Study on the Drag Force Coefficient between Gas and Liquid on the Gas-Lift System to Recover the Methane Hydrate*


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Abstract

The gas-lift method is expected as a method with the small running cost in order to recover the methane hydrate from the seabed. The methane gas injection makes the upward flow of the sea water, and it is necessary to establish the model able to estimate the sea water flow rate to the injected gas. Since the intended system is too large to conduct the experiment and multi-dimensional direct numerical simulation, this paper focuses on the calculation of the multi-phase flow in the one-dimension with the proper model equation for external forces. A drag force coefficient between gas and liquid is derived considering the liquid flow by the surrounding hydrostatic pressure. As a result, calculation with the proposed equation fairly agrees with the gas-lift experiment, compared with results with the other proposed model equations.

Key words: Methane Hydrate, Gas-Lift System, Two-Fluid Model, Modeling of the Drag Force

1. Introduction

Recent survey has revealed that there is the amount around $3.0 \times 10^{15}$ methane gas on the global seabed, which is in the form of the gas hydrate. Therefore, the methane hydrate is expected as a new natural energy source being replaced to the natural gas. In order to recover the methane hydrate, the gas-lift method is focused, though several concepts has been proposed.

Numbers of experimental studies have been conducted to understand the behavior of the actual multi-phase flow on the gas-lift system by the experiment or calculation. The work by Saito et. al., for example, also has successfully summarized their experimental results with the non-dimensional numbers. Yoshinaga et. al. derived semi-empirical equations to estimate the steady flow on the apparatus.

As for numerical approach, there are two kind of approaches, as the example with the numerical simulation, which are of the drift-flux model or two-fluid model. Hatakeyama and Kajishima et. al. conducted the estimation of the multi-phase flow on the gas-lift system with the drift-flux model. Hatta et. al. compared experimental data with the calculation for the behavior of the multi-phase flow with the two-fluid model, and equations of the drag-force model between gas and liquid proposed by Tomiyama. They obtained the coefficient of drag force by the experiment of the gas injection on the chamber. Other model equations for the drag force between two phases were also
derived to estimate the behavior of boiling bubbles in the nuclear reactor.\(^{(19)}\) It also focuses on the behavior of gas bubbles in the static liquid phase. Thus, the typical previous model for the drag force is derived for the system in which the liquid flow is occurred by only the convection by bubbles.

However, the actual gas-lift system has the upward flow by the hydrostatic pressure. Therefore, the system derived the previous typical model can not be directly applied to the intended system, and it may be better to use the drag force coefficient obtained by the experiment with the actual device. In this paper, the experiment is conducted to make the liquid upward flow by the gas injection, and the drag force coefficient on bubbles is proposed for the two-fluid model.

2. Equations and system

The numerical simulation was conducted under the following assumptions.

(i) Riser pipe has a uniform cross sectional area.
(ii) System is isothermal.
(iii) Gas and liquid phase are under the same pressure.
(iv) The gas is same to the ideal gas, then its density is obtained by the equation of the state, and the liquid is incompressible.

Following equations are basic equations.

\[
\frac{\partial}{\partial t}(\rho_k \alpha_k) + \frac{\partial}{\partial x}(\rho_k \alpha_k v_k) = 0
\]  

\[
\rho_k \alpha_k \left( \frac{\partial v_k}{\partial t} + v_k \frac{\partial v_k}{\partial x} \right) = -\alpha_k \frac{\partial p}{\partial x} - (\text{inter})(v_k - v_e) - (\text{fric},k) v_k - \rho_k \alpha_k g
\]  

\[
\sum_k \alpha_k = 1
\]

Eq. (1) and eq. (2) represent the conservation of the mass and momentum for each phase, respectively. Eq. (3) is the definition of the volume fraction. The \(v_k\), \(\alpha_k\), and \(p\) are unknown parameters. Terms (\text{inter}) and (\text{fric},k) are determined by the experiment illustrated in the following sections, which appear in the right hand of eq. (2).

Fig. 1 is a conceptual image of the calculation domain on the apparatus. The simulation was conducted limiting to the upper region than the injection point. The influence of the deeper part was considered as the section where the liquid is affected by the wall friction.
3. Experiment

3.1 Experiment

The gas-lift experiment was conducted in the system with two riser pipes of different diameters and around $5m$ in height as Fig. 2 shows, which was the available maximum length in our laboratory. The pipe diameters were set in 0.05$m$ (Type-A) and 0.23$m$ (Type-B), considering of the geometrical ratio in the actual system and to obtain the dependence of bubble behavior on the diameter. These conditions are summarized in Table 3. The dried air and tap water were used as the gas and liquid phase. The water circulates between the riser pipe and downcommer in Fig. 2. The device of Type-A especially has a quick shut-off valve, to measure the void fraction in rise pipe. The flow rate of the gas and liquid are recorded with the mass flowmeter and electro magnetic flowmeter, respectively. The air was injected with the oil-free compressor through the orifice-like injection hole after being dehumidified by the drier. The area-equivalent diameter of bubbles, relative velocity of gas to the liquid, gas volume fraction, and rising liquid volume flux were recorded as a function of the gas volume flux in order to derive the drag force coefficient between gas and liquid. The relationship between $U_g$ and $U_l$ in Type-B were compared between experimental and numerical results.

![Fig. 2 Schematic of apparatus](image)

Table 3 Profile of apparatus

<table>
<thead>
<tr>
<th></th>
<th>Type-A</th>
<th>Type-B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipe length $L$ [m]</td>
<td>5.50</td>
<td>5.73</td>
</tr>
<tr>
<td>Inlet position $L_{inj}$ [m]</td>
<td>4.4</td>
<td>4.63</td>
</tr>
<tr>
<td>Pipe diameter $D$ [m]</td>
<td>0.05</td>
<td>0.23</td>
</tr>
<tr>
<td>Quick shut off valve</td>
<td>○</td>
<td>×</td>
</tr>
</tbody>
</table>

The drag force coefficient $C_d$ is derived under the assumption that the drag force per the unit volume between two phases is given by eq. (4).

$$\left(\text{inter} \right) v_{g,rel} = \frac{1}{8} C_d \rho_a \left| v_{g,rel} \right| v_{g,rel}$$ (4)

Fig. 4 shows the concrete flowchart to derive $C_d$. It is firstly assumed that the $C_d$ can be defined as a function of $v_{g,rel}$ and $d_{50}$ then $v_{g,rel}(U_g)$ and $d_{50}(U_g)$ are derived in the next
chapter 3.2 and 3.3, respectively. Therefore, $C_d(v_{g,rel}, d)$ can be specified as a function of $U_g$. Finally, the relationship between $U_g$ and $\alpha_g$ will be obtained in chapter 3.4. As a result, $C_d$ is represented with the $\alpha_g$, which is the one of basic variables in the simulation. The coefficient is derived under the following assumptions:

(i) Bubbles rise through the liquid with a constant upward velocity, keeping spherical shapes.
(ii) The motion of bubbles is always in the steady state.

Bubbles are regarded as a single spherical bubble with the Sauter mean diameter in order to derive $C_d$. However, few bubbles rise without deforming, the break-up or coalescence in the actual phenomenon. This discrepancy between the model and actual is treated to regard $a$ as a function of $\alpha_g$. The following equations are given by Ishii, Kurul and Podowski.(20)(21)

$$a = \begin{cases} \frac{6\alpha_g}{d_s} & \alpha_g < 0.25 \\ \frac{6\alpha_g}{d_s} \left(1 - \alpha_g \right) + \frac{4.5}{D} \left( \frac{\alpha_g - \alpha_{g'}}{1 - \alpha_{g'}} \right) & 0.25 \leq \alpha_g \end{cases}$$

$$\alpha_{g'} = \begin{cases} \alpha_g & \alpha_g < 0.25 \\ 0.3929 - 0.5714\alpha_g & 0.25 \leq \alpha_g < 0.6 \\ 0.05 & 0.6 \leq \alpha_g < 1 \end{cases}$$

3.2 The gas relative velocity to the liquid

The velocity of gas phase is measured at two different points; 0.46$m$ and 2.85$m$ from the injection point in the Type-A apparatus with the high speed camera. The velocity is recorded with the distance between two constant points and time bubbles take to pass. Fig. 5 represents the relationship between $v_g$ and $U_g$. The gas volume flux is given by $U_g = 4Q_g/\pi D^2$. From these experimental data, eq. (7) was obtained.

$$v_g = 3.4U_g^{0.42}$$

The liquid velocity $v_l$ is indirectly measured as the volume flux of the liquid. $U_l$ depends on $U_g$ as Fig. 6 shows, and the following relationship is obtained.

$$U_l = 1.9U_g^{0.36} - 0.11$$

The definition of $U_l$ derives $v_l$ as eq. (9).
The gas relative velocity to the liquid was obtained as a function of $\alpha_g$ and $U_g$ with eq. (7), eq. (8) and eq. (9), which is defined by eq. (10).

$$v_{g,rel} = v_g - v_l$$

### 3.3 Measurement of the surface-area-equivalent diameter of bubbles

Rising bubbles are captured with the high-speed camera in the Type-A apparatus. The flow rate of the injected gas is set in 1.0, 3.0, 6.0 and 9.0 L/min. The procedure is listed as follows to derive the surface-area-equivalent diameter of bubbles (Sauter mean diameter).

The overestimation of the measured bubble diameter by the lens effect caused by the curved side wall of the riser was not considered at the present. This correction of this lens effect is our future plan. It has the actual diameter about 1.3 times larger at the maximum according to the examination with a plastic sphere of the known diameter.

1. Long and short axis of captured bubbles are measured within $10^{-3} \text{ m}$ accuracy. $V_b$ is determined by eq. (11) with $r$ and $R$. Fig. 7 is the example of the captured image.

$$V_b = \frac{\pi}{12} r R (r + R)$$

2. The mean diameter $d$ is determined by eq. (12).

$$d = \sqrt[3]{\frac{6V_b}{\pi}}$$

3. The bubble is classified by their diameter at $\Delta d = 1.0 \text{ mm}$ interval and $N_i$ is recorded to the $d_i$. Fig. 8 shows the example of the distribution of bubble number. $f_{N_i}$ is defined by eq. (13) with $\bar{d_i}$.

$$f_{N_i} = \frac{\bar{d}_i^2 N_i}{\sum_j \bar{d}_j^2 N_j}$$
4. $d_s$ is determined for the each $U_g$ by the following equation with $f_{Ni}$.

$$d_s = \sum_i (\tilde{d}_i \cdot f_{Ni})$$  \hspace{1cm} (14)

The relationship between $d_s$ and $U_g$ are determined from these procedures as shown in Fig. 9. Therefore, $d_s$ was given by eq. (15) as a function of $U_g$.

$$d_s = 0.031U_g^{0.11}$$  \hspace{1cm} (15)

3.4 Measurement of gas volume fraction

The gas volume fraction is measured by the procedure shown in the following procedures and Fig. 10 with $Q_g$ changed 1.0 to 80.0 L/min.:

1. The quick-shut valve is closed and the gas injection is stopped after the flow on the pipe is steady.

2. $L'$ is recorded after the all bubbles disappeared in the liquid.

3. $\alpha_g$ is obtained by $\alpha_g = L'/L_{adj}$

Fig. 10 shows the relationship between $\alpha_g$ and $U_g$ obtained by the procedure mentioned above. $\alpha_g$ uniformly increases with $U_g$.

$$U_g = 5.1\alpha_g^{1.5}$$  \hspace{1cm} (16)
3.5 Derivation of inter-phase drag coefficient

$C_d$ is determined with eq. (10) and eq. (15). Eq. (17) shows the equation of the motion for a single bubble rising on the liquid medium.

$$\frac{1}{6} \pi d_s^3 \rho_g \frac{dv_{g,rel}}{dt} = -\frac{1}{6} \pi d_s^3 \rho_g \dot{g} + \frac{1}{3} \pi d_s^3 \rho_l \dot{g} = AC_d \frac{1}{2} \rho_l v_{g,rel}^2$$  \hspace{1cm} (17)

The first, second and third terms in the right hand of the equation represent the gravitational force, buoyant force and the drag force from the liquid, respectively. Since a bubble is assumed as in the steady state, the left hand of equation is zero.

$$0 = \frac{1}{6} \pi d_s^3 \left( \rho_l - \rho_g \right) \dot{g} - AC_d \frac{1}{2} \rho_l v_{g,rel}^2$$  \hspace{1cm} (18)

Therefore, eq. (19) is derived from eq. (18).

$$C_d = \frac{4}{3} \frac{d_s \left( \rho_l - \rho_g \right) g}{\rho_l v_{g,rel}^2}$$  \hspace{1cm} (19)

Eq. (20) is obtained from eq. (19), applying eq. (7) ~ eq. (10) to $v_{g,rel}$ and eq. (15) to $d_S$.

$$C_d = \frac{4}{3} \frac{\left( \rho_l - \rho_g \right) g d_S}{\rho_l v_{g,rel}^2} = 13 \frac{0.031 U_{g}^{0.11}}{3.4 U_{g}^{0.42} \left( \frac{1}{1 - \alpha_g} \right)^2}$$  \hspace{1cm} (20)

Therefore, $U_g$ can be expressed with $\alpha_g$ by eq. (16), and eq. (21) is a final form of $C_d$.

$$C_d = 0.48 \frac{\left( 1 - \alpha_g \right)^2 \alpha_g^{0.17}}{\left( 6.8 \alpha_g^{0.65} - 6.8 \alpha_g^{1.10} - 3.4 \alpha_g^{3.54} + 0.11 \right)^2}$$  \hspace{1cm} (21)

Eq. (21) is valid only when $\alpha_g$ is less than 0.25 because the present experiment were conducted under the condition of $\alpha_g < 0.25$ as Fig. 10 shows. Following equations are used for other regions of $\alpha_g$, which are the offset expression based on the form obtained by Ishii et al.\(^{(20)}\)

$$C_d = 0.042 \left( \frac{1 - \alpha_g}{1 - \alpha_{gr}} \right)^3 \hspace{1cm} 0.25 < \alpha_g < 0.75$$  \hspace{1cm} (22)

$$C_d = 0.011 \left( \frac{1 - \alpha_g}{1 - \alpha_{gr}} \right)^2 \hspace{1cm} 0.75 < \alpha_g < 1$$  \hspace{1cm} (23)

These equations of $C_d$ are intended for a system with the upward flow of the liquid.
phase by the pressure difference between top and bottom of the pipe.

4. Evaluation of the proposed equation model

In order to verify the applicability of the obtained $C_d$ to the experimental data, $U_l$ was calculated to various $U_g$ with the proposed equation model and following equations for the inter-phase drag force and wall friction respectively. Then the result was compared with the experiment. The wall friction is defined by eq. (24) per the unit volume.

$$ \left( \text{fric, } k \right) v_k = \frac{4 f_m^k}{D} \left( \frac{1}{2} \rho_k \alpha_k w_k \right) \left| v_i \right| v_k $$

(24)

Here, $f_m$ and $w_k$ are defined by eq. (25), eq. (26) and eq. (27).$^{(22)(23)}$

$$ f_m = \begin{cases} \frac{16}{Re_m} & Re_m \leq 2300 \\ 0.0791mRe_m^{0.25} & 2300 < Re_m \end{cases} $$

(25)

$$ w_k = \begin{cases} 0 & \alpha_g \leq 0.9 \\ \left(10\alpha_g - 9\right) \left(21 - 20\alpha_g\right) & 0.9 < \alpha_g \end{cases} $$

(26)

$$ w_i = 1 $$

(27)

Where,

$$ m = \begin{cases} 4.8 + 0.4 \ln \alpha_g & \alpha_g \leq 0.2 \\ 4.2 & 0.2 < \alpha_g \end{cases} $$

(28)

$$ Re_m = \frac{\left(\alpha_g \rho_g v_g + \alpha_i \rho_i v_i\right) D}{\left(\alpha_g \mu_g + \alpha_i \mu_i\right)} $$

(29)

Fig. 12 and Fig. 13 represent the comparison of $U_l$ to $U_g$ between the experimental and calculation results for the each apparatus, type-A and B. $U_l$ increased with increasing $U_g$. In these figures, $U_l$ is shown for the each submerged ratio. $\gamma$ is ratio between the pipe length and static surface level under the condition of overflowing, and defined by eq. (30). $L_s$ is static surface level as shown in Fig. 2.

$$ \gamma = \frac{L_s}{L} $$

(30)

Calculation results tended to overestimate the experimental results in the system of Type-B, though they were qualitatively similar.

Results with other proposed model equations were examined to the both of Type-A and Type-B, and the error of estimated $U_l$ was evaluated to $U_g$. Fig. 14 shows the plot of the corresponding $U_l$ by calculation to the experiment's under the condition of $\gamma = 0.99$. Baseline represents an ideal line assumed the calculation perfectly had agreed with the experiment. Results by the preset model equations fairly agreed with the experimental result, compared to the other model for the both of system. According to Fig. 14-(a), the calculation with the model by Tomiyama et. al. could also estimate $U_l$ within 10% error to the $U_{l, \exp}$ as far as $U_l < 1.5 \text{ m/sec.}$ for the system of the small diameter. However, Fig. 14-(b)
shows Tomiyama’s model overestimated \( U_l \), compared to the calculation with eq. (21). Therefore, it revealed that model equations for the drag force between two phases for Type-A of \( D = 0.05 \text{m} \) could estimate \( U_l \) more accurately to Type-B of \( D = 0.23 \text{m} \) than other models.

**Fig. 12 Dependence of \( U_l \) on \( U_g \) (Type-A)**  
**Fig. 13 Dependence of \( U_l \) on \( U_g \) (Type-B)**

**Fig. 14 Validation of the accuracy of the simulation model at \( \gamma = 0.99 \)**  
(a) Type-A, (b) Type-B

5. **Summary**

In this study, the gas-lift experiment was conducted in the gas-liquid two-phase system with two different diameter pipes. The dependence of \( d_s \) and \( V_{g, rel} \) on \( U_g \) was measured, and they are summarized as a function of \( \alpha_g \) with \( U_j(\alpha_g) \). Then the equation model was obtained for the drag force between gas and liquid under the condition of overflowing by the surrounding hydrostatic pressure. Only \( \alpha_g \) was used to represent \( C_d \). It revealed that the simulation with the proposed model equation better agrees with the experiment than other equations, especially in the system of the large diameter.
Nomenclature

Alphabets

- $a$: concentration of the inter-phase area between gas and liquid, m²/m³
- $A$: cross sectional area of the bubble, m²
- $C_d$: coefficient of the drag force between two phases, –
- $d$: diameter of bubbles, m
- $\bar{d}_i$: the mean diameter at $i$th class, m
- $d_S$: Sauter mean diameter, m
- $D$: diameter of the riser pipe, m
- $f_{in}$: distribution of bubbles to their surface area, –
- $f_w$: wall friction factor, –
- $(fric,k)$: wall friction on the each phase, kg/(m³ . sec.)
- $g$: gravitational acceleration, m/sec².
- $(inter)$: drag force between two phases, kg/(m³ . sec.)
- $L$: pipe length, m
- $L_{inj}$: injection depth, m
- $L_s$: submerged ratio, –
- $m$: fitting parameter for the wall friction, –
- $N_i$: number of bubbles in the $i$th class, –
- $p$: pressure, Pa
- $Q_k$: volume flow rate, m³/sec.
- $r$: short axis of the bubble, m
- $R$: long axis of the bubble, m
- $Re_m$: Reynolds number of the mixture flow, –
- $t$: time, sec.
- $U_k$: volume flux, m/sec.
- $v_k$: velocity, m/sec.
- $v_{g,rel}$: gas relative velocity to the liquid phase, m/sec.
- $V_b$: volume of the bubble, m³
- $w_k$: distribution factor of the wall friction, –
- $x$: axis directed to the sea surface, m
- $\alpha_k$: volume fraction, m³/m³
- $\alpha_{gs}$: averaged gas volume fraction in the slug flow, m³/m³
- $\gamma$: submerged ratio, m/m
- $\Delta d$: interval to classify the bubble diameter, m
- $\rho_k$: density, kg/m³

Subscripts

- $calc.$: calculation
- $exp.$: experiment
- $g$: gas phase
- $k$: general phase
- $l$: liquid phase

References


(19) Ishii, M., One-Dimensional Drift-Flux Model and Constitutive Equations for Relative Motion Between Phases in Various Two-phase Flow Regimes", (1977)


