Application of Bias Factor Method with Use of Exponentiated Experimental Value to Prediction Uncertainty Reduction in Coolant Void Reactivity of Breeding Light Water Reactor*

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Abstract
We have applied the bias factor method to a coolant void reactivity of a breeding light water reactor with use of FCA-XXII-1 experiment with introducing a concept of exponentiated experimental value into the bias factor method. We have formulated the prediction uncertainty reduction by the use of the bias factor method extended by the concept of the exponentiated experimental value. From the numerical results, it is verified that the present method can reduce the prediction uncertainty in the design calculation value while the conventional bias factor method cannot reduce it. The present method overcomes a problem caused by the conventional bias factor method in which the prediction uncertainty increases in the case that the experimental core has the opposite reactivity worth and the consequent opposite sensitivity coefficients to the real core. It is concluded that the introduction of exponentiated experimental value can effectively utilize experimental data and extend applicability of the bias factor method.

Key words: Bias Factor Method, Exponentiated Experimental Value, Coolant Void Reactivity, Uncertainty, Sensitivity Coefficient, Critical Experiment, FCA

1. Introduction

An innovative water-cooled reactor concept aiming at an achievement of a high conversion ratio of about 1.1 and a high burn-up up to 100GWD/t has been investigated at JAEA for the effective fuel utilization through plutonium multiple recycling with mixed oxide (MOX) fuel. Such a high conversion ratio can be attained by reducing the moderation of neutrons, i.e. reducing the water fraction in the core. Another important design target is to achieve a negative void reactivity especially from the safety point of view. For the core designing of such a breeding light water reactor (LWR) core, it is important to accurately predict core characteristics such as criticality, a breeding performance and a void reactivity and improve the prediction accuracy of them. Through the conceptual designing, we have constructed a tentative core design concept\(^1\) under a BWR-type concept with relatively high core void fraction.
For MOX fueled tight lattice LWR cores, critical experiments were carried out using the fast critical facility, FCA, in order to obtain experimental data and to evaluate the prediction accuracy of core characteristics of such cores. Up to the present, we have performed critical experiments using FCA-XV and FCA-XXII-1 series cores with different plutonium fissile enrichments and different void fractions of moderator. Major items of the experiments are criticality, reaction rate ratios and coolant void reactivity. Using the experimental results, we can generally improve the prediction accuracy by correcting the design calculation value and reduce the prediction uncertainty based on the bias factor method. In the conventional bias factor method, the prediction uncertainty due to cross section errors can be reduced corresponding to a similarity of the sensitivity coefficients between the real core and the experiment core, which is evaluated by the ratio of relative change of core characteristic to relative change of cross section.

In the previous study, we have preliminarily investigated applicability of the FCA-XXII-1 and FCA-XV series cores to criticality and breeding ratio of the real core based on the conventional method. In the present study, we have investigated applicability of the FCA-XXII-1 series cores to a coolant void reactivity. In the real core, void tube assemblies are settled into the core to secure a negative void reactivity by means of neutron leakage through them at a power rise, because the core essentially has a positive void reactivity without them. On the other hand, the experimental values were negative. These are mainly because the real and critical experiment cores have positive and negative spectral components of void reactivity, respectively. The adjoint flux in the real core becomes larger with increasing neutron energy above 10keV. This yields a large positive spectral component above 10keV. On the other hand, the adjoint flux in the critical experiment core does not become so larger with increasing neutron energy by the effect of large neutron leakage due to the small experiment core. This yields a very small positive spectral component above 1MeV and a large negative one below 1MeV. Consequently, the spectral component becomes negative in the experiment core. Without the void tube assemblies, the real core essentially has the opposite sensitivity coefficients to the experiment core. In such a case, improvement of prediction accuracy cannot be achieved by the conventional method, because it can be attained with the similarity of the sensitivity coefficients between the real and experiment cores.

To overcome this problem, we have introduced a concept of an exponentiated experimental value into the bias factor method. With a negative exponent, the exponentiated experimental value will have the similar sensitivity coefficients to the real core. Consequently, the prediction uncertainty due to cross section errors will be reduced by use of such an exponentiated experimental value.

In the present paper, we review a theory of the conventional bias factor method in Chap. 2. In Chap. 3, we introduce the concept of the exponentiated experimental value. In Chap. 4, we briefly give a conceptual design of the breeding light water reactor and the FCA experiment. Then, we present numerical results and show the effectiveness of the newly introduced concept of the exponentiated experimental value in Chap. 5.

2. Conventional Bias Factor Method

Let us briefly describe a theory of the conventional bias factor method. The bias factor $f_m$ is evaluated by the ratio of the experimental value $E_m$ to analysis value $C_m$ for a mock-up critical experiment as

$$f_m = \frac{E_m}{C_m}.$$  \hspace{1cm} (1)

Then, the design calculation value $R^C_R$ is corrected into the design prediction value $R^m_R$.
by multiplying the bias factor;
\[ R_R^m = R_R^C \times f_m \quad . \tag{2} \]

The analysis value of the experiment and the design calculation value have uncertainty due to the cross section errors and the method errors. Let us denote the deviation of the cross section by \( \Delta \sigma \) and the sensitivity coefficient of \( R_R^C \) by \( S_R \). The relative error of \( R_R^C \) due to the method errors is expressed by \( \delta M_R \). Neglecting a correlation between the cross section and method errors, we can express the design calculation value as
\[ R_R^C = R_R^O (1 + \delta M_R + S_R \Delta \sigma) \quad , \tag{3} \]

where \( R_R^O \) is the true value for the real core. In the same way as for the real core, we can express the analysis value of the experiment as
\[ C_m = R_m^O (1 + \delta M_m + S_m \Delta \sigma) \quad , \tag{4} \]

where \( R_m^O \) is the true value for the critical experiment core. The experimental value has the experimental error and is expressed as
\[ E_m = R_m^O (1 + \delta E) \quad . \tag{5} \]

As a result, the design prediction value is expressed by
\[ R_R^m = R_R^C \times E_m \quad , \tag{6} \]

\[ C_m = \frac{R_R^O (1 + \delta M_R + S_R \Delta \sigma)(1 + \delta E)}{(1 + \delta M_m + S_m \Delta \sigma)} \quad . \]

Assuming that the above errors are small compared to unity and independent of one another, it is written as
\[ R_R^m = R_R^O \left\{ 1 + (S_R - S_m)\Delta \sigma + (\delta M_R - \delta M_m) + \delta E \right\} \quad . \tag{7} \]

Consequently, the variance of the design prediction value is composed of the uncertainty due to the cross section errors, the method errors and the experimental error and given by
\[
V(R_R^m / R_R^O) = (S_R - S_m)V_\sigma(S_R - S_m)^t + V(\delta M_R - \delta M_m) + V(\delta E) \\
= (S_R - S_m)V_\sigma(S_R - S_m)^t \\
+ V(\delta M_R) + V(\delta M_m) - 2 \text{Cov}(\delta M_R, \delta M_m) + V(\delta E) \quad , \tag{8} \]

where \( V_\sigma \) is a covariance matrix of cross sections and \( \text{Cov}(\delta M_R, \delta M_m) \) denotes covariance between the design calculation value and analysis value of the experiment with respect to the method errors. As for the variance in Eq. (8), \( V(\delta E) \), for example, denotes the variance of the experimental error. On the other hand, in the case that we do not use any experimental result, the original variance of the design calculation value is expressed by
\[ V(R_R^C / R_R^O) = S_R V_\sigma S_R^t + V(\delta M_R) \quad . \tag{9} \]

The uncertainty-reduction factor by the bias factor method has been introduced to
evaluate the effectiveness of the critical experiment for the improvement of the prediction accuracy and defined by the relative reduction of the variance as follows:

\[ UR^m = 1 - \frac{V(R_m^m / R_R)}{V(R_m^m / R_R^m)} . \]  

(10)

3. Concept of Exponentiated Experimental Value

In the present study, we have introduced a concept of an exponentiated experimental value as follows,

\[ E_{ex} = (E_m)^w . \]  

(11)

An analysis value corresponding to the exponentiated experimental value is also defined by an exponentiated analysis value with the same exponent as the exponentiated experimental value:

\[ C_{ex} = (C_m)^w . \]  

(12)

A bias factor is defined by a ratio of the exponentiated experimental value to the exponentiated analysis value:

\[ f_{ex} = \frac{E_{ex}}{C_{ex}} = \frac{(E_m)^w}{(C_m)^w} . \]  

(13)

The design calculation value is corrected into the design prediction value by multiplying the bias factor:

\[ R_{R}^{ex} = R_{R}^C \times f_{ex} . \]  

(14)

As a result, the corrected value for the real core is expressed by

\[ R_{R}^{ex} = R_{R}^C \times \frac{(E_m)^w}{(C_m)^w} = R_{R}^Q \times \frac{(1 + \delta M_R + S_R \Delta \sigma)^w}{(1 + \delta M_m + S_m \Delta \sigma)^w} \times \frac{(1 + \delta E)^w}{(1 + w \ln (1 + \delta E)} . \]

(15)

Assuming that the errors are small compared to unity, the above equation is rewritten as

\[ R_{R}^{ex} = R_{R}^Q (1 + \delta M_R + S_R \Delta \sigma) \times \frac{1 + w \ln (1 + \delta E)}{1 + w \ln (\delta M_m + S_m \Delta \sigma)} \times \frac{1 + w \delta E}{1 + w \delta M_m + w S_m \Delta \sigma} . \]

(16)
Finally, the design prediction value is expressed by

\[ R^*_R = R^0_R \left[ 1 + (S_R - wS_m) \Delta \sigma + \left( \delta M_R - w \delta M_m \right) + w \delta E \right]. \quad (17) \]

Therefore, the variance of the design prediction value is given by

\[ V(R^*_{R}) = \left( S_R - wS_m \right) V_{\sigma} \left( S_R - wS_m \right)' + V(\delta M_R - w \delta M_m) + V(w \delta E) \]
\[ = S_R V_{\sigma} S_R' - w S_R V_{\sigma} S_m' - w S_m V_{\sigma} S_R' + w^2 V(\delta M_m) \]
\[ - 2w \text{Cov}(\delta M_R, \delta M_m) + w^2 V(\delta E) . \quad (18) \]

In order to minimize the variance, the exponent is determined so as to make the first-order derivative of the above equation be zero:

\[ \frac{\partial}{\partial w} V(R^*_{R}) = 2w S_m V_{\sigma} S_m' - S_R V_{\sigma} S_R' - S_m V_{\sigma} S_R' + 2w V(\delta M_m) - 2 \text{Cov}(\delta M_R, \delta M_m) + 2w V(\delta E) = 0 . \quad (19) \]

The second-order derivative is found to be positive as

\[ \frac{\partial^2 V(R^*_{R})}{\partial w^2} = 2 S_m V_{\sigma} S_m' + 2 V(\delta M_m) + 2 V(\delta E) > 0 , \quad (20) \]

And therefore, the determined exponent makes the variance globally minimum. Finally, the determined exponent can be expressed as

\[ w = \frac{S_R V_{\sigma} S_R' + S_m V_{\sigma} S_m' + 2 \text{Cov}(\delta M_R, \delta M_m)}{2 S_m V_{\sigma} S_m' + 2 V(\delta M_m) + 2 V(\delta E)} . \quad (21) \]

As a result, the uncertainty-reduction factor in the present method is expressed as

\[ UR^* = 1 - \frac{\left\{ (S_R - wS_m) V_{\sigma} (S_R - wS_m)' \right\} + V(\delta M_R - w \delta M_m) + V(w \delta E)}{S_R V_{\sigma} S_R' + V(\delta M_R)} . \quad (22) \]

We discuss the mechanism of the present method to reduce the prediction uncertainty by considering two typical cases where the exponents equal 1 and -1. We assume negligibly small method errors and experimental error, for simplicity. When the exponent equals 1, Eq. (13) is the same as Eq. (1). This is an apparent feature of the present method that it includes the conventional method. Equation (8) shows that the uncertainty due to cross section errors
4. Core Design and Experiments

4.1 Breeding Light Water Reactor

In order to achieve the high conversion ratio, the volume of the moderator, i.e. water, should be reduced. For this purpose, the tight-lattice fuel rod configuration is adopted using a triangular lattice with a narrow gap between the fuel rods and the core void fraction is increased compared with that of the conventional BWR core. The fuel rod consists of MOX pellets with the average fissile Pu content of about 10% in the seed regions and the depleted UO₂ pellets in the blanket regions.

The tentative core concept has an axial heterogeneous core configuration as shown in Fig. 1. The core is divided into the upper and lower ones of 0.8m height each by the internal blanket of 0.1m height. And the upper and lower blankets of 0.5m height each are set above and below the core. The core average void fraction is set around 60% to reduce the water volume. The void fraction exceeds 80% at the upper blanket region. The void fraction changes largely from 0% to more than 80%. As a result, atomic number ratios of hydrogen to heavy metal (H/HM) in the upper and lower core regions are approximately 0.24 and 0.47, respectively. To satisfy the requirement of the negative void reactivity coefficient, neutron leakage should be increased when void is generated or increased in the core. The

![Fig. 1 Core configuration of breeding light water reactor](image-url)
axial heterogeneous core configuration with a combination of two cores and three axial blankets is adopted. Additionally, a new device of the void tube assembly is introduced to promote neutron leakage effect by the neutron streaming. Therefore, the core itself has essentially a positive void reactivity without the void tube assembly. The void reactivity worth is evaluated to be 0.0141%dk/kk’ with a 1-dimensional calculation neglecting the void tube assemblies.

4.2 FCA-XXII-1 Critical Experiments

In the critical experiments, three cores, FCA-XXII-1 (45V, 65V & 95V) with different void fractions of moderator (45%, 65% and 95% void fraction polystyrene foam) were constructed. The FCA-XXII-1 cores are coupled systems of a central test zone and a surrounding driver zone. The test zone is composed of a combination of uranium plates, plutonium plates, depleted UO₂ plates, alumina plates and polystyrene foam plates. The principal parameters of the test zones of the series cores are summarized in Table 1. The cell averaged enrichment of the test zones are 16 atom % of fissile plutonium and H/HM is systematically changed from 0.09 to 0.8.

The cross sectional views of the FCA-XXII-1 (65V) core, for example, are shown in Fig. 2. The numbers of the unit drawers for the test zones are 61, 69 and 77 in the FCA-XXII-1 (45V), (65V) and (95V) cores, respectively. The height of the test zone is 91 cm in the series cores. Upper and lower axial blanket zones of 30cm thickness containing natural uranium metal are placed to cover the test zone. In radial direction, the test zone is surrounded by the SUS buffer zone, the enriched U driver zone and two radial blanket zones; an inner blanket zone of 30cm thickness contains a significant amount of depleted uranium dioxide and sodium, and an outer blanket zone of 15cm thickness contains only depleted uranium metal.

![Fig. 2 Schematic views of the FCA-XXII-1 (65V) core](image)
We utilize the experimental results on void reactivity in the (45V) and (65V) cores. The measurement for coolant void reactivity was made by changing the void fraction of the polystyrene plates for nine unit cells in the central drawer from 45% to 95% in the (45V) core and from 65% to 95% in the (65V) core. The experimental values of the void reactivity worth were -9.17x10^{-4} dk/kk' ± 0.9% and -5.04x10^{-4} dk/kk' ± 1.3% in the (45V) and (65V) cores, respectively (2). The experimental values are opposite in sign to that of the real core.

4.3 Uncertainties in Design Calculation, Experiment and Experimental Analysis Calculation on Void Reactivity

The design calculations were carried out by means of a deterministic method (2). They include the uncertainties due to the cross section errors and the method errors. The uncertainty due to the cross section errors was evaluated by the first term of right hand side of Eq. (8). The sensitivity coefficients were obtained by a version of SAGEP (3) based on the generalized diffusion perturbation theory. The covariance data of cross sections of JENDL-3.3 (6) was used. The uncertainty due to the method errors was evaluated by a comparison with detailed calculations by the MVP continuous-energy Monte Carlo code (7). In the design calculations, four core calculations were carried out with different void distribution patterns. Therefore, there were three calculations for void reactivity. We adopted the standard deviation of the three results on the ratio of the design calculation value and the detailed calculation value as the method error, and consequently the mean value of the ratios is considered as another bias factor for the design calculation value. In evaluating the method error, the statistical errors of the MVP Monte Carlo calculations are taken into account.

The experimental errors (2) have been evaluated in the experimental study.

As for the experimental analyses, detailed analysis calculations have been carried out with MVP and JENDL-3.3. The analysis values include uncertainty due to the cross section errors and the method errors. The uncertainty due to cross section errors were evaluated by the same way as the design calculation results. With respect to the method errors, we considered statistical errors of the MVP Monte Carlo calculations and experimental analysis modeling errors (2) in the experimental analysis results. In the experimental analysis modeling errors, we took into account the uncertainty due to deviations in plate weight and in isotope composition. There was assumed to be no correlation between the experimental analysis and the design calculation values with respect to the method errors.

The evaluated uncertainties in the design calculations, experiments and experimental analysis calculations are summarized in Table 2. The uncertainty due to the cross section errors and the method errors in the real core were 3.5% and 1.4%, respectively. Both of the experimental errors and method errors in the analysis values of the experiments are smaller but not negligibly small compared with the uncertainty in the design calculation value.

<table>
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<tr>
<th>Table 2</th>
<th>Standard deviations in design calculation value, experimental value and analysis value of experiment on void reactivity</th>
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<tr>
<td>Cross section error</td>
<td>Method error</td>
</tr>
<tr>
<td>Real Core</td>
<td>3.5%</td>
</tr>
<tr>
<td>FCA-XXII-1(45V)</td>
<td>3.0%</td>
</tr>
<tr>
<td>FCA-XXII-1(65V)</td>
<td>3.8%</td>
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5. Numerical Results

We compare numerical results of prediction uncertainty reduction on a void reactivity between the conventional method and the present method introducing the exponentiated experimental value in Table 3.

By the conventional method, both of the experimental results cannot improve the prediction accuracy as they give the negative uncertainty-reduction factor. A major reason is the increase of the uncertainty due to cross section errors compared with the original one in the design calculation value. This increase is caused by opposite profiles of the sensitivity coefficients between the real and the experiment cores, which can be seen in Fig. 3.

By the present method, both of the experiments improve the prediction accuracy with the negative exponents of about -0.6. The negative exponents change their signs of the sensitivity coefficients to produce similar sensitivity coefficients to those of the real core, and reduce the uncertainty due to the cross section errors. In addition, in order to minimize the uncertainty as a whole, the present method gives the exponents whose absolute values are smaller than unity and suppresses the contribution from the method and experimental errors though it does not fully utilize potentials to reduce the uncertainty due to cross section errors. After all, the prediction uncertainty can be reduced up to 40 or 50% by the use of the experimental results in FCA-XXII-1 cores by the present method. From the present numerical evaluation, it can be said that both FCA-XXII-1 (45V) and (65V) cores are effective for the prediction uncertainty reduction on the coolant void reactivity of the real core and the effectiveness of the (65V) core is superior to that of the (45V) core.

<table>
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<th>Table 3</th>
<th>Summary of application results of prediction uncertainty reduction on a void reactivity</th>
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<tr>
<td></td>
<td>Uncertainty-reduction factor by the conventional method</td>
</tr>
<tr>
<td>FCA-XXII-1(45V)</td>
<td>-2.135</td>
</tr>
<tr>
<td>FCA-XXII-1(65V)</td>
<td>-3.125</td>
</tr>
</tbody>
</table>

We investigate the error introduced in the present method approximating exponential function as shown in Eq. (16). The absolute values of the exponents are about 0.6 and are smaller than unity as shown in Table 3. As a result, the approximation errors are less than 0.1%. They are considerably small compared with the standard deviations of experimental and analysis values shown in Table 2. Consequently, it can be said that the approximation error hardly affects the final numerical results.
6. Conclusions

In the present study, we have introduced a concept of an exponentiated experimental value into the conventional bias factor method. To extend the applicability of the bias factor method, we have formulated the prediction uncertainty reduction by the use of the bias factor method extended by the concept of the exponentiated experimental value. From the numerical evaluation, it has been shown that the prediction uncertainty due to cross section errors has been reduced by the use of such an exponentiated experimental value. The negative exponent overcomes a problem caused by the conventional bias factor method in which the prediction uncertainty increases in the case that the experimental core has the opposite reactivity worth and the consequent opposite sensitivity coefficients to the real core. It is concluded that the introduction of the exponentiated experimental value can effectively utilize experimental data and extend applicability of the bias factor method.

As for the applicability of FCA-XXII-1 series core to the coolant void reactivity worth of the breeding light water reactor core, FCA-XXII-1 series cores are found to have an ability to reduce the prediction uncertainty and the variance of the prediction uncertainty is reduced by about 50%.

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