define Nu (Christensen, 1985a) and Ra_T for time-dependent and internally heated convection as

\[ \text{Nu} = \frac{3Q_{\text{sur}}}{8T} \left( \frac{\text{mean temperature by conduction for } Q_{\text{sur}}}{\text{mean temperature by convection}} \right) \]  

(15)

and

\[ \text{Ra}_T = \frac{\rho_o \omega (0.5) g T^3 h^3}{\kappa (0.5) \eta (T, 0.5)} \]  

(16)

where \( Q_{\text{sur}} \) is the non-dimensional surface heat flux and \( T \) is the non-dimensional mean temperature of the mantle. The treatment of temperature difference to calculate \( \text{Ra}_T \)
is the same as the "top local Rayleigh number" proposed by Honda (1996). The value of the viscosity which determines $Ra_T$ is close to that of the mantle at the middle depth (about 900 km). The $Nu-Ra_T$ relationship is fit by a power-law curve:

$$Nu = cRa^\beta,$$

where $c$ and $\beta$ are constants. For all the obtained values, the curve is fit with $\beta$ of 0.44. This value reflects a rapid change in $Nu$ in Period I. Only for Period I, is $\beta$ of 0.46 adequate. These are larger than 1/3, which is the $\beta$ for steady-state convection with constant viscosity (Christensen, 1986), and agrees well with a dynamical thermal history model (Honda and Iwase, 1995). The speed of the sinking slab in the model depends on the viscosity of the deep mantle rather than the averaged viscosity of the whole mantle. The deep mantle controlling the subduction speed is cooled locally by the subducted slab and is always faster than the whole mantle. The viscosity at shallower depths controls the sinking slab as the deep mantle is cooled. The effective thickness of the convection layer becomes smaller. This is the reason why $\beta$ becomes larger. Values of $\beta$ larger than 1/3 are also found in convection models with pressure- and mean-temperature-dependent viscosity and cooling (Yuen et al., 1996). In Period II, $\beta$ is 0.26. This value is slightly smaller than that for constant viscosity convection. For
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Fig. 4. Snap shots for (a) 2,600 Myr and (b) 2,700 Myr. Contour intervals are the same as those of Fig. 1(b). The arrow shows the location of initiation of subduction.

Fig. 5. Relationship between internal Rayleigh number (Ra_r) and Nusselt number (Nu). The horizontal axis shows Ra_r and the vertical axis shows Nu. The solid curved line shows Nu for Ra_r in Period I and the dashed line shows that in Period II. The straight line shows power-law fitting to all the obtained values of Periods I and II.

Both periods, the obtained values are larger than β (<0.1) for convection with the high-viscosity lid due to the temperature-dependent viscosity (Christensen, 1984a, 1985a). The viscosity of the mantle interior controls heat removal from the mantle in our model.
4. Discussion and Conclusions

Using a 2-D convection model with stress-, temperature-, and pressure-dependent rheology, we have shown that plate velocity depends strongly on the internal viscosity of the mantle. Heat removal, therefore, depends strongly on internal viscosity. Because of the temperature dependence of the viscosity, heat removal strongly couples with the temperature of the mantle. The heat flow at the surface becomes much smaller with the cooling of the mantle, and quickly catches up when internal heat sources decrease. These characteristics are consistent with the conclusion by Gurnis (1989), although some more arguments may be required to justify whether our model is appropriate to the Earth's mantle or not.

In our model, the plate subducts smoothly in the initial 1,300 Myr (Period I), in which plate motion driven by subduction takes place. This does not mean that the model predicts that the present condition of the Earth's mantle is like Period II. The present mantle is probably in Period I, because subduction-driven plates exist (Forsyth and Uyeda, 1975). If employing higher Rayleigh numbers than are used in our model, a solution with Period I surviving for a longer time is obtained. However, we have not yet successfully simulated free convection with the subducting lithosphere in the case of applying higher Rayleigh numbers. This is a future subject of study for us.

Our model demonstrates that plate velocities are determined by the viscosity of the lower mantle. Because stress-dependent rheology weakens the strength of the plate upon subduction, resistance by lower mantle viscosity against the slab is more important in determining plate velocity than the strength of the plate. This may be changed, when the pressure dependence of the viscosity is weak or when the deep slab has viscosity so low that it cannot work as a stress guide (Moresi and Gurnis, 1996). We may have to change our argument also when slab penetration into the lower mantle is prevented by a phase transition with the negative Clapeyron slope. However, we believe that the viscosity of the lower mantle still plays an important role when the slabs penetrate it, even if they temporarily stagnate at the phase boundary as several authors pointed out (Christensen and Yuen, 1984, 1985; Machetel and Weber, 1991; Honda et al., 1993; Tackley et al., 1993). More studies are required when layered convection occurs in the mantle. This may be important for the thermal evolution of the early Earth (Steinbach and Yuen, 1993; Honda and Yuen, 1994).

Strong buffering between mantle temperature and heat removal occurs when both plate motion and subduction occur. This results in the slow cooling rate of the Earth as mentioned above. Many parameterized convection studies showed slow cooling assuming strong thermal buffering (e.g., Davies, 1980; Christensen, 1985b). In our model, the cooling rate of the mantle is 50 K/Gyr. This is consistent with empirical thermal history inferred from petrological data (Abbott et al., 1994). In our model, the Urey ratio (the portion of heat released by the Earth to the internal heating of the mantle; Christensen, 1985b) is about 0.6. This value is higher than that estimated from geochemical data (about 0.4; Christensen, 1985b). If this is the case, additional internal heat is required somewhere in the Earth or the Earth needs to have some mechanisms to reduce the Urey ratio; for example, growth of continental crust (Spohn and Breuer, 1993).
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Past plate motion is not clear, but some paleomagnetic data indicate that the velocity of continental plates, at least the minimum velocity, has not changed significantly (Ulrich and van der Voo, 1982). The continental plates usually have slower velocities than oceanic plates (Forsyth and Uyeda, 1975) because of the absence of subduction in the continental plates. The data for the velocity of oceanic plates are required to justify our results, which show that the existence of subduction significantly affects the velocity of the surface plate.

More calculations are required to arrive at definitive conclusions. We need to account for more realistic properties of the mantle such as Rayleigh number, rheology, three dimensionality, and sphericity in future studies. In this study, we have shown that coupled stress- and pressure-dependent rheology have an important role in the heat transport of the mantle, just as temperature dependence has. The heat transfer of the Earth’s mantle with subducting plate strongly couples with the temperature when the viscosity depends strongly on the pressure as well as the temperature.

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REFERENCES


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