Stability Estimation of ABWR on the Basis of Noise Analysis

Masahiro FURUYA**, Takanori FUKAHORI***, Shinya MIZOKAMI† and Jun YOKOYA††

** Central Research Institute of Electric Power Industry (CRIEPI)
2–11–1 Iwado-kita, Komae, Tokyo 201-8511, JAPAN
E-mail: furuya@criepi.denken.or.jp

*** Global Nuclear Fuel-Japan Co., Ltd. (GNF-J)
2–3–1 Uchikawa, Yokosuka, Kanagawa 239-0836, JAPAN
E-mail: Takanori.Fukahori@GNF.com

† Tokyo Electric Power Co., Inc (TEPCO)
1–1–3 Uchisaiwai-cho, Chiyoda, Tokyo 100-0011, JAPAN
E-mail: mizokami.shinya@tepco.co.jp

†† Electric Power Development Co., Ltd (J-POWER)
6–15–1 Ginza, Chuo-ku, Tokyo 104-8165, JAPAN
E-mail: Jun.Yokoya@jpower.co.jp

Abstract
In order to investigate the stability of a nuclear reactor core with an oxide mixture of uranium and plutonium (MOX) fuel installed, channel stability and regional stability tests were conducted with the SIRIUS-F facility. The SIRIUS-F facility was designed and constructed to provide a highly accurate simulation of thermal-hydraulic (channel) instabilities and coupled thermal-hydraulics-neutronics instabilities of the Advanced Boiling Water Reactors (ABWRs). A real-time simulation was performed by modal point kinetics of reactor neutronics and fuel-rod thermal conduction on the basis of a measured void fraction in a reactor core section of the facility.
A time series analysis was performed to calculate decay ratio and resonance frequency from a dominant pole of a transfer function by applying auto regressive (AR) methods to the time-series of the core inlet flow rate. Experiments were conducted with the SIRIUS-F facility, which simulates ABWR with MOX fuel installed. The variations in the decay ratio and resonance frequency among the five common AR methods are within 0.03 and 0.01 Hz, respectively. In this system, the appropriate decay ratio and resonance frequency can be estimated on the basis of the Yule-Walker method with the model order of 30.

Key words: ABWR, Void-Reactivity Feedback, Regional Stability, Channel Stability, AR Model, Time-Series Analysis

1. INTRODUCTION

In Boiling Water Reactors (BWRs), the reactor power is suppressed when the amount of steam bubbles (voids) increases in the core (void-reactivity feedback). The void-reactivity feedback is one of the inherent safety features of BWRs. The feedback may cause self-sustained oscillations depending on the core design and operating conditions, particularly in high-power and low-flow conditions. Consequently, BWRs must be designed and operated with a sufficient stability margin to suppress the oscillations.

There are three types of instabilities closely related to the two-phase flow in the core: (1) channel instability, (2) core-wide instability, and (3) regional instability. Channel instability is a thermal-hydraulic instability, known as density-wave oscillations. The core-wide and regional instabilities are related to thermal-hydraulics-neutronics coupling, i.e., void-reactivity...
The subcooled water enters at the bottom of the core. At the core exit, approximately 15% of the coolant mass is converted into steam. This steam-water mixture flows through the steam separators and dryers exiting the core. The steam exits the vessel and drives the turbines.

In core-wide instability, neutron flux oscillations are in-phase at all streamwise locations in the core. In regional instability, neutron flux on one side of the core fluctuates in the opposite phase to that on the other side. The observed regional instability in BWRs, neutron flux oscillation mode was a first azimuthal mode.

Early test programs at the Peach Bottom(1),(2) and the Vermont Yankee(3) reactors have provided a large quantity of data on core-wide stability and the onset of limit cycle oscillations in the natural circulation region of the power/flow map.

Regional oscillations in the out-of-phase mode were first seen at the Caorso plant in 1982. Subsequently, another test program was set up in October 1983(4) to study out-of-phase mode oscillations. The stability test performed during the startup of the Leibstadt plant in 1984 also resulted in regional oscillations under minimum pump speed conditions. A few occurrences of regional oscillations have been observed at other plants as well.

To analyze the stability, linear stability analysis is a useful tool for calculating the decay ratio, or damping coefficients, as a stability index. Most of the linear stability analysis codes have been validated with plant operation data in BWRs. These data are applied for licensing analysis to meet stability criteria. However, stability experiments have not explored the effects of system parameters such as core thermal power, flow rates, fuel types, void-reactivity feedback coefficients, or others. No test facility was available to simulate regional stability at that time. Consequently, the authors designed and constructed the SIRIUS-F facility so that they could build an experimental database on regional stability to validate analysis codes and confirm design confidence(5). To simulate regional stability in the SIRIUS-F facility using electric heater in place of the core in real plant, the void fraction is measured in a boiling two-phase loop of the SIRIUS-F facility, and is used for the artificial void-reactivity feedback to the power from the heaters that are installed in the thermal-hydraulic loop.

In recent years, some BWRs have been operating with mixture oxides of uranium and plutonium (MOX) fuels installed. The absolute value of the void-reactivity feedback coefficient of the core with MOX fuels installed becomes larger than that of the core when uranium dioxide fuels are used. Consequently, the decay ratio for the core with MOX fuels becomes
This study is intended to verify the noise analysis for the stability estimation of the core with MOX fuel installed and, thereby, to test the regional stability using the SIRIUS-F facility.

2. SIRIUS-F EXPERIMENTAL FACILITY

2.1. Design Concept

The SIRIUS-F facility is a full-height (13m-high) facility used for simulating channel, core-wide, and regional stabilities of the Advanced Boiling Water Reactor (ABWR). The facility has the following features:

1. Artificial neutronic feedback implementation

   Void-reactivity feedback is a crucial phenomenon in core-wide and regional instabilities. To simulate void-reactivity feedback, which does not exist in the test facilities, void fractions are measured over the core sections and, subsequently, used as input for real-time simulations of void-reactivity feedback and modal point kinetics. The power supplies are controlled in accordance with the simulation output.

2. Closed transfer function

   Figure 3 illustrates sets of transfer functions in terms of core-wide and regional stabilities. The upper part of the figure describes the closed transfer function of a BWR, including void-reactivity feedback. For instance: perturbations in void fractions feedback through void reactivity, generated power density, surface heat flux, and thermal-hydraulic transfer functions. To reproduce core-wide and regional stabilities, one must perform a real-time simulation of the void-reactivity feedback. This simulation requires void fractions over the core sections as an input and it controls the power supplies accordingly. The lower part of Figure 3 illustrates a block diagram representing the closed-loop transfer functions for the void-reactivity feedback. The fuel pellets in a BWR are made of ceramic, and the time constant, in terms of gap conductance and heat conduction in cladding and fuel pellets ranges from 4 to 8 s, depending upon the composition and diameter of the pellet, and the pellet-to-cladding gap conductance. In contrast, the typical time constant for a metallic heater, which is commonly used in the out-of-pile facility, is approximately 1 s. Therefore, two additional transfer functions should be implemented to cancel out these differences: the inversed transfer function of thermal conduction for the metallic heater and a transfer function
of thermal conduction for the target fuel. One must note that distortion of the set of transfer functions occurs upon installing measurement and controlling systems. This should be canceled out by including inversed transfer functions, as illustrated in Figure 3. Although the illustrated transfer functions comprise one single channel, neutron diffusion in space and the multichannel effect of thermal hydraulics should also be considered in a BWR configuration. In this study, two simulated fuel channels are used, representing two half-cylindrical cores, to demonstrate the fundamental mode, i.e., core-wide stability and the first-azimuthal mode, i.e., regional stability. Interactions between two simulated fuel channels are solved numerically to account for reactivity exchange between them\(^6\). A detailed formulation of the reactor neutronics is described in Ref. (7).

(3) \textbf{Real-time analysis for the artificial neutronic feedback} In this study, the void fraction in the core section was calculated from the differential pressure over the simulated fuel channel section, inlet velocity, and inlet temperature. The void fraction obtained was used for void-reactivity feedback to volumetric heat generation in the fuel rod. Void-reactivity feedback was analyzed by reactor neutronics, including eigenvalue separation of the higher-harmonic neutronic mode. The heat generated in the fuel was transferred to a coolant with a certain delay time. This delay, caused by thermal conduction in the fuel rod, can be formulated using point kinetics. Formulations of the real-time analysis are described in Ref. (7).

(4) \textbf{Fuel rod time constant} A verification test was conducted to obtain the fuel rod time constant of the heater rods used in the SIRIUS-F facility. The temperature response was acquired from thermocouples embedded in the heater rod surface. Figure 4 shows a typical response of heater surface temperature to a stepwise increase in heater power. The temperature response curves closely agreed with the first-order time delay function of 1.1 s, as shown in Figure 4. Therefore, a time constant of 1.1 s was assumed.

(5) \textbf{Time constant of measurement system} The differential pressure measurement system (DPMS) has a certain delay in its response to differential pressure changes. A high-response differential pressure measurement system was developed for the SIRIUS-F facility. Figure 5 shows the response of the DPMS to a stepwise increase in differential pressure. All 16 curves followed the first-order delay function of 0.19 s, as Figure 5 illustrates. Time delays of 0.19 s for the DPMS and 1.1 s for the heater rod were added to the fuel rod time constant in the real-time simulation. Therefore, the fuel rod time constant indicated as an experimental parameter, \(\tau_f\), included these time delays.

### 2.2. Boiling Two-Phase Flow Loop

Figure 2 is a schematic of the thermal-hydraulic loop of the SIRIUS-F facility. It is
Fig. 6  Axial pressure profile measured with the SIRIUS-F facility and analyzed on the basis of the ABWR design specification code. The measured pressure profiles agree well with those of the ABWR, because local pressure losses such as tie-plates and orifices, are accurately simulated in the SIRIUS-F facility.

Table 1  Experimental Conditions

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated thermal Power</td>
<td>3 926 MWt / 872 / 74 × 2</td>
</tr>
<tr>
<td>Rated core flow</td>
<td>145 000 kg/s / 872 / 74 × 2</td>
</tr>
<tr>
<td>Radial power distribution</td>
<td>1.0</td>
</tr>
<tr>
<td>Axial power distribution</td>
<td>1.70N3</td>
</tr>
<tr>
<td>λ-mode eigenvalue separation</td>
<td>0.00318</td>
</tr>
<tr>
<td>Target operating conditions</td>
<td>Natural circulation line</td>
</tr>
<tr>
<td></td>
<td>(21% flow rate)</td>
</tr>
<tr>
<td>Fuel</td>
<td></td>
</tr>
<tr>
<td>- Type</td>
<td>STEP-III (9×9 fuel) Type-A</td>
</tr>
<tr>
<td>- Active fuel length</td>
<td>3 708 mm</td>
</tr>
<tr>
<td>- Orifice loss coefficient</td>
<td>40K FABLE</td>
</tr>
<tr>
<td>- Spacer</td>
<td>Ferrule type</td>
</tr>
<tr>
<td>- Gap conductance</td>
<td>8 520 W/m²K</td>
</tr>
<tr>
<td>- Fuel rod time constant</td>
<td>3.0 s</td>
</tr>
<tr>
<td>Void-reactivity coefficient</td>
<td>0.093 $ at 40 % void fraction</td>
</tr>
<tr>
<td>Total $β$</td>
<td>0.0043</td>
</tr>
</tbody>
</table>

a full-height facility (13 m high). The thermal-hydraulic loop consists of a lower plenum, two simulated fuel channels, a bypass channel, an upper plenum separator, a downcomer, a subcooler, and a preheater. A heater pin is installed concentrically in each simulated fuel channel. The heated length, \( l_c \), is 3708 mm, which is the same as the fuel rod in the ABWR. The SIRIUS-F facility can be operated up to 9 MPa. The water working fluid is passed through an ion exchange resin.

Because of the importance of the dimensions and local pressure losses of orifices and tie plates in boiling two-phase flow phenomena, the seven spacers and two tie plates were inserted at the same elevation, having the same local pressure loss coefficients as commercial fuel, called STEP-III (9×9 fuel) Type-A. Local pressure-loss coefficients of these spacers and tie plates were determined in a single-phase forced circulation experiment for various flow rates.

Figure 6 shows the axial pressure profile measured with the SIRIUS-F facility and analyzed on the basis of the design specification code for the ABWR. Eight differential pressure measurement systems were installed in each simulated fuel channel. Each differential pressure measurement region corresponds to one local pressure loss, each with a lower tie plate and
seven spacers. The axial pressure distribution of the boiling two-phase flow measured with the SIRIUS-F facility agreed closely with that analyzed on the basis of the design specification code for the ABWR.

Eight Type-K thermocouples (0.5-mm OD) were embedded in, and silver brazed to, the surface of the sheath heater rod. The flow rate was measured using an orifice flowmeter attached to a downcomer. System pressure, $P_s$, refers to the pressure in the separator dome.

The decay ratios and the resonance frequency were estimated with the help of a noise analysis\(^7\). Artificial void-reactivity feedback control was initiated after the thermal-hydraulic conditions, i.e., subcooling and flow rate at the channel inlet, system pressure, and channel heat flux, were steady at the specified values. These boundary conditions can be maintained even when instabilities occur within a certain amplitude of the limit cycles because vapor volume in the separator and the volume of the preheater are sufficiently large.

### 2.3. Experimental Procedure

Experiments were conducted as follows:

1. Deaerate the thermal-hydraulic loop.
2. Heat the simulated fuel and bypass channels to induce boiling and natural circulation flow.
3. Start the recirculation pump when the system pressure is high enough to protect the pump from cavitation damage.
4. Pressurize the thermal-hydraulic loop at the specified system pressure by automatically controlling the heater outputs and the condenser performance.
5. Control the channel inlet temperatures, heat fluxes, and channel inlet flow rate by changing performance of the subcoolers and preheater.
6. After steady state conditions are confirmed for the boiling two-phase flow, activate the void-reactivity feedback system with the specified reactor physics constants and fuel time constants.
7. Acquire experimental data for four minutes.
8. Deactivate the void-reactivity feedback system. Wait until the heat flux reaches a constant value.
9. Repeat the experiments using the next set of boundary conditions, following the same procedure (6).

### 3. CONSTITUTIVE EQUATIONS OF REAL-TIME SIMULATION

#### 3.1. Thermal-Hydraulics

A locally measured void fraction has often been used for void-reactivity feedback simulation\(^8\). The phase and amplitude of the locally measured void fraction are, however, different from those required for the void-reactivity feedback model\(^9\). In this study, the void fraction in the core section was calculated from differential pressure over the channel section, inlet velocity, and inlet temperature. Assumptions for the analysis are as follows:

1. The flow is one-dimensional and steady-state. This assumption is valid only if the integration time in the acquisition system (10 ms) is sufficiently shorter than the flow oscillation period ($\tau_{fo} > 10$ s).
2. The following pressure losses are considered: acceleration, gravitation, wall friction, and local pressure losses.
3. Each measurement region is divided into a liquid single-phase flow part and a boiling two-phase flow part, separated by a boiling boundary.
4. Thermo-physical properties are referred to those under saturated conditions at the system pressure, except for the liquid density in the connecting pipes of the differential pressure measurement system (DPMS), which is determined by an experimental correlation.
5. Subcooled boiling is neglected, since the void fraction created by subcooled boiling is relatively small in this experimental range to determine the average void fraction.
The momentum equation is described as follows, thereby taking the above-mentioned assumptions into account.

\[
\frac{d}{dz} \left[ \frac{1}{2} \alpha \rho u_i^2 + \frac{1}{2} (1 - \alpha) \rho u_l^2 \right] + \frac{d}{dz} P + M_{\text{wall}} + M_{\text{local}} + \left[ n_p g + (1 - \alpha) \rho \right] g = 0
\]

(1)

\( \alpha \) is the void fraction, \( \rho \) the density, \( P \) the pressure and \( g \) the gravitational acceleration. The subscripts \( g \) and \( l \) denote vapor and liquid phases respectively. The terms on the left-hand side of the equation correspond to acceleration, total pressure, wall friction, local pressure losses, and gravitational losses respectively. The wall friction loss, \( M_{\text{wall}} \), is given as follows.

\[
M_{\text{wall}} = \begin{cases} \frac{\delta}{2} \varphi \rho u_{in}^2, & \text{if single-phase} \\ \frac{\delta}{2} \alpha \rho g u_l^2 + \frac{\delta}{2} (1 - \alpha) \rho u_l^2, & \text{if two-phase} \end{cases}
\]

(2)

The wall friction coefficient is estimated on the basis of Blasius’s correlation for liquid single-phase flow (\( f_{l1} \)), and Martinelli-Nelson-Jones’ correlation for two-phase flow (\( f_{2g} \)). The local pressure loss due to the orifices and changes of the flow area \( M_{\text{local}} \) is expressed as follows.

\[
M_{\text{local}} = \begin{cases} \sum \frac{\delta}{2} \alpha \rho g u_l^2 \delta (z - z_i), & \text{if single-phase} \\ \sum \left[ \frac{\delta}{2} \alpha \rho g u_l^2 + \frac{\delta}{2} \alpha \rho g u_l^2 \right] \delta (z - z_i), & \text{if two-phase} \end{cases}
\]

(3)

The locations and coefficients of the orifices and changes of the flow area are determined experimentally and summarized in a previous paper(10). Assuming thermal equilibrium conditions, the non-boiling length (distance between channel inlet and the boiling boundary) and the boiling length are expressed as

\[
Z_{1\theta} = \frac{\rho_l C_p \Delta T_{\text{sub},in} u_{in} A_c}{\pi Dq''},
\]

(4)

\[
Z_{2\theta} = l - Z_{1\theta}.
\]

(5)

The \( u_{in} \) denotes the velocity at the channel inlet, \( A_c \) is the flow area of the channel, \( D \) denotes the heater diameter and \( q'' \) refers to the heat flux, \( l \) is length of measurement region. The liquid and vapor velocities are determined by the drift flux model as a function of the void fraction. The differential pressure of each region, \( P_{ex} - P_{in} \), is related to the measured value of the DPMS, \( \Delta P_{\text{DPMS}} \) as follows:

\[
\Delta P_{\text{DPMS}} = P_{ex} - P_{in} + \rho_{\text{DPMS}} g \left( Z_{1\theta} + Z_{2\theta} \right),
\]

(6)

where \( \rho_{\text{DPMS}} \) is the liquid density in the connecting pipes of the DPMS, which is higher than that under saturated conditions since the liquid flow is stagnant and cooled due to heat loss. Neglecting this effect results in a large error in the estimated void fraction, especially when the void fraction is small and the system pressure is high, as in this study. Therefore, \( \rho_{\text{DPMS}} \) was determined experimentally as a function of ambient temperature and the local saturation temperature.

The void fraction in the two-phase region, \( \alpha_{2\theta} \), is obtained by integrating equation (1) over the liquid single-phase and two-phase regions, thereby using equation (6) as a restraint. This process requires an iterative method, since the coefficients in the momentum equation are functions of the void fraction. The total void fraction in each measurement region of DPMS, \( \alpha_{\text{total}} \), is then calculated as follows.

\[
\alpha_{\text{total}} = \frac{Z_{2\theta}}{Z_{1\theta} + Z_{2\theta}} \alpha_{2\theta}
\]

(7)
3.2. Neutronics

Hashimoto\textsuperscript{(6)} derived the linearized formulation of reactor neutronics including the eigenvalue separation of the higher-harmonic neutronic mode, as follows:

\[
d\rho_m^F(t) = \frac{\rho_m^F(t) - \beta}{\Lambda_m} n_m(t) + \Lambda_c m(t) + \frac{\rho_m^F(t)}{\Lambda_m} N_0 + \sum_{n=0}^{\infty} \frac{\rho_m^F(t)}{\Lambda_m} n_m(t),
\]

(8)

\[
dc_m(t) = \frac{\beta}{\Lambda_m} n_m(t) - \Lambda_c m(t).
\]

(9)

\(N_0\) is the density of prompt neutrons at criticality. \(n\) and \(c\) are variations in precursor number densities of prompt and delayed neutrons, respectively. \(\Lambda\) is the decay constant of the delayed neutron precursor, \(\beta\) denotes the delayed neutron fraction and \(\Lambda\) the generation time of prompt neutrons. Subscripts \(n\) and \(m\) indicate the order of the higher harmonic mode. \(\rho_m^F(t)\) is the adjoint function.

The macroscopic fission cross section is influenced via changes of the neutron spectrum. We assume that the change in neutron absorption is dominating the core-wide instability and regional instability. Using first-order perturbation theory\textsuperscript{(11)} a change of macroscopic absorption cross section \(\delta \Sigma_{\alpha}(\vec{r}, t)\) leads to a reactivity change \(\rho_m^F(t)\) as:

\[
\rho_m^F(t) \approx - \frac{dF_{\alpha}^m(\vec{r})}{d\Sigma_{\alpha}(\vec{r}, t)} \delta \Sigma_c(\vec{r}, t) \equiv \Lambda_m.
\]

(11)

where \(\alpha\) is the \(m\)-th mode eigenvalue. \(\rho_m^F(t)\) is the excitation reactivity of the \(m\)-th mode.

When liquid turns into vapor (‘void’) the neutron balance is affected via changes of the neutron absorption and scattering cross-sections, leading to a reactivity change. In fact, even the neutron fission cross section is influenced via changes of the neutron spectrum. We assume that the change in neutron absorption is dominating the core-wide instability and regional instability. Using first-order perturbation theory\textsuperscript{(11)} a change of macroscopic absorption cross section \(\delta \Sigma_{\alpha}(\vec{r}, t)\) leads to a reactivity change \(\rho_m^F(t)\) as:

\[
\rho_m^F(t) \approx - \frac{dF_{\alpha}^m(\vec{r})}{d\Sigma_{\alpha}(\vec{r}, t)} \delta \Sigma_c(\vec{r}, t) \equiv \Lambda_m.
\]

(11)

where \(\delta \Sigma_c(\vec{r}, t)\) is the neutron flux of the \(m\)-th harmonic mode, the superscript ‘\(*\)’ indicates its adjoint function. \(v\) denotes the average number of neutrons released per fission and \(\delta \Sigma_{\alpha}(\vec{r}, t)\) is the macroscopic fission cross section.

Differential pressures were measured at seven different regions to estimate the void reactivity. In addition, the axial power profile is bottom-peaked (which is often used for licensing reasons since this profile destabilizes the regional mode). The void reactivity changes of the different regions to estimate the void reactivity.

In order to estimate core-wide and regional stabilities, the fundamental mode \((m=0)\) and the first azimuthal mode \((m=1)\) are considered. Neglecting other higher harmonic modes yields\textsuperscript{(12)}:

\[
dn_0(t) = \frac{\rho_{00}^F(t) - \beta}{\Lambda_0} n_0(t) + \frac{\rho_{00}^F(t)}{\Lambda_0} N_0 + \frac{\rho_{01}^F(t)}{\Lambda_0} n_1(t) + \Lambda c_0(t),
\]

(14)

\[
dn_1(t) = \frac{\rho_{01}^F(t)}{\Lambda_1} (N_0 + n_0(t)) + \frac{\rho_{11}^F(t)}{\Lambda_1} n_1(t) + \epsilon - \frac{\beta}{\Lambda_1} n_1(t) + \Lambda c_1(t),
\]

(15)

\[
dc_0(t) = \frac{\beta}{\Lambda_0} n_0(t) - \lambda c_0(t),
\]

(16)
The $\lambda$-mode eigenvalue separation of the first azimuthal mode is rewritten as $\epsilon = \rho^2(t) = 1 - 1/\lambda_1$. $n_0$ and $n_1$ can be obtained by solving the differential equation (14) - (17). The heat generation rate per unit volume in channel 1 ($q'''_{ch1}$) and in channel 2 ($q'''_{ch2}$) are determined as follows:

\[
q'''_{ch1} = C_{ch1} \left[ N_0 + n_0(t) + n_1(t) \right]
\]

\[
q'''_{ch2} = C_{ch2} \left[ N_0 + n_0(t) - n_1(t) \right]
\]

$C_{ch1}$ and $C_{ch2}$ are proportional constants of the number density to the power generation rate per unit volume in the simulated fuel channels 1 and 2 of the SIRIUS-F facility.

3.3. Thermal Conduction in the Fuel Rod

Thermal conduction in the fuel rod is formulated as point model. The average temperature of the fuel rod, $T_f$, is expressed using surface heat flux of the fuel rod.

\[
\rho_f C_p f A_f \frac{dT_f}{dt} = A_f q'' - P_h q''.
\]

The surface heat flux is derived from the overall heat transfer coefficient, $h_{overall}$, and the degree of superheat, $T_f - T_{sat}$,

\[
q'' = h_{overall} \left( T_f - T_{sat} \right).
\]

The fuel rod time constant, $\tau_f$, is defined as

\[
\tau_f = \frac{\rho_f C_p f A_f}{h_{overall} P_h}.
\]

Using the above expressions yields the nondimensional form of equation (20),

\[
\frac{d\hat{q}''}{dt} = \frac{1}{\tau_f} (\hat{q}''' - \hat{q}'').
\]

The symbol “$'$” means nondimensionalized variables, following

\[
\hat{\epsilon} = \frac{\epsilon}{\hat{q}'},
\]

\[
\hat{\epsilon}'' = \frac{\epsilon''}{\hat{q}''_0}
\]

\[
\hat{\epsilon}''' = \frac{\epsilon'''}{P_h \hat{q}''_0}
\]

Here, $q''_0$ is the surface heat flux under the critical condition.

4. STABILITY ESTIMATION METHOD ON THE BASIS OF NOISE ANALYSIS

The stability of the BWR is usually qualified by its decay ratio and resonance frequency, as well as the distance of operating conditions to the stability boundary. In this section, we introduce a method to analyze the decay ratio and resonance frequency from time series of the signals obtained in the experiments for studying the stability characteristics.

The closed-loop transfer function describing the BWR dynamics has several poles, which are real and/or complex conjugate numbers. Each decay ratio ($DR$) and resonance frequency ($f$) can be found by using a pole ($s = \sigma + i\omega$) as follows:

\[
DR = e^{\frac{\pi \sigma}{2\pi}}
\]

\[
f = \frac{|\omega|}{2\pi}
\]

The least stable pole (the largest $\sigma/|\omega|$) determines the stability of the system.

In this study, noise analysis was performed to obtain the decay ratio and the resonance frequency by using a time-series of the measured inlet channel flow rate as follows:
Fig. 7 ABWR Power-flow operating map and SIRIUS-F experimental range. Experiments were conducted along the natural circulation line: the core flow is fixed at 21% and the core thermal power ranges from 20% to 40%.

1. A linear trend is removed from the time-series of acquired data on the basis of the best straight-line fit.

2. A transfer function in the z-plane, \( G(z) \), is estimated by auto-regressive (AR) model:

\[
G(z) = \frac{c_2}{1 + a_2z^{-1} + a_3z^{-2} + \cdots + a_mz^{-m}}
\]

where \( c_2 \) and \( a_2, a_3, \cdots, a_m \) are constants.

3. Poles, \( p_1, p_2, \cdots, p_m \), can be obtained from a partial-fraction expansion of the transfer function:

\[
G(z) = \frac{r_1}{1 - p_1z^{-1}} + \frac{r_2}{1 - p_2z^{-1}} + \cdots + \frac{r_m}{1 - p_mz^{-1}}
\]

The magnitude of the residues, \( r_1, r_2, \cdots, r_m \), indicates the importance of the corresponding pole.

4. Decay ratios and frequencies are estimated from the most dominant pole in the s-plane converted from the z-plane by the equations (26) and (27).

5. STABILITY ESTIMATION AND DISCUSSIONS

5.1. Experimental Parameters

Forced circulation BWR is in its least-stable condition when its power-to-flow ratio is high. Figure 7 shows an ABWR power-flow operating map. The ABWR plant was selected as the target plant for stability investigation because its power-to-flow ratio is the highest under natural circulation conditions among the operating BWRs in Japan.

The experimental range is represented in the figure by a hatched rectangle. Region is laid out with a natural circulation (NC) line including the maximum power point. The term “natural circulation” refers to the state in which all internal pumps are not in operation, i.e., the rotors are fixed.

When the operating condition moves to the high power/flow region, after the pump trip event in the ABWR, a predetermined set of control rods is inserted in the core to suppress the core thermal power, i.e., selected control rods run-in (SCRRI). In the present study, the core thermal power was assumed to remain the maximum power point because of a malfunction in the SCRRI, therefore producing a conservative estimate of the stability.

Experimental conditions are summarized in the Table 1. The target fuel type is called STEP-III Type-A. Figure 8 shows the axial power profile, which is the relative thermal power profile along the heated section. The profile is a bottom skew profile (1.70N3) and usually applied in a licensing analysis. The Part Length Rod (PLR) design of STEP-III Type-A fuel...
was implemented in the SIRIUS-F facility, and analyzed as a change in flow area. The \( \lambda \)-mode eigenvalue separation was set as 0.00318 for the natural circulation line to provide a conservative estimate, i.e., the largest value under all conditions that met the design criteria.

5.2. Signal Time Traces

Core-wide and regional stability experiments were conducted for the natural circulation line with simulated void-reactivity feedback.

Figure 9 shows time traces of signals measured at inlet channel in terms of the core thermal power. The level of the flow rate is fixed at 21% of the rated core flow rate. The sampling frequency is 10 samples/s. As indicated in the figure 9, the amplitude of fluctuations in the inlet flow velocity enlarges with increasing the core thermal power. The regional instability and core-wide instability become less stable with increasing thermal power, since these instabilities are driven by density-wave oscillations overlaid with void-reactivity feedback. Nevertheless, the amplitude of fluctuations is small and the flow is stable in all cases of the core thermal power.

5.3. Effect of Auto-Regressive Model

In order to investigate the effect of auto-regressive models, the following five common models are applied to the signals.

(a) **least squares model** The sum of squared forward prediction errors is minimized.

(b) **forward-backward model** The sum of a least squares criterion for a forward model and the analogous criterion for a time-reversed model is minimized.

(c) **Burg’s lattice-based model** The lattice filter equations are solved using the harmonic mean of forward and backward squared prediction errors.

(d) **geometric lattice model** The lattice filter equations are solved using the geometric mean of forward and backward squared prediction errors. This is as in Burg’s method, but the geometric mean is used instead of the harmonic one.

(e) **Yule-Walker model** The Yule-Walker equations, formed from sample covariances, are solved.

Figure 10 show the decay ratio and the frequency estimated based on the five different auto-regressive models. In the SCRRI event, the core thermal power is suppressed at 20%. The channel stability decay ratio was less than 0.1 in all models. As the power increased tentatively from 20 % to 40 %, the decay ratio increased monotonically. The analysis estimated the decay ratio conservatively at higher power levels. Even at 40 % power, the decay ratio was less than
The amplitude of fluctuations in the inlet flow velocity enlarges with increasing the core thermal power.

Fig. 9 Time traces of signals measured at inlet channel in terms of the core thermal power. The amplitude of fluctuations in the inlet flow velocity enlarges with increasing the core thermal power.

Fig. 10 Effect of Auto-Regressive model. Variations in the decay ratio and the frequency among five different model are within 0.03 and 0.01 Hz, respectively.

unity, indicating that the ABWR is stable even if the SCRRI malfunctioned.

The auto-regressive model order and the number of data are fixed to be 30 and 2048, respectively. Variations in the decay ratio and the frequency among five different models are within 0.03 and 0.01 Hz, respectively. De Hoon et al. (13) reported that the Yule-Walker model gives a lower decay ratio than Burg’s lattice-based model does when those are applied for less stable signals. The experimental results are consistent with the finding that the Yule-Walker model gives a lower decay ratio than Burg’s lattice-based model does for high thermal power, which is less stable conditions. Nevertheless, the difference in the decay ratio between these two models is less than 0.03 within the experimental range. In this experimental series, the Yule-Walker method was selected, since it gives appropriate coefficients from relatively stable signals (13).

5.4. Effect of Auto-Regressive Model Order

One may determine an auto-regressive model order, N, according to Akaike’s Information Criterion (AIC) (14). In order to identify relatively low frequency signals, parameter survey can, therefore, give better estimation of N, since AIC counts on all frequencies in a signal to optimize N. Figure 11 shows the decay ratio, DR, and the frequency, f, as a function of the core thermal power on the basis of the Yule-Walker scheme. The number of data is fixed to be 2048. For a parametric study, the auto-regressive model order N was varied from 10 to 30. Smaller N gives lower estimate of DR, and large degree of divergence in f for the lower thermal power. The estimation accuracy is low due to the small signal to noise ratio where the system is more stable for lower thermal power, as shown in the figure 9. The appropriate value of N is 30 within these experimental ranges, since the decay ratios and the frequencies for N = 26 and N = 30 are almost identical. One must note that when N is much larger than 30, the most dominant pole will separate into more stable poles, resulting in an inaccurate estimation of decay ratio.

5.5. Effect of Data Size

In order to gain reliability of stability estimation, one must acquire a sufficient number of data. In this study, the effect of data size was investigated parametrically. Figure 12 shows the effect of a number of the data on the stability estimation. The number of data was varied from 256 to 2048. The auto-regressive model used in this experiment is the Yule-Walker model.
Fig. 11 Effect of model order. Smaller auto-regressive model order gives lower estimate of DR, and a large degree of divergence in f for the lower thermal power.

Fig. 12 Effect of Data Size. Small number of data gives lower estimate of DR, and a large degree of divergence in f for the lower thermal power.

The model order is fixed to be 30. In general, a small number of data gives a lower estimate of DR, and a large degree of divergence in f for the lower thermal power. The estimation accuracy is low due to the small signal to noise ratio where the system is more stable for the lower thermal power, as shown in the figure 9. The appropriate value of n is 2048 within these experimental range, since the decay ratios and the frequencies for n = 1024 and n = 2048 are almost identical.

6. CONCLUSIONS

In order to quantify the regional stability of BWR, a time series analysis was performed to calculate decay ratio and resonance frequency from a dominant pole of a transfer function by applying AR methods to time-series of the core inlet flow rate. Experiments were conducted with the SIRIUS-F facility, which simulates the Advanced Boiling Water Reactors (ABWRs) with an oxide mixture of uranium and plutonium (MOX) fuel installed.

Experimental range covers a wide range along maximum power points on the natural circulation line including the target power level after a SCRRI of the ABWR. The decay ratios obtained in this range are less than unity, even at elevated power levels.

The variations in the decay ratio and resonance frequency among five common AR methods are within 0.03 and 0.01 Hz, respectively. In this system, the appropriate decay ratio and resonance frequency can be estimated on the basis of the Yule-Walker method with the model order of 30 and with 2048 of data series.

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