Comparison of Q_C^{-1} Estimates from Coda Envelopes Constructed from One- and Multi-Component Seismograms

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S-wave coda is a useful tool to investigate high-frequency seismic wave attenuation in the lithosphere. The generation and amplitude decay of S-wave coda were introduced by a single scattering model proposed by Aki and Chouet (1975). The amplitude decay rate with time is defined by the quality factor of the S-wave coda (Q_C^{-1}), which is investigated widely at various regions by the single scattering model.

It is only one-component seismograms that have been used to estimate Q_C^{-1} in most studies. This is based on the *assumption* that the decay of the S-wave coda amplitude recorded on a one-component seismometer is equivalent to the decay of the S-wave coda amplitude which is introduced by the single scattering model. In the single scattering model, S-wave coda is represented by the superposition of single scattered waves which are generated by scatterers distributed randomly and come from any direction to the station. We hereafter call the S-wave coda amplitude expected by the single scattering model the “*real coda amplitude*.” The ground motion of the S-wave coda portion, which consists of the superposition of the scattered waves, has various directions with time. Since the amplitude of the S-wave coda part on a one-component seismogram represents only the projection of the real coda amplitude in the direction of the component, the amplitude on the seismogram depends on the direction of the ground motion. Therefore, it is not trivial that the amplitude decay of a one-component seismogram for a coda part gives the decay of the real coda amplitude (i.e., the *assumption*). If we have three-component seismograms, we can estimate the real coda amplitude more exactly by vector addition of the amplitudes of the three-component seismograms (hereafter “*total vector-amplitude*”). In this report, we will check the *assumption* using real three-component seismic data.

We estimate Q_C^{-1} from a total vector-amplitude and amplitudes of one-component seismograms, and compare them to investigate Q_C^{-1} differences among these amplitudes. Here, we adopt 5 kinds of amplitudes to estimate Q_C^{-1} as a one-component amplitude: three of them are the amplitudes of each original component and the other two are the amplitudes of the radial and transverse components, which are constructed by horizontal two-component amplitudes. Furthermore, we produce the *horizontal vector-amplitude*, by vector addition of the amplitudes for, horizontal two-component seismograms, and measure Q_C^{-1} . This Q_C^{-1} is also compared with the Q_C^{-1} measured from one-component amplitudes or total vector-amplitude.

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Now, we refer to the data used in this analysis. We use seismograms recorded by three-component velocity type seismometers. Each seismometer records up-down (UD), north-south (NS) and east-west (EW) components of the ground motion. The events analyzed in this report are 78 local ones with magnitudes ranging from 3.0 to 4.6, which occurred in the period from January 1985 to December 1990 in the Hidaka area of southern Hokkaido. The seismograms were collected at the seismic stations of the Research Center for Earthquake Prediction (RCEP) of Hokkaido University by short-period (2 Hz) velocity seismometers with digital recording at 46 samples per second. In Fig. 1, we show the locations of the four stations and the epicenters of the events used for this study. The epicenters were determined by RCEP.

The amplitudes of the radial and transverse components, and horizontal and total vector-amplitudes are constructed from the three-component seismograms. These amplitudes are given by

$$A_R = A_N \cos \theta + A_E \sin \theta \quad (1)$$

Fig. 1. A map of epicentral distribution of the events used in this report (determined by RCEP). Symbols plotted by square, circle, cross, and diamond represent the epicenters analyzed at KMU, MUJ, ERM, and MYR stations, respectively. These stations are also plotted by the solid triangles with station codes.

for the radial component,

$$A_T = -A_N \sin \theta + A_E \cos \theta \quad (2)$$

for the transverse component,

$$A_H = \sqrt{A_R^2 + A_T^2} \quad (3)$$

for the horizontal vector-amplitude and

$$A_t = \sqrt{A_Z^2 + A_R^2 + A_T^2} \quad (4)$$

for the total vector-amplitude; where A_Z , A_N , and A_E represent the amplitudes of the UD, NS, and EW components, respectively. Here, θ is an azimuthal angle defined by the angle between vectors pointed to the north and to the station measured clockwise from the epicenter. The azimuthal angle (θ) is necessary to estimate the amplitudes of the radial and transverse components in Eqs. (1) and (2). For estimating the azimuthal angle, we use the epicentral location of each event determined by RCEP. On the other hand, for the horizontal and total vector-amplitudes, since the relation

$$A_R^2 + A_T^2 = A_E^2 + A_N^2 \quad (5)$$

holds, we can estimate these amplitudes without estimation of the azimuthal angles. We calculate A_H and A_t by use of Eq. (5).

We use the single isotropic scattering model extended by Sato (1977) for the configuration that a station and hypocenter are at different places. Based on this model, the logarithm of the coda amplitude is represented in terms of the frequency and lapse time measured from the event origin time as

$$\begin{aligned} \ln[A(\omega, t)r/\sqrt{K(V_S t/r)}] &= \ln(C(\omega)) - (\omega/2Q_C)t \\ K(x) &= (1/x) \ln[(x+1)/(x-1)] \quad \text{for } x > 1; \end{aligned} \quad (6)$$

where $A(\omega, t)$, ω , t , r , V_S and $C(\omega)$ are bandpass-filtered coda amplitude, angular frequency, lapse time, source-receiver distance, S-wave velocity, and the term related to source, respectively. $A(\omega, t)$ for the UD, NS, and EW components are estimated by filtering the original seismograms for a narrow frequency band centered at 1.0, 2.0, 4.0, 8.0, and 16.0 Hz with a bandwidth of an octave, and calculating RMS (the root mean square) on a temporary window of 4 cycles which shifts at steps of 1 cycle throughout the S-wave coda at each frequency band. For the radial and transverse components, after constructing a seismogram for each component by Eqs. (1) or (2), $A(\omega, t)$ is calculated by RMS mentioned above. For the $A(\omega, t)$ of horizontal and total vector-amplitudes, we apply a bandpass filter to the original three-component seismograms, estimate each amplitude by Eqs. (3), (4), and (5), and calculate $A(\omega, t)$ by RMS. After all, we get 7 values of Q_C^{-1} for each event at each station, which are estimated from the amplitudes of the UD, NS, EW, radial and transverse components, and horizontal and total vector-amplitudes.

We consider two cases for the time-window selection to estimate Q_C^{-1} . In the first case, we choose the time window from twice the S-wave arrival time to the time when

amplitude decays to three times the noise level for each frequency range. This time-window regime is widely used in the study of Q_C^{-1} analysis (Aki and Chouet, 1975; Tsujiura, 1978; Sato *et al.*, 1988). In this study, the noise level is determined from the amplitude for 5 s before the P-wave arrival. In the second case, the time window is fixed at a lapse time measured from event origin time for all events, taking into account Q_C^{-1} dependence on the lapse time. The details for this time-window selection for each station will be mentioned in the following.

Figure 2 shows the envelopes of S-wave coda given by the term of the left-hand side of Eq. (6), which have been constructed from 5 kinds of one-component amplitudes, and horizontal and total vector-amplitudes, respectively. We can see that the envelope fluctuations of the horizontal and total vector-amplitudes are smaller than those of individual one-component amplitude, and that the envelopes for the total vector-

Fig. 2. Example of envelopes of S-wave coda given by the term of the left-hand side of Eq. (6) for an event observed at KMU. The envelopes are represented for five frequency bands (1.0, 2.0, 4.0, 8.0, and 16.0 Hz). There are seven traces for each frequency band, which are the envelopes of the UD, NS, EW, radial, and transverse components, and horizontal and total vector-amplitudes from top to bottom. Each trace is normalized by the corresponding maximum value of the envelope of the total vector-amplitude for each frequency band, and shifted along the vertical axis. The lapse time measured from the event origin time is shown at the bottom of the figure. One should notice that the envelope fluctuations of the horizontal and total vector-amplitudes are smaller than those of the amplitude for any one-component seismogram. Especially, the fluctuation of the total vector-amplitude is the smallest among all the kinds of amplitudes at all frequency ranges.

amplitude decay smoothly as expected by the single scattering model. Since Q_C^{-1} can be evaluated by fitting the right-hand side of Eq. (6) to the envelopes shown in Fig. 2, we expect that the coda envelopes measured from the total vector-amplitudes give more stable Q_C^{-1} (i.e., the standard deviation of Q_C^{-1} is smaller) than those measured from any other amplitude.

Figure 3 shows Q_C^{-1} estimated from all events for each kind of amplitude at KMU where the number of analyzed events is the largest among all stations. Here the time window has been taken by the first case of time-window selection. The end of the time window for the horizontal or total vector-amplitude has been decided by the time when the RMS amplitude for individual (horizontal or total) bandpass-filtered amplitude of coda part decays to three times the RMS amplitude for individual bandpass-filtered amplitude before P-wave arrival. For the same event, the time windows for all kinds of amplitudes were almost the same after all. Figure 3 indicates that Q_C^{-1} estimated from the total vector-amplitudes tends to be less scattered than that from only one-component amplitudes. The maximum value of Q_C^{-1} is, at most, about twice the minimum value of Q_C^{-1} for the total vector-amplitude. On the contrary, the maximum value is always greater than twice the minimum value for some one-component amplitudes.

Fig. 3. Distribution of the estimated Q_C^{-1} for the five frequency ranges at KMU station. The labels from A to G along the horizontal axis correspond to the amplitudes of the UD, NS, EW, radial, and transverse components, and the horizontal and total vector-amplitudes, respectively. Each symbol represents the Q_C^{-1} value estimated from each event. The time window has been defined by first time-window selection (see text). One should notice that the estimated Q_C^{-1} for the total vector-amplitude tends to be less scattered than that for any other amplitude, which was also observed at the other stations.

Table 1. Ratios of the standard deviations of Q_C^{-1} to the estimated Q_C^{-1} averaged over all events and stations. The unit is percent. "Horizontal" and "Total" in the first column mean the horizontal and total vector-amplitudes, respectively.

	1.0 Hz	2.0 Hz	4.0 Hz	8.0 Hz	16.0 Hz
UD	13.1	8.8	5.2	3.5	2.3
NS	14.2	8.1	5.2	3.2	2.2
EW	14.3	7.5	5.4	3.0	2.0
Radial	15.5	8.1	5.1	3.2	2.1
Transverse	14.4	8.3	5.8	3.3	2.2
Horizontal	10.2	7.1	4.1	2.2	1.4
Total	9.3	6.2	3.2	2.1	1.2

Comparing Q_C^{-1} estimated from the 5 kinds of one-component amplitudes, there are differences in the distribution among them. Such differences in the distribution of the estimated Q_C^{-1} for different kinds of amplitudes were also found at the other stations.

To check the stability of the estimated Q_C^{-1} from every event for each kind of amplitude, we calculate the fraction of the standard deviation in estimated Q_C^{-1} for every event, and compare the fractions among all kinds of amplitude. The fraction is estimated as follows. First, we measure the ratio of the standard deviation of Q_C^{-1} to the estimated value of Q_C^{-1} for each event in each kind of amplitude. Next, we average the ratios over all events for each kind of amplitude. These averaged ratios may represent the goodness of the model fitting for each kind of amplitude. The results are shown in Table 1. It shows that the ratios for the vector-amplitude are much smaller than those for the 5 kinds of one-component amplitudes for all frequency ranges. In some cases, the ratio for the total vector-amplitude is approximately only half the ratios for the one-component amplitudes. This result shows that, for every event, Q_C^{-1} given by the total vector-amplitude is more stable than that by any other amplitude. It also means the coda envelopes obtained from the one-component amplitudes suffer from much more fluctuation than that from the total vector-amplitude. The ratios of the standard deviations of Q_C^{-1} to the values of Q_C^{-1} estimated from the horizontal vector-amplitudes are also shown in Table 1. They are larger than those for the total vector-amplitude, but smaller than those for the one-component amplitudes. In other words, the envelope fluctuation can be reduced much more by the use of the horizontal vector-amplitude than by the use of any one-component amplitude. However, the fluctuation is still larger than that for the total vector-amplitude. Thus, we recognize that the envelope fluctuation for the total vector-amplitude is the smallest among that for all of the amplitudes used in this study.

When we evaluate Q_C^{-1} in any region, it is usual to average Q_C^{-1} over many events. We estimate the mean value of Q_C^{-1} , μ , by the following equation:

$$\mu = \frac{\sum_{i=1}^N w_i x_i}{\sum_{i=1}^N w_i}, \quad (7)$$

and the standard deviation of Q_C^{-1} , σ , by

$$\sigma = \sqrt{\frac{\sum_{i=1}^N w_i (x_i - \mu)^2}{\sum_{i=1}^N w_i} \times \frac{N}{N-1}}; \quad (8)$$

where N is the number of samples to be averaged. x_i and w_i represent

$$x_i = (Q_C^{-1})_i$$

and

$$w_i = \frac{1}{\sigma_i^2};$$

where $(Q_C^{-1})_i$ and σ_i are the value of Q_C^{-1} and standard deviation of Q_C^{-1} estimated from the seismic records of the i -th event, respectively. The result is shown in Fig. 4. From this figure, we find the differences of the mean Q_C^{-1} among all kinds of amplitudes are small. The difference between the largest and smallest mean values of Q_C^{-1} among the 7 kinds of amplitudes is much less than a tenth of the estimated mean value of Q_C^{-1} in each frequency band. For example at KMU, the orders of the differences are 10^{-4} at 1.0 and 2.0 Hz bands, and 10^{-5} at the other frequency bands. These are less than 10% of the estimated mean values of Q_C^{-1} for any kind of amplitude. On the other hand, the standard deviations of Q_C^{-1} range between 10 and 30% of the mean values of Q_C^{-1} . For the other stations, the results are almost the same as that for KMU. Therefore, in the first case of time-window selection, we can conclude that the values of Q_C^{-1} estimated from different kinds of amplitudes are almost equal within the standard deviations.

Comparing the magnitudes of the standard deviations of Q_C^{-1} estimated from the 5 kinds of one-component amplitudes, they are different depending on which kind of amplitude we select to estimate them. Moreover, if we select the component for which the standard deviation is the smallest among the 5 kinds of one-component amplitudes, the selected component differs with frequency. For example at MYR, the component giving the smallest standard deviation is the radial component (labeled with D in Fig. 4) for 1.0 Hz band, EW component (labeled with C) for 2.0 and 16.0 Hz bands, and UD component (labeled with A) for 4.0 and 8.0 Hz bands. This means that if we use a one-component amplitude to estimate Q_C^{-1} , the accuracy of the estimated Q_C^{-1} value depends on which kind of amplitude we select among the 5 kinds of one-component amplitudes. On the contrary, the standard deviation for the total vector-amplitude is almost the same or less than the smallest one among the 5 kinds of one-component

Fig. 4. Mean Q_c^{-1} and the standard deviations estimated by Eqs. (7) and (8) at the four stations. The labels from A to G along the horizontal axis are described in the caption to Fig. 3. The time window has been defined by first time-window selection. Note that although the estimated Q_c^{-1} among all the kinds of amplitudes are almost the same within the standard deviations, the standard deviation for the total vector-amplitude is smaller than that for any other amplitude.

amplitudes for all frequency bands. As a result, Q_c^{-1} measurement using the total vector-amplitude is the most reliable for all frequency bands.

Next, we attend to Q_c^{-1} measured by the second case of time-window selection. In the first case, the time window for Q_c^{-1} measurements was different for every event. There are some studies which show the selection of time window can affect the estimated Q_c^{-1} value. Some researchers found that Q_c^{-1} estimated from an early part of the coda are greater than the Q_c^{-1} from the later part (Rautian and Khalturin, 1978; Roecker *et al.*, 1982; Pulli, 1984; Kosuga, 1992). The same phenomenon can be also predicted numerically, since the later part of the coda is more affected by multiple scattering than the early part (Gao *et al.*, 1983; Hoshiba, 1991). In order to restrict the effects of time-window selection on the estimation of Q_c^{-1} as much as possible, we now measure

Q_C^{-1} with a common time window for all events (i.e., the estimation of Q_C^{-1} by the second time-window selection).

A time window common to all records at each station is selected so that the largest number of events in Fig. 1 could be analyzed under the condition that the coda portion used to estimate Q_C^{-1} could be taken for at least 20 s. As a result, the selected time windows for Q_C^{-1} measurements were from 25 to 45 s measured from the event origin time at KMU and from 20 to 40 s at MUJ. Since a small number of events remained if we selected the common time window for all events at ERM and MYR, we did not evaluate Q_C^{-1} at these stations. The result at KMU is plotted in Fig. 5. The scatter of measured values of Q_C^{-1} for the events is less for the total or horizontal vector-amplitude than for the one-component amplitudes as well as for the first case of time-window selection. The averaged Q_C^{-1} over all events and standard deviations of Q_C^{-1} at each station are shown in Fig. 6. This figure shows that Q_C^{-1} for the one-component amplitudes and total vector-amplitude are almost the same within the standard deviations again. However, the standard deviation for the total vector-amplitude is the smallest among any kind of amplitude in most frequency bands, and it is from half to two-thirds that of the largest one among the 5 kinds of one-component amplitudes. It was found that total vector-amplitude can give more stable Q_C^{-1} than any one-component amplitude for the second case of time-window selection as well as the first case.

Fig. 5. Distribution of the estimated Q_C^{-1} for the five frequency ranges at KMU station. The labels from A to G along the horizontal axis are described in the caption to Fig. 3. The time window used for the Q_C^{-1} estimation is from 25 to 45 s measured from the event origin time for all events. One should notice that the estimated Q_C^{-1} for the total vector-amplitude tends to be less scattered than that for any other amplitude, as can be seen in Fig. 3.

Fig. 6. Mean Q_C^{-1} and standard deviations estimated by Eqs. (7) and (8) at stations KMU and MUJ. The labels from A to G along the horizontal axis are described in the caption to Fig. 3. The time windows used for the Q_C^{-1} estimation are from 25 to 45 s for KMU and 20 to 40 s for MUJ measured from the event origin time. Note that although the estimated Q_C^{-1} among all the kinds of amplitudes are almost the same within the standard deviations, the standard deviation for the total vector-amplitude is the smallest among them in most frequency bands.

Comparing the distribution of Q_C^{-1} for each kind of amplitude in the second case of time-window selection with that in the first case (Figs. 3 and 5), the former is about twice larger than the latter. This fact is seen even for the total and horizontal vector-amplitudes. We should notice that time-window selection is important to evaluate stable Q_C^{-1} even if we use the coda envelope for the total vector-amplitude.

Now we discuss the result that the envelope fluctuation of the total vector-amplitude is smaller than that of individual one-component amplitude as mentioned before. Kennett (1993) has employed the total vector-amplitude to investigate the dependence of the amplitude of not S-wave coda but the Pn, Sn, and Lg phases as a function of range from the source to 700 km with a linear array for an explosion source in water.

He showed the amplitude behaviors for these phases, as a function of range, can be discerned in cartesian components but are clearer in vector representation, because the amplitude distribution versus epicentral distance in vector representation is less scattered. He pointed out, for this reason, that the amplitude given by the vector measurement does not depend on the incident direction of these phases, since its amplitude is invariant for the rotation of the coordinate. This means that even if we cannot have the knowledge of the azimuthal or dip angle of the incident wave (i.e., the knowledge of the direction of the motion of the incident wave), the amplitude of the incident wave can be measured correctly by the total vector-amplitude of three-component seismograms in the rectangular coordinate system. Turning now to S-wave coda, the amplitude can always be measured by total vector-amplitude more correctly than by one-component amplitude, since the motion of S-wave coda may have any direction with time. On the other hand, the horizontal vector- or one-component amplitude in a direction represents only the projection of the coda amplitude on the horizontal plane or direction. Hence, these kinds of amplitudes depend on not only the coda amplitude but also the direction of the motion of the coda waves. The apparent fluctuation resulting from the variation of the direction of the S-wave coda motion must be included in the envelopes of these kinds of amplitudes.

We now summarize the results made clear in this study.

(1) The coda envelopes obtained from the one-component amplitudes suffer from much more fluctuation than that from total-vector amplitude.

(2) There are more variations in Q_C^{-1} estimated from one-component amplitudes than those estimated from total vector-amplitudes.

(3) The mean values of Q_C^{-1} , which are obtained by averaging overall events at each station, for the horizontal and total vector-amplitudes and one-component amplitudes are almost the same within the standard deviations. However, the standard deviation for the total vector-amplitude is the smallest among all the kinds of amplitudes in most frequency bands.

The above results suggest that the result of Q_C^{-1} estimated from a one-component seismogram is reliable, however we should use total vector-amplitude to measure more stable and reliable Q_C^{-1} if three-component records are available.

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