On the Frequency Distribution of Rupture Lengths of Earthquakes Synthesized from a One-Dimensional Dynamical Lattice Model

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A one-dimensional BK dynamical lattice model (Burridge and Knopoff, 1967) is applied to simulate earthquakes for the study of the scaling relation between frequency and rupture length of earthquakes. Velocity-dependent friction controls the motion of mass elements. The distribution of the breaking strengths (i.e., static friction) is considered to be a fractal function. Simulation results show that the fractal dimension of the distribution of the breaking strengths is a minor factor in affecting the scaling of frequency versus rupture length. A fast velocity-weakening process from static friction to dynamic friction and a slow velocity-hardening one from dynamic friction to static friction are appropriate for interpreting the scaling of the frequency-rupture length (FL) relation. The frictional drop rather than the level of the breaking strength affects the FL scaling. Hence, the friction drop ratio (g) which determines the minimum value of the dynamic frictional force, is an important factor in influencing the FL relation. Smaller g (which a large friction drop) leads to a smaller scaling exponent value in the regime of localized events than larger g (with a smaller friction drop). The stiffness ratio, which is defined as the ratio of the stiffness of the coil spring to that of the leaf spring of the model, is also a significant parameter affecting the FL distribution. Nevertheless, simulation results show that small s is unable to produce a power-law FL relation.

1. Introduction

Gutenberg and Richter (1944) first reported a relation between log N and M, where M is the earthquake magnitude and N is the cumulative or discrete frequency of earthquakes with magnitude ≥ M, in the form of log N = a − bM. The earthquake magnitude relates to seismic energy (E) in the form of M = log E^β, and thus, N relates to E in a power-law function: N ∝ E^−θ, where β = bη. Since then, numerous power-law relations of earthquakes and faults have been reported. From field observations in the San Andreas fault system, Aviles et al. (1987) and Okubo and Aki (1987) stressed fractal characterization of the faults. Cowie et al. (1993), Gudmundsson (1987), Henderson et al. (1994), Marrett (1994), Marrett and Allmendinger (1991), Sahimi et al. (1993), Scholz
and Cowie (1990), Scholz et al. (1993) and Walsh and Watterson (1992) stated that the fault populations obey fractal geometry in a form of $N \sim l^{-d}$, where $N$ is the cumulative number of faults having lengths $\geq l$ and $d$ is the scaling exponent. The observed scaling exponent value is 0.89 for the fractures on the Reykjanes, southwest Iceland (Gudmundsson, 1987), 2.1 for intraplate faults in Japan (Scholz and Cowie, 1990), 1.8 for several data sets obtained by Marrett and Allmendinger (1991), 1.91 for basin and range faults and 1.24 at Yucca Mountain, Nevada (Marrett, 1994) and 1.3 for the volcanic tableland faults (Scholz et al., 1993). For earthquakes occurring in the Geysers geothermal field in northeast California, Sahimi et al. (1993) obtained a value of about 1.9. From laboratory experiments to simulate continental collisions, Sornette et al. (1990, 1993) and Davy et al. (1990) reported that the value of $d$ is in the range from 0.7 to 1.6. Nevertheless, Davy (1993) and Davy et al. (1995) suggested that a Gamma function in the form of $N(l) = Al^{-a+1} \exp(-l/l_c)$, where $A$ is a constant, is more appropriate than other functions to fit the discrete frequency-length distribution obtained from the San Andreas fault system and those from experimental results. They also stated that the characteristic length ($l_c$) is close to the thickness of the brittle crust. However, it is noted that in the above-mentioned observations, the frequency-length power-law scaling ranges only an order of magnitude in length.

Burridge and Knopoff (1967) proposed a one-dimensional (1-D) dynamical lattice model (called the BK model in this study) to approximate the fault dynamics. This model has been widely used to study the frequency-magnitude scaling law (Carlson and Langer, 1989; Carlson et al., 1991; Knopoff et al., 1992; de Sousa Vieira et al., 1993; Wang, 1994, 1995a, 1996; Xu and Knopoff, 1994), the source rupture duration (Carlson et al., 1991; Wang, 1993), the relation between rupture length and earthquake moment (Carlson et al., 1991; Wang 1995b), seismicity patterns (Shaw et al., 1992; Pepke et al., 1994), and the scaling of source spectra (Shaw, 1993). In addition, a simplified form of the 1-D BK model and its two-dimensional (2-D) extension were also used for seismicity simulations based on the cellular automaton concept (cf. Ito, 1992).

In this study, an attempt is made to explore the effects of the model parameters on the frequency-rupture length (FL) relation of earthquakes based on a comparison between the simulation results from the 1-D BK model and the observed results. Most of the above-mentioned observed results were obtained from 2-D geological structures and the faults in consideration include both tectonic and earthquake faults. Although the present model is 1-D and only represents an earthquake fault, we still assume that the simulation results from the model can provide significant information for understanding the effects on the scaling of the population of faults due to the model parameters. In the modeling, velocity-dependent friction is taken to control the motion of mass elements. The distribution of the breaking strengths (i.e., static friction) is considered to be a fractal function. In addition, a comparison of the simulation results with the observed results will not only help us to examine the applicability of the 1-D dynamical lattice model to seismicity simulations but also provide constraints on the determination of the acceptable range of the model parameters.
2. Theory

The 1-D BK dynamical lattice model consists of a chain of \( N \) mass elements of equal mass \( (m) \) and \( N-1 \) springs with each mass element being linked by a coil spring of stiffness \( K \) with two other neighbors and each mass element also being pulled by a leaf spring of stiffness \( L \) on a moving plate with constant velocity \( V \). Originally all mass elements rest in an equilibrium state, and the spacing between two mass elements is “\( a \)”.

The \( n \)-th mass element is located at position \( u_n \), measured from its initial equilibrium position, along the \( x \)-axis. This fault model is schematically illustrated in Fig. 1. Each mass element is subjected to a velocity-dependent frictional force \( (F(v_n)) \) where \( v_n \) \( (=\frac{\partial u_n}{\partial t}) \) is the sliding velocity of the \( n \)-th mass element. The equation of motion of the \( n \)-th mass element of the system is

\[
m\frac{\partial^2 u_n}{\partial t^2} = K(u_{n+1} - 2u_n + u_{n-1}) - L(u_n - V) - F(v_n).
\]

It is noted that the spacing between two mass elements is not an explicit parameter in Eq. (1). \( K \) and \( L \) are two important parameters representing the coupling between two mass elements and that between a mass element and the moving plate, respectively. The ratio of \( K \) to \( L \) is a significant factor in controlling the seismicity pattern and frequency-magnitude relation (Carlson and Langer, 1989; Carlson et al. 1991; Wang, 1994, 1995a). This ratio is called the stiffness ratio and denoted by \( s \) by Wang and \( l^2 \) by Carlson and her co-authors. A detailed study of the effect on the frequency-magnitude relation due to the stiffness ratio can see Wang (1995a).

When the sum of the driving force on a certain mass element from the moving plate and the spring forces exerted from its neighbors exceeds its breaking strength, the mass element is accelerated and starts to slide. After a while, the mass element decelerates because of either an increase in the spring forces due to the change of relative positions of it and its neighbors or the rise of the dynamic frictional force when the sliding velocity is greater than the critical velocity \( (v_c) \) as described below. Finally, the mass element stops. The driving force from the moving plate, which loads the system continuously, increases the force on the mass element, thus pushing it to slide again.

In this study, a periodic boundary condition is specified at the two ends of the system for solving Eq. (1).

Friction is a very complicated physical process and has been studied based on theoretical and experimental approaches for a long time. Burridge and Knopoff (1967)
first suggested a velocity-dependent friction law, including a weakening process at low velocities and a hardening one at high velocities. Ruina (1983) proposed a rate- and state-dependent function to describe friction. From laboratory experiments, Shimamoto (1986) reported a very interesting velocity-dependent friction law. A complete description was reported by Blanpied et al. (1987). The matter whether velocity-dependent friction is velocity-weakening or velocity-hardening depends upon the decrease or increase in the dynamic frictional force when the sliding velocity is increased. The rate- and state-dependent friction law is quite complex. For simplifying numerical computations based on the BK model, Wang and Knopoff (1991) considered a stepwise linear velocity-dependent friction law:

\begin{align}
F(v) &= F_o - rv & (v \leq v_c) \; ; \\
&= gF_o + \gamma(v - v_c) & (v > v_c) .
\end{align}

In Eq. (2), the dynamic frictional force is velocity-weakening when \(v \leq v_c\) and velocity-hardening when \(v > v_c\). Although the friction law is quite simplified, we still consider that it is a good first-order approximation of the velocity-dependent friction law. The decreasing rate \(r\) and increasing rate \(\gamma\) for the variation of the dynamic frictional force with the sliding velocity are two parameters of the friction law. When \(v = v_c\), the dynamic frictional force reaches the minimum value \(gF_o\), where \(g\) is the friction drop ratio and the third parameter of the friction law. The value of \(g\) is positive, yet smaller than 1. Wang (1996) studied, in detail, the effect on the motion of mass elements and the frequency-magnitude relation due to velocity-weakening friction based on the 1-D BK model. He stressed that the friction law with large \(r\) as well as \(r = \infty\), which radically is the classic static/dynamic friction law, is more appropriate for interpreting the Gutenberg-Richter-type frequency-magnitude relation than that with small \(r\). For simplicity, the law with \(r = \infty\) is mainly considered below. Figure 2 schematically shows

![Fig. 2. A linearly velocity-dependent friction law. \(F_o\), the breaking strength; \(g\), the friction drop ratio.](image-url)
the friction law mainly used, for which $v_e = 0$. Figure 2 also shows that no backward motions of mass elements are allowed.

Earthquake fault zones are usually quite complex. Field observations (Scholz and Aviles, 1986; Aviles et al., 1987; Okubo and Aki, 1987) and laboratory experiments (Brown and Scholz, 1985; Power et al., 1987) showed a fractal distribution of fault surface roughness. Wang and Knopoff (1991) first used a fractal distribution of the breaking strengths for seismicity simulations. Such a fractal distribution is also used in this study. The Midpoint Displacement Method developed by Saupe (1988) is applied to yield a fractal distribution of the breaking strengths. This method can produce a set of fractals with $2^i + 1$ discrete points after $i$ computational steps, where the exponent $i$ is an integer. The complicated distribution of the breaking strengths makes the problem intractable analytically, and thus, only the numerical solution is dealt with in the following.

The displacement of a mass element is measured from its new equilibrium position to the one where it sticks after sliding. The new position is a new equilibrium for the next stage of motion. Since several connected mass elements may slide almost simultaneously within a short time span, the sum of the displacements of such connected mass elements in terms of time shows the history of the displacement. Such a time history is considered to be an event.

Generally, the scaling exponent of the power-law scaling relation, for instance the Gutenberg-Richter-type frequency-magnitude relation, is assumed to be independent of the selection of the cumulative frequency or discrete frequency. However, from seismic observations, Davy (1993) stressed the importance of plotting both discrete (density) and cumulative frequency data before the type of curve fit is chosen. For the Gutenberg-Richter-type frequency-magnitude relation, Main (1992) and Wang (1995a) also pointed out the difference in the scaling exponents estimated from the discrete and cumulative frequency distributions. From the observations by Main (1992) and simulation results by Wang (1995a), the scaling exponent from the discrete frequency is usually larger than that from the cumulative frequency. However, in this study only the cumulative FL relation and its scaling exponent are taken into account.

3. Numerical Results

From Eqs. (1) and (2), the major model parameters are (1) the fractal dimension ($D$) of the geometrical distribution of the breaking strengths; (2) the decreasing rate ($r$) and increasing rate ($\gamma$) of friction; (3) the friction drop ratio ($g$) of the velocity-weakening process; (4) the velocity of the moving plate ($V$); (5) the stiffness ratio ($s = K/L$); and (6) the mass ($m$) of a mass element. As mentioned above, for simplification, the values of $m$ and $L$ are set to one. Hence, in this study the value of $s$ is exactly equal to that of $K$. According to a comparison between the observed data and simulation results, Carlson and Langer (1989) and Carlson et al. (1991) suggested that the value of $s$ must be large. Their theoretical analyses showed that the crossover magnitude ($M_c$) from localized to de-localized events equals $\ln(4s^{1/2}a/\theta)$, where $-\theta$ is the initial slope of a general velocity-weakening friction law. In general, $M_c$ is about 6 (Carlson and Langer, 1989), so that $s \approx 10^4\theta^2$ when $a = 1$ length unit. Obviously, $s$ is very large when $\theta > 1$.
Nevertheless, Carlson and her co-workers often used small values of $s$ ranging from 10 to 100 for numerical simulations. By comparing the 1-D BK equation with a finite-difference equation, which is an approximation of a 2-D plane strain-type wave equation in the neighborhood of a fault plane, Yamashita (1976) reported that when the spatial units of the finite-difference equation along two axes are equal, $K = 2(\mu + v)(\beta/\alpha)^2$ and $L = \mu$, where $\mu$ and $v$ are the Lame's constants of the materials, $\beta$ is the related S-wave velocity and $\alpha$ is the related P-wave velocity. From Yamashita's relations, $s = [2(\mu + v)/\mu](\beta/\alpha)^2$. For the regular crustal materials, $\mu$ approximately equals $v$ and $\beta/\alpha$ is about $3^{-1/2}$, thus leading to a value of $s$ of about 1.33. For some models with a small number of mass elements, the value of $s$ was taken to be 4.0 by Rundle and Jackson (1977) and approximately 2.33 by Cao and Aki (1986). For a very simplified model consisting of two frictional sliders, Ruff (1992) considered the values of $s$ to be in the range from 0.06 to 0.40, for which the coupling between the plate and the slider is much stronger than that between two sliders. However, Wang (1994, 1995a) stressed that only values of $s$ in the range from 20 to 120 are appropriate for the simulations of earthquakes in order to have a Gutenberg-Richter-type frequency-magnitude relation over a large magnitude range. The relation of $L = \mu$ derived by Yamashita would be questionable for the BK model. The coupling between the fault zone and the background moving plates must be reduced due to some degrees of separation between them, so that the relation between $L$ and the material constants cannot be deduced directly from an elastic continuum. Anyway, at the moment, it is still difficult to determine a precise value of $s$ for the earthquake fault zone. Five values of $s$ (i.e., 10, 20, 50, 80, and 100) are used in the following simulations.

From the observed data, the velocity of the moving plate should be a very small value, about 50 mm/yr or $1.6 \times 10^{-9}$ m/s. However, this value is too small for the numerical simulations because a very large computational time is required. Hence, in this study, a larger value for $V$ is taken into account. Wang (1995a) stated that the seismicity pattern and the frequency-magnitude relation obtained from the model with $V = 10^{-3}$ velocity units is very similar to those from the model with $V = 10^{-4}$ velocity units. For a two degree-of-freedom block-spring model, Brun and Gomez (1994) claimed that the driving velocity of the moving plate is a significant factor in affecting seismicity. In their modeling, the value of $s$ is 1.2, which is small and leads to an almost equal action on the seismicity due to the coupling between two mass elements and that between a mass element and the moving plate. Their simulation results show a variation of dynamical behavior (chaotic and periodic) of the system with the velocity. Schmittbuhl et al. (1993) defined $\Theta = vN$, where $v (= V(m/K)^{1/2}/F_0)$ is a normalized velocity, as a parameter controlling dynamical behavior. Their simulation results with large values of $s$ show that for $\Theta \leq 4$, the system is characterized by a chaotic-type solution, while at $\Theta = 8$ (the first local minimum) a one-solitary-wave solution is observed. Of course, a higher-mode solitary-wave solution can be observed at the larger local minimum. At $\Theta = 4$, the normalized driving velocity is $4/N$. Thus, the system only shows chaotic behavior when $v < 4/N$. Although the friction law used by them is different from the present one, we assume that their result can still be taken as a reference for the present modeling. In this study, the critical driving velocity must be $1.55 \times 10^{-2}$ velocity units.
when \( N = 129 \), \( m = 1 \) mass unit, \( K = 100 \) stiffness units and \( F_o = 5 \) force units. Since the driving velocity (\( = 10^{-4} \) velocity units) used in this study is, at least, 15 or 150 times smaller than the above-mentioned critical value, only non-periodic behavior can be observed here. Hence, \( V \) must be a minor factor in affecting seismicity simulations based on the present values of model parameters. To save the memory of storage, a value of \( 10^{-4} \) velocity units is selected in the following computations.

The fractional distribution of the breaking strengths is controlled by the \( D \) value. Essentially, there is no direct measured value of fractal dimension of the distribution of the breaking strengths over a fault surface. On the other hand, the \( D \) values of the geometrical structures of many faults have been measured. Such values are, in general, less than 1.5 for the single faults (cf. Aviles et al., 1987; Okubo and Aki, 1987); whereas, for a 2-D system composed of a large number of faults, the \( D \) values can be greater than 1.5 (cf. Marrett, 1994). Such kind of fractal dimension cannot be directly taken to be the fractal dimension of the distribution of the breaking strengths. Nevertheless, we assume that the fractal geometry of a fault has a strong relationship with the fractal, heterogeneous distribution of the breaking strengths over the fault. Thus, it might be reasonable to consider that the fractal dimension of the distribution of the breaking strengths over a single fault is less than 1.5. Of course, for a 2-D system, the \( D \) value can be greater than 1.5. For this study, since only a single fault is considered, the use of two \( D \) values (i.e., \( D = 1.1 \) and \( D = 1.5 \)) for simulations would be appropriate.

Although Wang (1995a) stated that the maximum breaking strength (i.e., \( F_{o, \text{max}} \)) mainly affects the inter-event time and does not cause any direct effect on the frequency-magnitude relation, the FL relations for two values of \( F_{o, \text{max}} \) (i.e., 5 and 10 force units) are also constructed. The time step for computation is 0.1 units and the computational time for each case is several \( 10^7 \) time steps. Simulation results obtained by Carlson et al. (1991) show that although the maximum size of delocalized events is affected by the number of mass elements, the frequency-magnitude relation for localized events is somewhat independent of such a number. In this study, for all cases, the number of mass elements \( (N) \) is 129 (for \( i = 7 \)), which is large enough to produce self-similar or localized events in the range of an order of magnitude in length. The number of simulated events for each case shown below is larger than 5,000, which makes the statistical analysis of simulation results significant.

As an example, the space-time pattern of synthetic events for a case with \( s = 100 \), \( D = 1.5 \), \( g = 0.8 \), \( r = \infty \), and \( F_{o, \text{max}} = 5 \) force units is shown in Fig. 3. The distribution of the breaking strengths is displayed on the left-hand side of the diagram. The line segment in the figure represents an event. The length is related to the number of connected mass elements which slide during an event. A longer line segment shows a larger event. However, two events whose lengths are equal may not have the same magnitude value because their values of the maximum displacement and/or the total force drop of the related mass elements are different. It is obvious that the time interval between any two consecutive larger events is greater than that between any two consecutive smaller ones. A detailed discussion about the space-time patterns of synthetic events can be found in a report by Wang (1995a).

Figure 4 shows a log \( N \) versus log \( l \) plot for two \( D \) values (denoted by open circles for \( D = 1.1 \) and plus for \( D = 1.5 \)). The rupture length \( (l) \) of an event is calculated...
Fig. 3. The space-time pattern of simulated events when $s=100$, $D=1.5$, $g=0.8$, $r=\infty$, and $F_{\text{max}}=5$ force units.

Fig. 4. The plots of $\log N$ versus $\log(l)$. Pluses for $D=1.5$ and open circles for $D=1.1$ when $s=100$, $r=\infty$, $g=0.8$, and $F_{\text{max}}=5$ force units. Two solid lines represent the lines with a slope value of $-1$ and $-1.5$, respectively.

directly from the number of connected mass elements. From Fig. 4, three kinds of events, small ones with $l<l_1=4$ length units ($\log(l_1)=0.6$), intermediate ones with $l$ in the range from $l_1=4$ length units to $l_2$ and large ones with $l>l_2$, can be delineated. The value of $l_2$ is about 20 length units (i.e., $\log(l_2)=1.2$) for $D=1.1$ and 32 length

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units (i.e., \( \log(l_2) = 1.5 \)) for \( D = 1.5 \). The former is slightly smaller than the latter. For small and intermediate events, the data points for the two values of \( D \) are close to each other. For large events, the data points of the two \( D \) values separate, and the rupture length for \( D = 1.5 \) is, in general, larger than that for \( D = 1.1 \). However, they follow a very similar falling trend. The distribution of data points shows three different scaling laws for the three kinds of events. Intermediate events remarkably show a power-law relation with a scaling exponent value of 1.5 (i.e., \( N \sim l^{-1.5} \)). The scaling exponent value is less dependent of the \( D \) value. On the other hand, small events as well as large ones show a power-law function with a larger scaling exponent, whose value is about 3 for small events and 8 for large ones. The two values are quite different from those obtained from field survey or laboratory experiments. In addition, the range of rupture length for small and large events is smaller than that for intermediate events. The different distributions of the breaking strengths for the two \( D \) values lead to a small difference in the distributions of \( \log N \) versus \( \log(l) \), especially for intermediate events. Hence, the \( D \) value is taken to be 1.5 in the following computations.

In the numerical simulations, the friction law of Eq. (2) is used. Since velocity-weakening friction causes fault instabilities and velocity-hardening friction makes the fault stable, the former is mainly taken into account in this study. However,
for comparison, the plots of log $N$ versus log$(l)$ for the two values of $\gamma$ (i.e., 1 and 5) when $r=\infty$, $s=100$, $D=1.5$, $g=0.8$ and $F_{\text{max}}=5$ force units are shown in Fig. 5. The case with $\gamma=1$ represents a slower hardening process, while the case with $\gamma=5$ denotes a faster one. It is obvious that the number of small events for $\gamma=5$ is much larger than $\gamma=1$. Meanwhile, the plot for $\gamma=5$ is quite different from that for $\gamma=1$, and there is not a range where a power law between $N$ and $l$ can be delineated. This shows that a fast hardening process does not seem to be appropriate for earthquake occurrences. Hence, the value of the increasing ratio ($\gamma$) is taken to be 1 for all cases below. When the increasing ratio is taken to be a fixed value, the two main parameters specifying a stepwise linear velocity-weakening friction law are $r$ and $g$. There is an extreme case of velocity-weakening friction for which the value of $r$ is $\infty$. Such a friction law shows a sudden drop in the frictional force from static to dynamic, and is the classic static/dynamic friction law. Wang (1995a, b) stated that such a friction law as well as a law with large $r$ is more appropriate for generating self-similar earthquakes than that with small $r$. For the purpose of comparison, velocity-weakening friction with $r=1$ is also taken into account. The plots of log $N$ versus log$(l)$ for the two values of $r$ when $s=100$, $D=1.5$, $g=0.8$, and $F_{\text{max}}=5$ force units are shown in Fig. 6. For $r=1$, the value of log $N$ decreases somewhat exponentially with increasing log$(l)$, and unlike those

![Figure 6](image_url)

**Fig. 6.** The plots of log $N$ versus log$(l)$. Pluses for $r=\infty$ and open circles for $r=1$ when $s=100$, $D=1.5$, $g=0.8$, and $F_{\text{max}}=5$ force units. Two solid lines represent the lines with a slope value of $-1$ and $-1.5$, respectively.
for $r = \infty$ the data points do not follow a power-law distribution in a particular range of length.

The friction-drop ratio ($g$) is directly related to the drop in the frictional force on a mass element after it starts to slide. Two values of $g$ (i.e., 0.6 and 0.8) are taken to study the effect on the FL relation due to the change of $g$. The values of the frictional force drop related to $g = 0.6$ and $g = 0.8$ are $0.4F_o$ and $0.2F_o$, respectively. The plots of log $N$ versus log($l$) for $g = 0.6$ (open circles) and for $g = 0.8$ (pluses) when $s = 100$, $D = 1.5$, $r = \infty$, and $F_{o \text{max}} = 5$ force units are shown in Fig. 7. The largest length of events for $g = 0.6$ is greater than that for $g = 0.8$. The number of greater events generated in a fixed computational time interval is larger for $g = 0.6$ than for $g = 0.8$. The size of the largest event for $g = 0.6$ is greater than that for $g = 0.8$. The range of data points with a power-law $N$-$l$ relation for $g = 0.6$ and $g = 0.8$ is from $l = 4$ to $l = 32$ length units. However, the scaling exponent of the FL relation for $g = 0.6$ is about 1 and smaller than that (=1.5) for $g = 0.8$. Since Wang (1995a) stated that larger $g$ (now 0.8) can produce a better-distributed Gutenberg-Richter-type frequency-magnitude relation than smaller $g$ (now 0.6), only $g = 0.8$ is considered in the following study. In addition, included also in Fig. 7 are the data points for $F_{o \text{max}} = 10$ force units when $s = 100$.

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Fig. 7. The plots of log $N$ versus log($l$). Pluses for $g = 0.8$ and $F_{o \text{max}} = 5$ force units; open circles for $g = 0.6$ and $F_{o \text{max}} = 5$ force units; and asterisks for $g = 0.8$ and $F_{o \text{max}} = 10$ force units when $s = 100$, $D = 1.5$ and $r = \infty$. Two solid lines represent the lines with a slope value of $-1$ and $-1.5$, respectively.
Fig. 8. The plots of log N versus log(l). Pluses for $s=100$, crosses for $s=80$, open circles for $s=50$, asterisks for $s=20$, and open squares for $s=10$ when $D=1.5$, $r=\infty$, $g=0.8$, and $F_{\text{omax}}=5$ force units. Two solid lines represent the lines with a slope value of $-1$ and $-1.5$, respectively.

$D=1.5$, $r=\infty$, and $g=0.8$. It is interesting that the distribution of data points for $F_{\text{omax}}=10$ force units and $g=0.8$ is very similar to that for $F_{\text{omax}}=5$ force units and $g=0.6$, except for the matter that the number of small events for the former is less than that for the latter. The scaling exponent value of the FL relation for $F_{\text{omax}}=10$ force units in the range from $l_1=4$ to $l_3=32$ length units is also about 1.

The plots of log $N$ versus log(l) for $s=10$, 20, 50, 80, and 100 are shown in Fig. 8. The distribution of data points for small $s$ is remarkably different from that for large $s$. When the value of $s$ is smaller than 50, no range of self-similar or localized events as mentioned above can be delineated. When the value of $s$ is 80 or 100, there is a range specified with a power-law function between frequency and rupture length.

4. Discussion

Simulation relations of frequency versus rupture length for several model parameters are shown in Figs. 4–8. Although the range to show a power-law relation between frequency and rupture length spans only one order or one-and-one-half orders of magnitude.
magnitude in length, the simulation results seem to be comparable with the observed results. Hence, the simulation results can be used for further discussion. Figure 4 shows three kinds of events in three ranges of length: small events with \( l < l_2 \) length units, intermediate ones with \( l \) in the range from \( l_2 \) to \( l_3 \), and large ones with \( l > l_3 \). The value of \( l_2 \) is about 20 length units for \( D = 1.1 \) and 32 length units for \( D = 1.5 \). The scaling exponent value of the power-law function for intermediate events is about 1.5; while that for small and large events is about 5 and 8, respectively. Either of the two values for small events is larger than the observed results. The scaling exponent value (=1.5) for intermediate events seems to be comparable with the observed results as mentioned above. Hence, only the scaling relation of intermediate events is considered to be significant, and intermediate events are assumed to be self-similar and are called localized events in this study. The similarity of the seismicity patterns and FL relations, especially for localized events, for \( D = 1.1 \) and \( D = 1.5 \) (as displayed in Fig. 4) shows that the fractal variation in the breaking strengths is a minor factor in affecting the FL scaling relation.

Figure 5 shows the plots log \( N \) versus log \( l \) for \( \gamma = 1 \) and \( \gamma = 5 \) when other parameters are fixed. It can be seen that for \( \gamma = 5 \), the value of log \( N \) decreases more or less exponentially with increasing log \( l \), and no localized events can be recognized. On the other hand, for \( \gamma = 1 \), there is a range, where a power-law relation between \( N \) and \( l \) can be recognized. The results indicate that a slow velocity-hardening friction law is more appropriate for generating localized earthquakes than a fast one.

Figure 6 shows the plots log \( N \) versus log \( l \) for \( r = \infty \) and \( r = 1 \) when other parameters are fixed. It can be seen that for \( r = 1 \), the value of log \( N \) decreases more or less exponentially with increasing log \( l \), and no localized events can be recognized. This indicates that velocity-weakening friction with \( r = \infty \) is more appropriate for generating localized earthquakes than that with \( r = 1 \). This result is similar to that obtained by Wang (1995a) for the frequency-magnitude relation and by Wang (1995b) for the rupture length-earthquake moment relation. There are at least two mechanisms to produce classic static/dynamic friction (i.e., \( r = \infty \)) and friction with a rapid decrease (i.e., large \( r \)) in frictional force from static to dynamic might be due to numerous possibilities. One of them is due to the existence of pore fluids (Sibson, 1973), and another is caused by the normal modes of ruptures (Brune et al., 1993). An increase in the pore pressure mainly reduces the shear stress along the fault plane. Although this causes a decrease in the frictional force, it does not necessarily cause a sudden drop in it. On the other hand, the normal modes along the fault plane could cause a sudden drop in friction. Evidence obtained by Heaton (1990) seem to support Brune’s normal mode faulting model. Nevertheless, Dieterich and Linker (1992) disputed this conclusion. However, the results by Wang (1995a, b) and the present simulation results seem to prefer Brune’s proposition.

Figure 7 shows that the length range of localized events for \( g = 0.6 \) is similar to that for \( g = 0.8 \) when other parameters are fixed. This is quite different from the fact that the magnitude range of \( g = 0.6 \) for the frequency-magnitude relation is smaller than that for \( g = 0.8 \) (Wang, 1995a). This might indicate that a drop in friction causes a larger effect on the energy release than on the number of connected slide mass elements for an event. In addition, Fig. 7 also displays that the scaling exponent value of localized
events is 1.5 for $g=0.8$ and 1 for $g=0.6$. In other words, the scaling law between $N$ and $l$ is in the form $N \sim l^{-3/2}$ for $g=0.8$ and $N \sim l^{-1}$ for $g=0.6$. Since the scaling exponent values of the two values of $g$ are both in the range of observed values, it is difficult to assure which one is more responsible for earthquake occurrences than the other. Figure 7 also shows that the plot of cumulative frequency versus rupture length for $F_{\text{omax}}=10$ force units and $g=0.8$ is very similar to that for $F_{\text{omax}}=5$ force units and $g=0.6$, yet different from that for $F_{\text{omax}}=5$ force units and $g=0.8$. The frictional drop $(F_o-gF_o)$ is 2 force units for the former two cases and 1 force unit for the third case. This indicates that the significant factor in affecting the FL relation is the amount of frictional drop rather than the level of the breaking strength. Besides, Wang (1995b) stressed that larger $g$ (with a smaller frictional drop) is more appropriate for generating localized earthquakes, which show a well-distributed frequency-magnitude relation, than smaller $g$ (with a larger frictional drop). However, although various values of $g$ yield different scaling exponent values in the regime of localized events, they result in similar patterns of cumulative frequency versus rupture length in the whole length range. Hence, the FL relation is not so sensitive to the frictional drop as the frequency-magnitude relation.

Figure 8 shows the plots of $\log N$ versus $\log(l)$ for $s=10, 20, 50, 80$, and 100 when other parameters are fixed. The data points do not show a linear distribution in the entire length range. It is obvious that localized events can be recognized only as the value of $s$ is increased. Localized events appear in the range from $l=4$ to $l=32$ length unit when $s=100$. For comparison, the plots of $\log N$ versus $l$ for the five values of $s$ are shown in Fig. 9. It is obvious that the largest number of mass elements of an event is about 8 for $s=10$ and 20 and almost 17 for $s=50$. On the other hand, the largest number of mass elements of an event is about 34 when $s=80$ and 100. It is remarkable that large $s$ is more capable of making a larger number of connected mass elements slide almost simultaneously than small $s$. In other words, it is easier to produce larger-sized events for large $s$ than for small $s$. Like the conclusion made by Wang (1994, 1995a), this indicates that large $s$ is more appropriate for generating localized events than small $s$.

The comparison of the present simulation results through a 1-D model with the experimental results based on 2-D physical models obtained by Cowie et al. (1993) and Sornette et al. (1993), as well as the field data by several authors (Davy, 1993; Davy et al., 1990, 1995; Marrett, 1994; Marrett and Allmendinger, 1991; Scholz and Cowie, 1990; Scholz et al., 1993; Sornette et al., 1990, 1993), can lead to a fact. It is that the distributions of cumulative frequency versus rupture length of synthesized earthquakes are similar to those of experimental and field data in the regime of intermediate and large events. However, there is a remarkable difference in the distributions of small events. For the field and experimental results, the pattern becomes flat; while for the simulation ones, it still becomes larger. This difference might be due to either the low capability of 1-D numerical models to interpret the 2-D results or an underestimate of frequency in laboratory experiments and field surveys because of problems in detecting small faults. The rupture duration of small, simulated events is generally small. These events would be yielded mainly in the nucleation processes. Thus, for the nucleation events, a power-law frequency-length scaling relation might not exist.
The Gamma function is more appropriate to describe the measured FL distribution of the San Andreas fault system than others. The estimated characteristic length ($l_o$) in the Gamma function ranges from 22 to 24 km and is close to the thickness of the brittle crust in California. Davy (1993) also inferred that Poissonian segmentation is a crustal process. Figure 8 displays that the FL distribution for small $s$ seems unable to be described by a power-law or a lognormal function. Figure 9 obviously shows that the distributions cannot be described by an exponential function. For the exponential function (i.e., $N(l) \sim \exp(-l/l_o)$), the initial slope must be related to $-1/l_o$. From Fig. 9, it can be seen that the initial slope values for small lengths are almost the same for all cases, and the whole patterns of the five distributions are quite different. This indicates that the exponential function would not be a good candidate for describing the simulated distributions for various values of $s$. Figure 9 also shows that the distribution for small $s$ seems able to be described as a Gamma function. In principle, it is able to estimate characteristic length $l_o$ from the simulation results. Based on the results shown in Davy (1993), the value of $l_o$ in the present distribution must be several length units rather than one unit. From the field data, characteristic length $l_o$ is close to the thickness of the brittle crust in California.
the brittle crust (Davy, 1993). However, the present simulations were based on a 1-D model, for which the thickness of the brittle crust is not included, and thus, the length of \( L_0 \) estimated from the simulation results cannot display such a geological character. Although the present results for small values of \( s \) are similar to the observed results obtained by Davy, it is not appropriate to make a conclusion that small \( s \) is more responsible for earthquake occurrences than large \( s \). Based on the power-law scaling relation between frequency and length from the field work done by the above-mentioned authors, the present results and the fact that there is a better fit of the frequency-magnitude distribution in a wide magnitude range for large \( s \) than for small \( s \) from the dynamical simulations by Wang (1995a), large \( s \) seems to be more appropriate than small \( s \) to produce a power-law distribution of the population of faults.

In addition, the FL distribution shown in Fig. 9 is very similar to the measured size distribution of radial cracks caused by the impact of steel sphere on a ceramic plate (Curran et al., 1987). The trend of variation of the pattern with \( s \) in Fig. 9 is also very similar to that with the plastic impression radius, which indicates the degree of indentation caused by the sphere, and hence is a measure of the stress applied to the sample. Henderson et al. (1994) suggested a cellular automaton model to approach the experimental results by Curran et al. (1987). Their size distribution of clusters of failed elements is very similar to that in Fig. 8. Their results also show that the number of small events increases with decreasing size and does not become constant when the size is less than a certain value. It is noted that, at present, we do not know if the mechanism used by them is equivalent to that considered in this study. Knopoff et al. (1973) first tried to approach the crack dynamics based on the 1-D BK model. Although they reported some significant results, it does not seem that an equivalence between the two models has made by them. Hence, it is difficult to directly relate the results from one model to those based on the other. Nevertheless, given results seem to indicate that there is something similar between the two models, even though they are not completely equivalent.

5. Conclusions

Simulation results show that the fractal dimension (\( D \)) of the distribution of the breaking strengths is a minor parameter affecting the scaling of FL distribution, especially for small and intermediate events. Nevertheless, larger \( D \) can produce longer events than smaller \( D \). The decreasing rate (\( r \)) and the increasing rate (\( \gamma \)) of the dynamic frictional force with sliding velocity and the friction drop ratio (\( g \)) of velocity-weakening friction are three parameters significantly affecting the scaling of the FL relation. Large \( r \) and small \( \gamma \) are more appropriate for generating a power-law FL relation for simulated events than small \( r \) and large \( \gamma \). Different values of \( g \) result in different scaling exponents of the FL relations in almost the same length range. Smaller \( g \) (with a larger frictional drop) can lead to a smaller scaling exponent value than larger \( g \) (with a smaller frictional drop). When other model parameters are fixed, the scaling exponent value is 1.5 for \( g=0.8 \) and 1.0 for \( g=0.6 \). In addition, the frictional drop rather than the level of the breaking strength (i.e., static friction) affects the FL distribution. The stiffness ratio (\( s \)) is also a significant parameter influencing the FL distribution. However, small \( s \) cannot
result in a power-law scaling relation of frequency versus rupture length.

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